Matter at Extreme Density

A Tale of Two Topics

Helmut Satz

Universität Bielefeld, Germany

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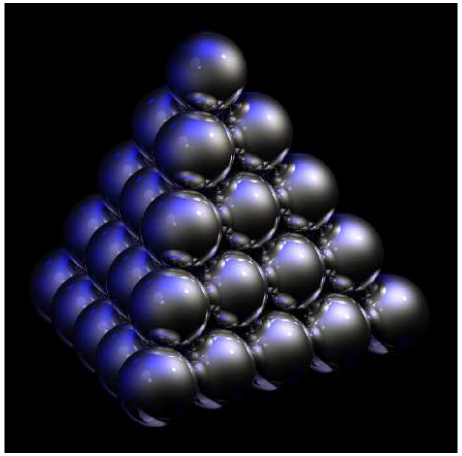


1. Denser than Nuclei:

Big Bang, Neutron Stars,

High Energy Collisions

How dense can matter get?

Sir Walter Raleigh / Thomas Harriot:

optimal stacking of cannon balls on ships?

Johannes Kepler's conjecture (1611):

orderly close packing $\pi/\sqrt{18} \simeq 74$ % filled

random close packing shaken not stirred $\simeq 64$ % filled

onset of "jamming" $\simeq 0.32$ % filled

On Earth, densest matter,

only strongly interacting matter:

nuclear matter, $n_0 \simeq 0.16 \text{ fm}^{-3}$, 0.41 % filled

nucleons can still rattle around, but are largely jammed

energy density of normal nuclear matter $\epsilon_0 \simeq 0.15 \text{ GeV/fm}^3$

energy density of a single nucleon $\epsilon_h \simeq 0.45 \text{ GeV/fm}^3$

Where and how can we get matter of higher density?

- Big Bang age of expanding universe vs. energy density:

$$t = 1/H(t) = \sqrt{3/(8\pi G\epsilon(t))}$$

so that for

$$t \leq t_h = t(\epsilon_h) = 10^{-5} \text{ sec},$$

in first ten microseconds, energy density of universe was greater than ϵ_n – pre-hadronic era of early universe

- **Neutron Stars**

  gravitational collapse of cold star

  $$\text{white dwarf} < \text{neutron star} < \text{black hole}$$

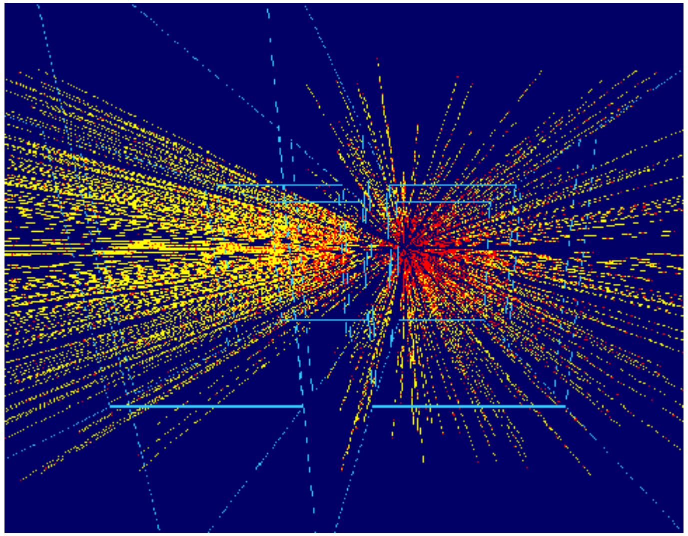
  core density $\sim\ 5\, n_0\ >$ **orderly close packing density**

  core medium too dense to be hadronic?

- **High Energy Collisions**

  collide two gold nuclei
  at $\sqrt{s} = 200$ **GeV**

  per unit rapidity,
  800 hadronic secondaries



energy density in central $A - A$ collision:

$$\epsilon \simeq \frac{p_0}{\pi R_A^2 \tau_0} \left(\frac{dN_A}{dy} \right)_A$$

$\Rightarrow 6 - 8$ GeV/fm^3 for Au-Au at $\sqrt{s} = 200$ GeV

twenty times the energy density inside a nucleon

Proposal ($\sim$ 1980)

study pre-hadronic matter through high energy nuclear collisions

Prelude

- hadronic matter
- hadronic substructure

2. All those Resonances:

Hagedorn's Vision

$\sim$ 1950

ultimate constituents of matter:

protons, neutrons $\longrightarrow$ nucleus;

nucleus + electrons $\longrightarrow$ atom

short range of nuclear force (Yukawa): $\exists\ \pi$ meson

collide nucleons to make pions: Pandora's box

$\pi, \eta, \rho, \omega, K, K^*, \phi, p, n, \Delta, N^*, \Lambda, \Sigma, \Xi, \Omega, ...$

so many "elementary" hadrons? $\Rightarrow$ two new developments...

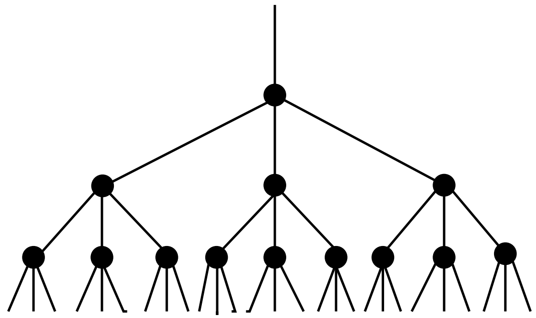
◦ subconstituents of hadrons → quarks, QCD

• limits of hadron thermodynamics

Hagedorn (1965)

self-similar composition/decay:

statistical bootstrap model



fireballs consist of fireballs

which consist of fireballs

which consist of fireballs...

excercise: partition integers into ordered sets of integers

2=1+1, 2 $\rightarrow \rho(2) = 2^1$

3=1+1+1, 1+2, 2+1, 3 $\rightarrow \rho(3) = 2^2$

4=1+1+1+1, 1+1+2, 1+2+1, 2+1+1, 1+3, 3+1, 2+2, 4 $\rightarrow \rho(4) = 2^3$

$$\rho(n) = 2^{n-1} = (1/2)\exp\{n \ln 2\}$$

number $\rho(n)$ of partitions grows exponentially with n

resonances: integers plus component's momentum

self-similar composition/decay: $\rho(M) \sim M^a \exp\{b\,M\}$

what happens to an ideal gas of such resonances?

partition function of an ideal pion gas (Boltzmann factor):

$$\ln Z_0(T) \sim V T m_0^2 K_2(m_0/T) \sim V\,(T m_0)^{3/2} \exp\{-m_0/T\}$$

ideal resonance gas

$$\ln Z(T) \sim V T \int dM\, M^{2-a} \exp\{b\,M\} K_2(M/T)$$
$$\sim V T^{3/2} \int dM\, M^{(3/2)-a} \exp\{M[b - (1/T)]\}$$

singular behavior for $T \geq T_H = 1/b$:

Hagedorn : ultimate temperature of matter

$\Updownarrow$

Cabibbo-Parisi : it's the $M^{-a} \rightarrow$ *hadronic* matter

$a \leq 7/2$: energy density diverges for $T \to T_H$

$a > 7/2$: energy density finite for $T \to T_H$,
phase transition

Hadron thermodynamics defines its own limits without knowing quark infrastructure: for $T > T_H$, new physics.

What is the value of T_H?

statistical bootstrap model: $T_H = 1/b \simeq 200$ **MeV**

b = **range of strong interaction force**

dual resonance model: $T_H \simeq (3/8\pi^2\alpha')^{1/2} \simeq 200$ **MeV**

α' = **Regge trajectory slope** $\simeq 1$ **GeV**$^{-2}$

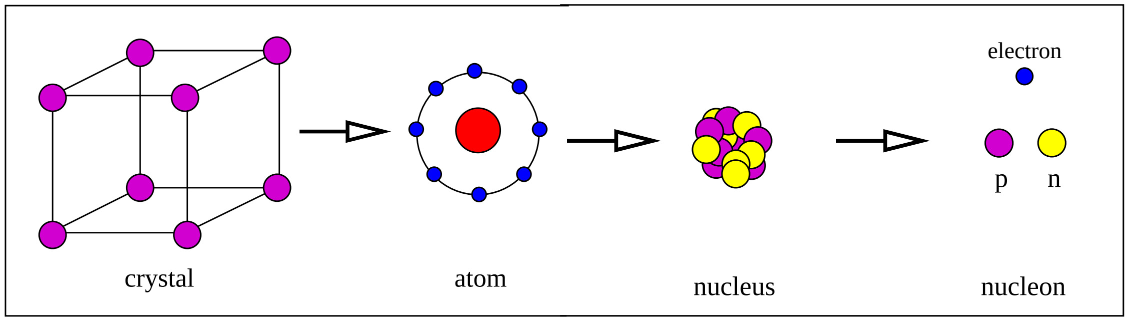
3. The Conjecture of Lucretius: Quarks

Reductionism, fundamental atoms (Democritos, 300 B. C.):

Complexity (the visible many-faceted world)

$\Updownarrow$

Simplicity (invisible simple building blocks)



atoms divisible, endless chain of ever smaller objects having an independent existence: is there an end to reduction?

Titus Lucretius Carus, 50 B. C.


(1)

T. LUCRETII CARI

DE

RERUM NATURA

Liber Primus.

AENEADUM genetrix, hominum divumque voluptas,
Alma Venus, coeli subter labentia signa
Quæ mare navigerum, quæ terras frugiferenteis
Concelebras: per te quoniam genus omne animantum
Concipitur, visitque exortum lumina solis:
Te, Dea, te fugiunt venti, te nubila coeli,
Adventumque tuum: tibi suaveis dædala tellus
Summittit flores, tibi rident æquora ponti,
Placatumque nitet diffuso lumine coelum.
Nam simul ac species patefacta 'st verna diei,
Et reserata viget genitabilis aura Favoni:
Aeriæ primum volucres te, diva, tuumque
Significant initum percussæ corda tua vi.
Inde feræ pecudes persultant pabula læta,
Et rapidos tranant amneis: ita capta lepore,
Illecebrisque tuis omnis natura animantum

B Te

So there must be an ultimate limit to bodies,
beyond perception by our senses.
This limit is without parts,
is the smallest possible thing.
It can never exist by itself,
but only as primordial part of a larger body,
from which no force can tear it loose.

2000 years later: Quantum Chromodynamics

hadrons consist of quarks,
interacting through exchange of gluon vector fields

quarks are confined, can only exist as constituents of hadrons

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} - \sum_f \bar{\psi}^\alpha_f \left[i\delta_{\alpha\beta}\gamma^\mu\partial_\mu - \frac{g}{2}(\lambda_a)_{\alpha\beta}\gamma^\mu A^a_\mu \right] \psi^\beta_f$$

$$F^a_{\mu\nu} = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^a_{bc} A^b_\mu A^c_\nu)$$

quarks carry non-Abelian color charge of strong interaction,
bind to form color-neutral hadrons: mesons as $q\bar{q}$ pairs,
baryons as quark triplets

quarks in fundamental representation of color $SU(3)$ give
all observed hadron state quantum numbers ☺

short distance limit:

asymptotic freedom ⇒ strong interactions become weak
⇒ perturbative QCD ☺

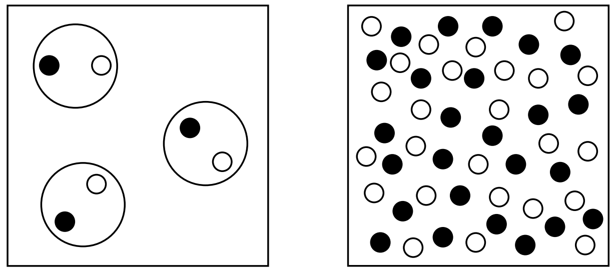
high energy limit? high parton densities ⇒ saturation?

What happens when strong interactions are strong?

- color confinement (Clay Institute Millenium Problem)
- hadron masses as bound states of (almost) massless quarks
- hadron-hadron scattering at low momentum transfer
- strongly interacting matter

What happens when hadrons are compressed to overlap?

confinement is a long range phenomenon!



at high densities, temperatures:

color screening overcomes confinement,

$\exists$ quark deconfinement,

color insulator to conductor transition

confined quarks acquire gluon dressing: constituent quarks
$\sim$ spontaneous chiral symmetry breaking

at high temperatures, dressing melts:
$\exists$ chiral symmetry restoration

strongly interacting matter at high temperatures, densities:

- quark deconfinement
- chiral symmetry restoration

- do they coincide?

- properties of the new deconfined, chirally symmetric state of matter, the QGP?

strong coupling regime of QCD is not amenable to analytic calculations ☹

we need a new way to address these problems!

5. Shift of Paradigm:

Computer Simulation of Lattice QCD

to study strongly interacting matter,

- ○ analytic approach:
  calculate partition function,
  derivatives → thermodynamic observables

- • computer simulation:
  represent phase space
  on computer, "measure"
  thermodynamic observables

M. Creutz S. Kahana

C. Rebbi K. Wilson

Lattice 1986

Brookhaven National Laboratory

- rewrite partition function $Z(\beta, V) = \mathrm{Tr}\,\exp\{-\beta \mathcal{H}\}$ as Euclidean path intgral

  $$Z_E(\beta, V) = \int \mathcal{D}A\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\ \exp\ \{- \int_V d^3x \int_0^\beta d\tau\ \mathcal{L}_E(A, \psi, \bar{\psi})\ \}$$

  $\tau \sim$ imaginary time (Feynman argument)

- discretize x, τ on a $N_\sigma^3 \times N_\tau$ lattice
- change variables from gluon fields $A(x, \tau)$

  to SU(3) matrices $U_{i,j}$ on lattice links between i, j
- integrate fermion fields $\psi, \bar{\psi}$ (Grassmann variables)
- obtain lattice partition function

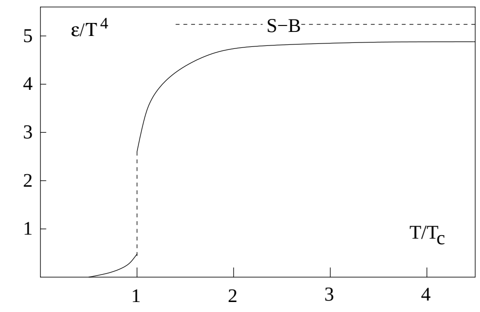
  $$Z(N_\sigma, N_\tau; g^2) = \int \prod_{\text{links}} dU \exp\{-[S_G(U) + S_Q(U)]\}$$

  with S_G, S_Q for gluon and quark actions
- S_G: globally $\longrightarrow$ locally invariant spin system (Wegner)

  $\longrightarrow$ gauge invariant SU(3) system, plaquettes

First results: pure gauge theory ($S_Q = 0$), here show SU(3)

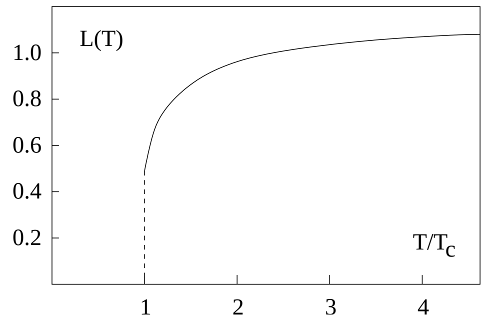
$\exists$ temperature T_c where energy density suddenly increases by *latent heat of deconfinement*



$\exists$ *deconfinement order parameter*

$L(T) \sim \exp\{-F_{Q\bar{Q}}/T\}$

identifying transition, universality class



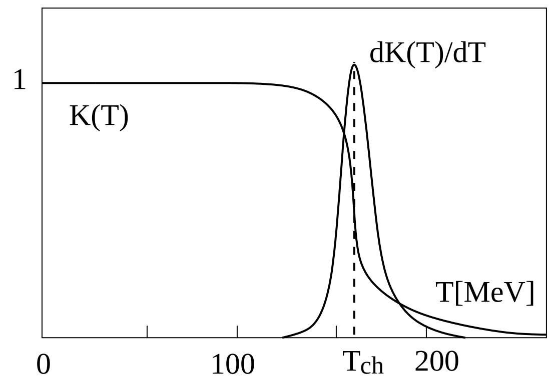
Subsequent lattice studies $\Rightarrow$ full QCD

with light dynamical quarks ($m_q \neq 0$):

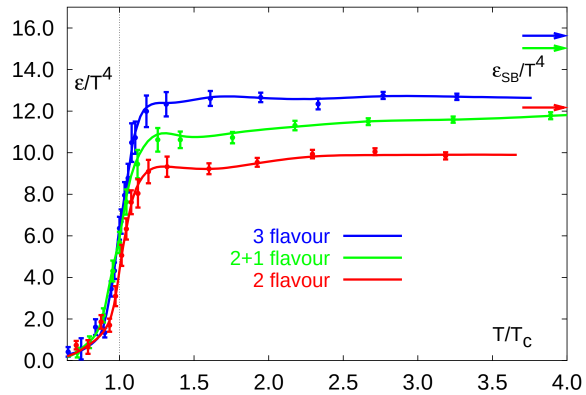
string breaking, broken chiral symmetry

nevertheless, sharp drop in chiral condensate, defines

$$T_{\rm ch} \simeq 160 \text{ MeV}$$



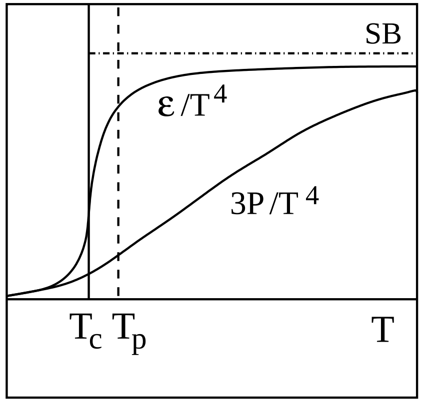
sharp increase in energy density at the same temperature, latent heat of deconfinement



<u>Conclusion</u> (NB: $\mu_B = 0$)

- $\exists$ unique transition from hadronic matter to QGP
- deconfinement and chiral symmetry restoration coincide

Interactions in QGP?

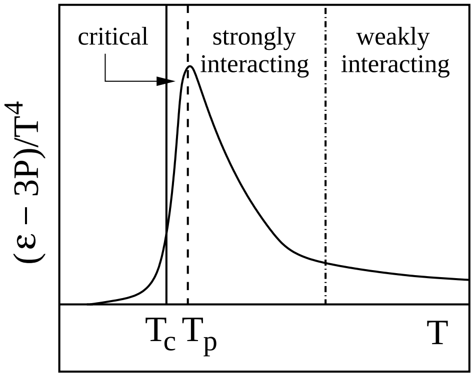


interaction measure: trace anomaly

$$\Delta(T) = \frac{\epsilon - 3P}{T^4}$$

on hadronic side: resonance gas

on QGP side?



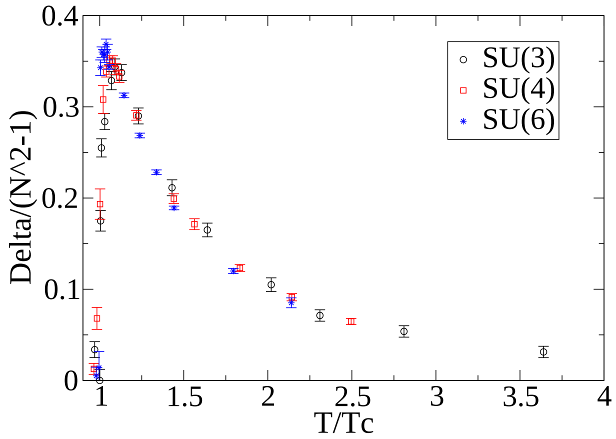
for $T \leq 2 - 3\ T_c$,
strong non-perturbative interactions

- $T_c \leq T \leq 1.2\ T_c$, critical region
  at T_c, infinite correlation length,
  correlation "clusters" decreasing with increasing T
- $1.2\ T_c \leq T \leq 2 - 3\ T_c$, massive colored constituents,
  $\sim$ temperature independent mass

- $T \geq 2\text{--}3\, T_c$, perturbatively dressed colored constituents, $m \sim T$, weak coupling regime, resummed pQCD (HTL)

understanding of non-perturbative QGP interactions?

beyond narrow critical region,
interactions "count"
degrees of freedom



So ∃ open ends, but nevertheless...

QCD predicts sharp transition between two distinct states:

- hadronic matter
- quark-gluon plasma

experiment?

5. The Little Bang: Making Matter in Collision

nuclei, as heavy as bulls,
in collision generate new states of matter

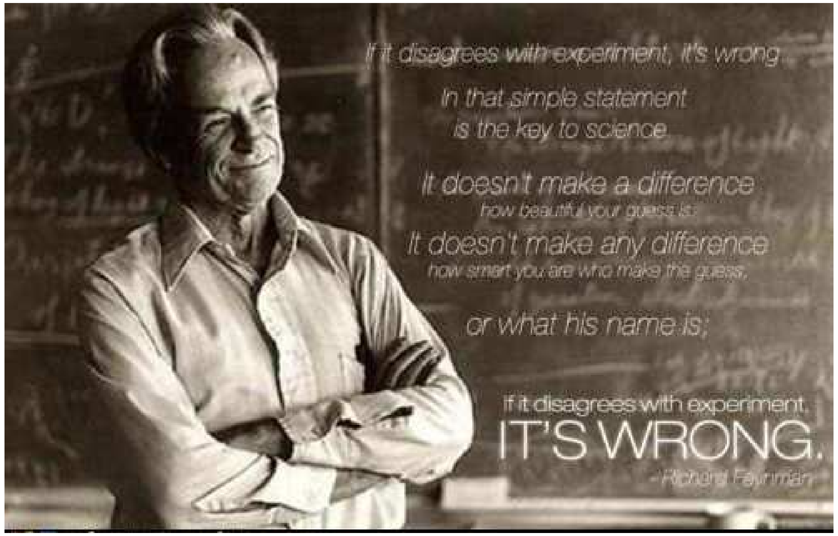
big bang: long ago

neutron stars: far away

is there a way to make strongly interacting matter on Earth?

collide two heavy nuclei:

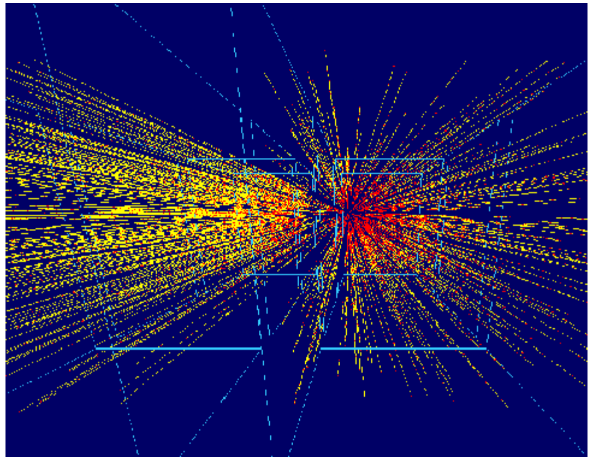
is it possible by colliding two nuclei with $A = 200$
to make something we can call matter?



Feynman's objection:

If I throw my watch
against the wall,
I get a broken watch,
not
a new state of matter.

- that was one principal problem
- the other: can nuclear collision
  experiments be analysed?
  can one measure and identify
  all those tracks?
  can one observe
  interpretable features?



1986: Start of BNL & CERN "heavy ion" programs

charge: produce & study QGP predicted by statistical QCD

light ion beams → heavy ion beams → heavy ion colliders

AGS (5 GeV) / SPS (20 GeV) → RHIC (200 GeV) → LHC (3 TeV)

tracks could be analysed and identified.

∃ one striking conclusion:

Feynman was wrong: his watch was not it...

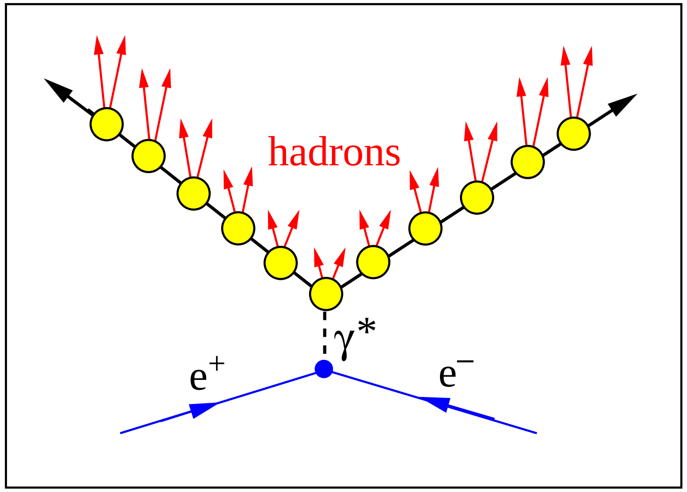
if I throw a watch against the wall, I get randomly distributed fragments.

if I collide two hadrons, I get new hadrons with a universal abundance pattern, corresponding to one specific formation temperature – there is thermal behavior.

6. The Abundance of the Species:

Universal Hadrosynthesis

hadronic & nuclear collisions,
e^+e^- annihilation:
$\Rightarrow$ passing color charges
disturb vacuum,
deposit bubbles of energy,
bubbles hadronize



boost-invariance: bubbles are identical, rapidity-independent

to study abundances of hadron species:

one effective global bubble hadronizes thermally what is "thermal"?

- equal *a priori* probabilities for all states in accord with a given local average energy $\Rightarrow$ temperature T;

- grand canonical partition function of ideal resonance gas

$$\ln Z(T) = V \sum_i \frac{d_i}{(2\pi)^3} \phi(m_i, T)$$

- Boltzmann factor

$$\phi(m_i, T) = \int d^3p \exp\{\sqrt{p^2 + m_i^2}/T\} \sim \exp -(m_i/T);$$

- relative abundances $$\frac{N_i}{N_j} = \frac{d_i \phi(m_i, T)}{d_j \phi(m_j, T)}$$

- let resonances decay according to PDG rates to get measured abundances

observe up to 30 species

$\boldsymbol{\pi, \eta, \rho, \omega, K, K^*, \phi, p, n, \Delta, N^*, \Lambda, \Sigma, \Xi, \Omega, \ldots}$

in various collision configurations

$\boldsymbol{e^+e^-,\ p-p, p-\bar{p},\ \pi-p,\ K-p,\ A-A}$

over a wide range of (high) energies from 10 to 1000 GeV

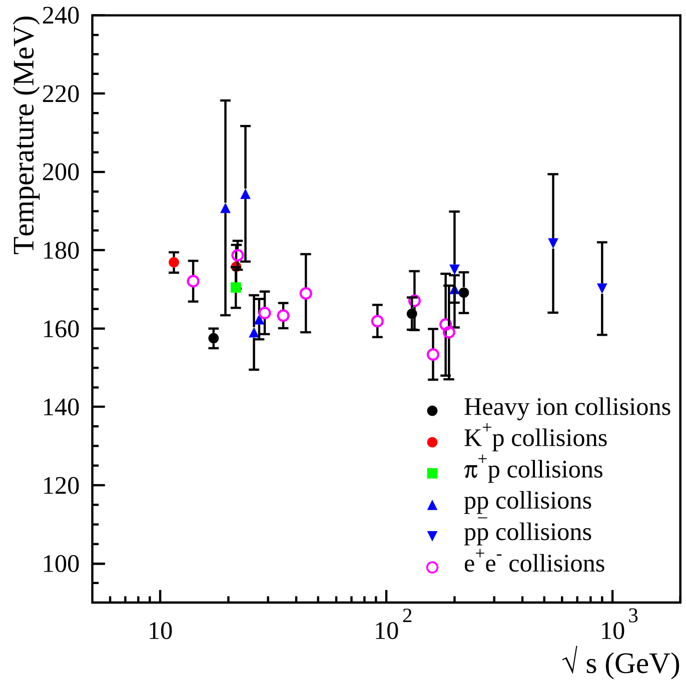
compare to resonance gas rates as function of temperature:

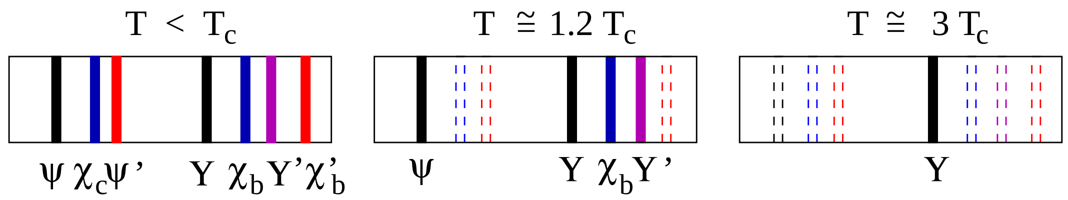
$\exists$ universal hadronization point

$$\boldsymbol{T_H \simeq 170 \pm 10 \text{ MeV}}$$

in accord with critical temperature from lattice QCD

so far the only case of quantitative agreement between data and statistical QCD...?





7. The Missing Quarkonia:

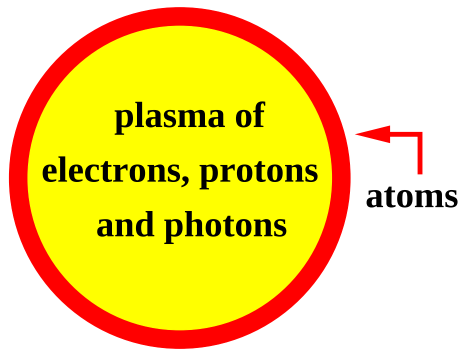
QGP Thermometer?

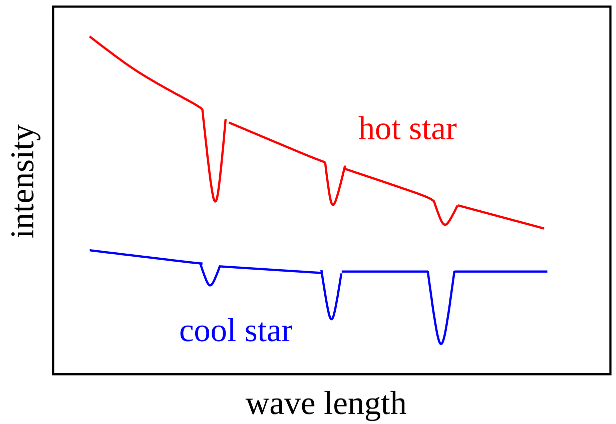
hadron abundances ⇒ hadronization stage of QGP

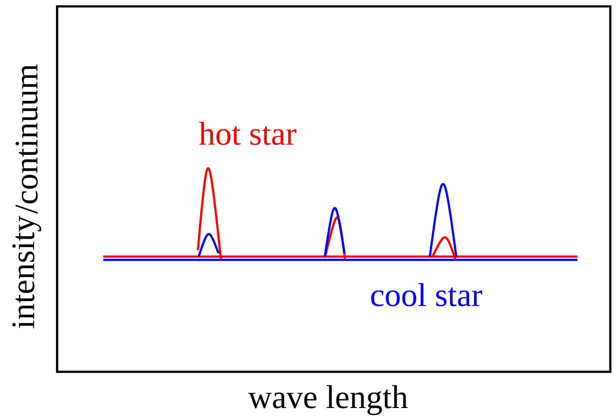
∃ probe of earlier, hot QGP, "smoking gun"?

better look at a shining star than for a smoking gun...

temperatures of stellar interiors?
photons from plasma core are emitted,
absorbed by atoms in crust, lead to
absorption lines in stellar spectra







- absorption lines indicate presence of atomic species
- absorption strength gives temperature of stellar interior

Conjecture: Quarkonia are the Spectral Lines of the QGP

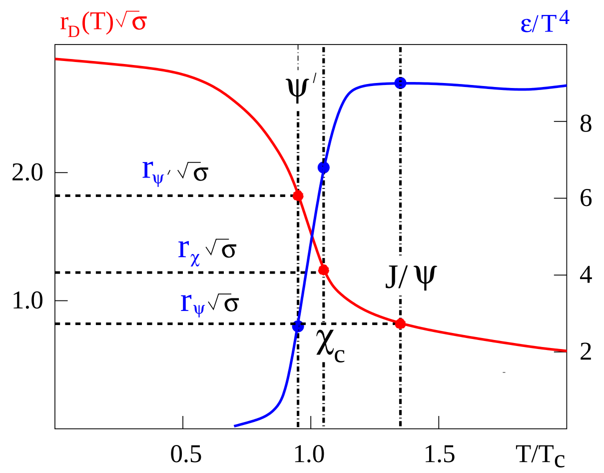
$\exists$ no crust of QGP, but early hard production of quarkonia

they're there when QGP appears,
and the QGP effect on different quarkonium states
tells how hot it is.

- quarkonia are unusual hadrons
  - very small:

    $r_{J/\psi} \simeq 0.25$ fm, $r_{\Upsilon} \simeq 0.14$ fm $\ll \Lambda_{QCD}^{-1} \simeq 1$ fm
  - very tightly bound:

    $2M_D - M_{J/\psi} \simeq 0.64$ GeV $2M_B - M_{\Upsilon} \simeq 1.10$ GeV
  - survive deconfinement, exist in QGP up to some T
- quarkonia melt in hot QGP through color screening
  - when screening radius $r_D(T)$ becomes smaller than binding radius r_i, quarkonium state i melts



- quarkonium melting points
  - their measurement determines temperature, energy density of QGP

Challenge to theory: quarkonium melting temperatures

- potential theory: large $m_Q \longrightarrow$ NR Schrödinger eq'n

$$\left\{2m_Q - \frac{1}{m_Q}\nabla^2 + V(r,T)\right\}\Phi_i(r,T) = M_i\Phi_i(r,T)$$

heavy quark lattice studies $\longrightarrow$ heavy quark binding
free or internal energy to specify potential?

- direct lattice studies: measure correlator

$$G_i(\tau,T) = \int d\omega\ \sigma_i(\omega,T)\ K(\omega,\tau,T)$$

invert integral transform to get spectrum $\sigma_i(\omega,T)$;
$G_i(\tau,T)$ not known for enough values of τ;
maximum entropy method (MEM) $\longrightarrow$ most likely result.

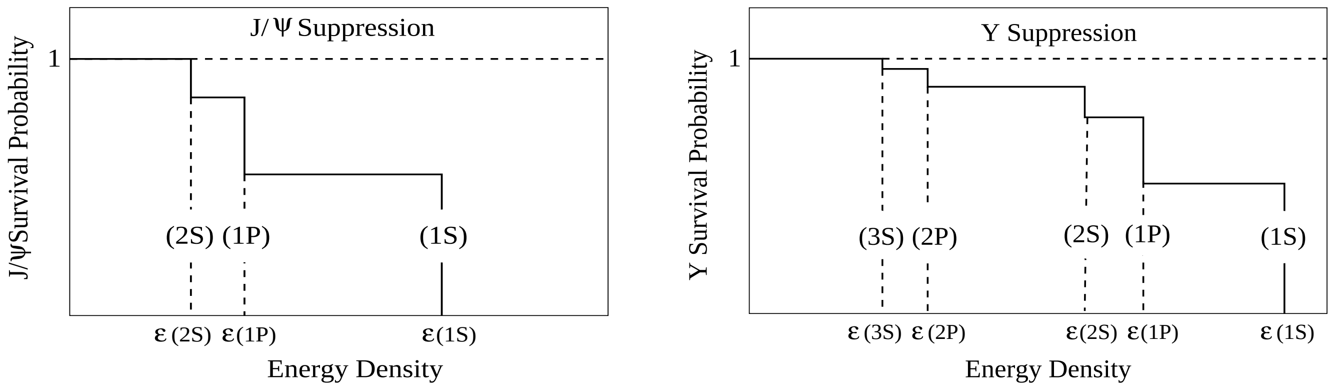
tentative result:

state	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ''
T_d/T_c	1.5 - 2.0	1.1	1.1	$>$ 4.0	1.8	1.6	1.2	1.2

Challenge to experiment:

measure quarkonium dissociation points

- feed-down: quarkonium ground states J/ψ and Υ only about 50 % direct, remainder from excited states decay
- decay outside interaction region, medium affects excited states
- result: sequential suppression

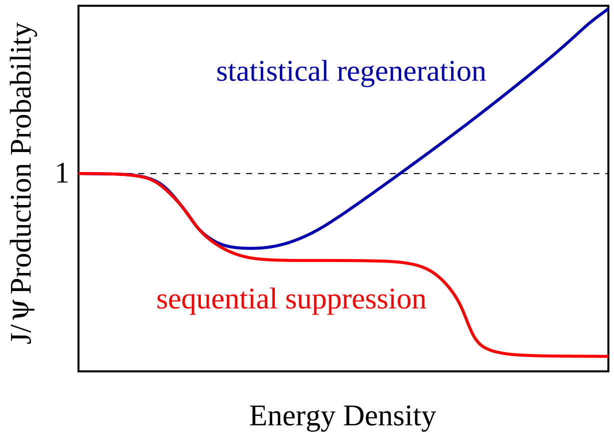


Compare ratios of suppression onsets in nuclear collisions with corresponding ratios calculated in statistical QCD.

⇒ Quantitative experimental check of statistical QCD in deconfinement regime ⇐

possible problem for charmonia: statistical regeneration

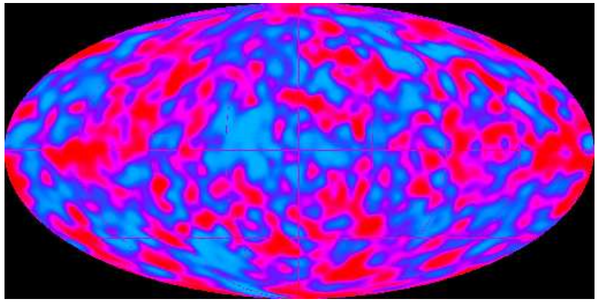
- initial primary charmonia dissolved
- if ∃ abundant c and $\bar{c}$ production, statistical pairing at hadronization can generate new secondary charmonia
- result: enhanced instead of suppressed J/ψ production relative to scaled pp rates



in that case,
sequential Υ production remains as experimental test of statistical QCD

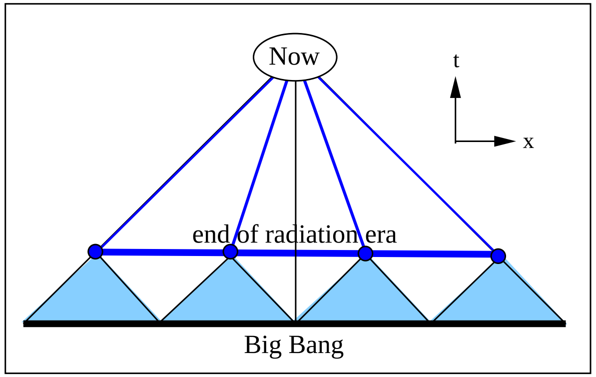
8. Horizons

cosmic background radiation



$T_{\mathrm{CBR}} = 2.752548 \pm 0.00057\ ^{\circ}\mathrm{K}$

cosmological horizon problem:
why is temperature so uniform?



"standard" explanation:

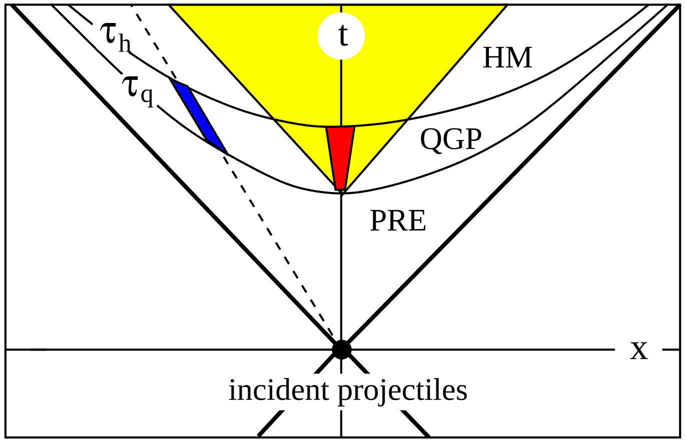
inflation, previous causal connection

NB: horizon problem also arises for nuclear scattering

[Becattini, Castorina & HS]

assume:
$\tau_q = 1$ fm, $\tau_h = 8$ fm

then QGP bubbles at rapidity
$\eta = 0$ and $\eta = 2$
are not causally connected



size of causally connected QGP region (needed for thermal equilibrium) is determined by equilibration time and QGP life time

why universal hadronization behavior?

- pre-equilibrium (CGC, glasma) $\sim$ inflation era
- hadronization is universal local phenomenon

$$\Downarrow$$

quark tunnelling confinement horizon?

The little bang may simulate more of the Big Bang

than we bargained for...