COMPARISON ANALYSIS OF UNCONDITIONAL AND CONDITIONAL BAYESIAN PROBLEMS OF TESTING MANY HYPOTHESES

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Summary

In Bayesian statement of hypotheses testing, instead of unconditional problem of minimization of average risk caused by the errors of the first and the second types, there is offered to solve the conditional optimization problem when restrictions are imposed on the errors of one type and, under such conditions, the errors of the second type are minimized. Depending on the type of restrictions, there are considered different conditional optimization problems. Properties of hypotheses acceptance regions for the stated problems are investigated and, finally, comparison of the properties of unconditional and conditional methods is realized. The results of the computed example confirm the validities of the theoretical judgments.

Key words: Bayesian problem, hypotheses testing, significance level, conditional problem, unconditional problem.

1. Introduction

In many branches of mathematical statistics, some of basic methods are the methods based on the Bayes theorem, which are called the Bayesian methods. The Bayesian methods are also widely used in the theory and practice of making the statistical decisions, in particular, in hypotheses testing. To the development of this method, a lot of scientific works are devoted [see, for example, 1-13]. Among different methods of testing of statistical hypotheses, the Bayesian approach is of primary importance as, under certain conditions (actually always fulfilled at solving practical problems), the class of Bayesian decisions is complete concerning Δ, where Δ is a set of all decision rules δ with bounded risk functions [14, 15]. As is known, at testing of statistical hypotheses errors of the first and the second types could be made [16-18]. The error of the first type corresponds to the case when a true hypothesis is rejected and the error of the second type corresponds to the case when an incorrect hypothesis is accepted. By choosing the loss function it is practically impossible to achieve that the decision made would be free of errors even of one type to a certain extent, for example, to obtain that the probability of correct hypothesis testing was not less than the given level, and, under such conditions, the probability of incorrect hypothesis testing was as minimum as possible. In classical Bayesian approach, the risk of total errors caused by the errors of the first and second types is minimized, and the exact ratio among them is unknown, i.e. we do not know which share of total risk is caused by the errors of one type and which – by another. For elimination of this drawback, instead of unconditional problem of minimization of average risk caused by the errors of the first and the second types, there is offered to solve the conditional optimization problem when restrictions are imposed on the errors of one type and, under such conditions, the errors of the second type are minimized. Depending on the type of restrictions, there are considered different conditional optimization problems. Properties of hypotheses acceptance regions for the stated problems are investigated and, finally, comparison of the properties of unconditional and conditional methods is realized.

2. Statement of the Problem

2.1. General Statement of the Bayesian Problem of Hypotheses Testing

Let us consider n-dimensional random observation vector $x^T=(x_1,...,x_n)$ with probability distribution density $p(x,\theta)=p(x_1,...,x_n;\theta_1,...,\theta_m)$, given on σ-algebra of Borellian set of space R^n $(x\in R^n)$, which is called the sample space. By $\theta^T=(\theta_1,...,\theta_m)$ is designated the vector of parameters of distribution. In general, $n\neq m$. Let in m-dimensional parametrical space Θ^m be given S possible values of considered parameters $\theta^{i^T}=(\theta_1^i,...,\theta_m^i)$, $i=1,...,S$, i.e. $\theta^i\in\Theta^m$; $\forall i: i=1,...,S$. On the basis of $x^T=(x_1,...,x_n)$ it is necessary to make the decision namely by which distribution $p(x,\theta^i)$, $i=1,...,S$, the sample x is generated. Let us introduce designations: $H_i:\theta=\theta^i$, is the hypothesis that the sample $x^T=(x_1,...,x_n)$ is born by distribution $p(x,\theta^i)=p(x_1,...,x_n;\theta_1^i,...,\theta_m^i)\equiv p(x\,|\,H_i)$, $i=1,...,S$; $p(H_i)$ is the a priori probability of hypothesis H_i; $D=\{d\}$ - a set of solutions, where $d=\{d_1,...,d_S\}$, it being so that

$$d_i=\begin{cases}1, & \text{if hypothesis } H_i \text{ is accepted},\\ 0, & \text{otherwise};\end{cases}$$

$\delta(x)=\{\delta_1(x),\delta_2(x),...,\delta_S(x)\}$ is the decision function that associates each observation vector x with a certain decision: $x \xrightarrow{\delta(x)} d \in D$.

Γ_j is the acceptance region of hypothesis H_j, i.e. $\Gamma_j=\{x:\delta_j(x)=1\}$. It is obvious that $\delta(x)$ is completely determined by regions Γ_j, i.e. $\delta(x)=\{\Gamma_1,\Gamma_2,...,\Gamma_S\}$. Let us introduce loss function $L(H_i,\delta(x))$, which determines the value of loss in the case when the sample has the probability distribution corresponding to hypothesis H_i, but, because of random errors, decision $\delta(x)$ is made.

When the decision is made that hypothesis H_i is true, in reality true could be one of the following hypotheses $H_1,...,H_{i-1},H_{i+1},...,H_S$, i.e. accepting one of hypotheses we risk to reject one of $(S-1)$ really true hypotheses. This risk is called the risk corresponding to hypotheses H_i and is equal to [3, 19]:

$$\rho(H_i,\delta)=\int_{R^n} L(H_i,\delta(x))\,p(x\,|\,H_i)dx\,.$$

For any decision rule $\delta(x)$, a complete risk, i.e. a risk of making the incorrect decision, is characterized by the function:

$$r_\delta=\sum_{i=1}^{S} p(H_i)\rho(H_i,\delta)=\sum_{i=1}^{S} p(H_i)\int_{R^n} L(H_i,\delta(x))\,p(x\,|\,H_i)dx\,, \tag{1}$$

which is called the risk function.

Decision rule $\delta^*(x)$, or, which is the same, $\Gamma_i^*,\ \ i=1,...,S$ - the regions of acceptance of hypotheses $H_i,\ \ i=1,...,S$, are called Bayesian if there takes place:

$$r_{\delta^*}=\min_{\{\delta(x)\}} r_\delta\,. \tag{2}$$

By solving task (2), we obtain [19, 20]:

$$\begin{gathered}\Gamma_j=\{x:\sum_{i=1}^{S}L(H_i,H_j)p(H_i)p(x\,|\,H_i)<\sum_{i=1}^{S}L(H_i,H_k)p(H_i)p(x\,|\,H_i);\\ \forall k:k\in(1,...,j-1,j+1,...,S)\},\quad j=1,...,S.\end{gathered} \tag{3}$$

2.2. Conditional Bayesian Tasks of Hypotheses Testing

Decision rule (3) minimizes risk function (2), which contains the errors of both kinds. The shares of these errors are unknown. As was mentioned above, at solving a lot of practical problems, it is necessary to have a guarantee that the error of one kind does not surpass a certain value, and, in such a situation, to minimize the error of other kind. For obtaining such decision rules, we introduce the statements of conditional Bayesian problems and develop the methods of their solution [21, 22].

The examples of practical problems when statements given below are necessary are: 1) air defense – the cost of incorrectly detected target and the missed one is different, and defence interests demand guaranteed detection of hostile flying vehicles; 2) identification of river water emergency pollution sources; 3) medicine production – the cost of overdosing and underdosing is not identical and the safety of patients requires guaranteed protection of prepared medicines against overdosing; 4) market investigation with the purpose of making recommendations about investments - guaranteed protection from the loss of invested credits; 5) revealing the fact of ship bending on the basis of the measurement results of special sensors; 6) the problem of sustainable development of production and so on.

2.2.1. Restriction on the averaged probability of acceptance of true hypothesis (Task 1)

As was mentioned above, the general function of losses consists of two components: the losses caused by incorrectly accepted and by incorrectly rejected hypotheses.

Let us designate by $\rho_f(H_i,\delta)$ and $\rho_p(H_i,\delta)$ the mathematical expectations of losses caused by incorrectly accepted and incorrectly rejected hypotheses, respectively, brought by decision rule $\delta(x)$ provided that hypotheses H_i is true:

$$\begin{aligned}\rho_f(H_i,\delta)&=E_x\left[\sum_{j=1,\,j\neq i}^{S}L(H_i,\delta_j(x)=1)\right],\\ \rho_p(H_i,\delta)&=E_x\left[\sum_{j=1,\,j\neq i}^{S}L(H_i,\delta_j(x)=0)\right].\end{aligned} \tag{4}$$

As the loss functions for rejection and incorrect acceptance of each hypothesis, we take the probabilities of these events. Then expression (4) takes the form:

$$\rho_f(H_i,\delta)=\sum_{j=1,j\neq i}^{S}\int_{\Gamma_j}p(x\,|\,H_i)dx,$$

$$\rho_p(H_i,\delta)=\int_{\overline{\Gamma}_i}p(x\,|\,H_i)dx=1-\int_{\Gamma_i}p(x\,|\,H_i)dx,$$

$$i=1,...,S.$$

The averaged value of probabilities of incorrectly rejected hypotheses given by decision rule $\delta(x)$ is determined as follows:

$$r_\delta=\sum_{i=1}^{S}p(H_i)\rho_f(H_i,\delta)=\sum_{i=1}^{S}p(H_i)\sum_{j=1,j\neq i}^{S}\int_{\Gamma_j}p(x\,|\,H_i)dx. \quad (5)$$

Trying to minimize r_δ by choosing $\delta(x)$, we shall demand from it that the averaged value of incorrectly accepted hypotheses was not higher than the set level α, i.e.

$$\sum_{i=1}^{S}p(H_i)\rho_p(H_i,\delta)=1-\sum_{i=1}^{S}p(H_i)\int_{\Gamma_i}p(x\,|\,H_i)dx\leq\alpha. \quad (6)$$

Let Δ be a set of those decision rules $\delta(x)$ which satisfy condition (2.6). Decision rule $\delta^*(x)$ is called optimum if

$$r_{\delta^*}=\min_{\delta\in\Delta}r_\delta, \quad (7)$$

Let us rewrite restrictions (6) as follows:

$$\sum_{i=1}^{S}p(H_i)\int_{\Gamma_i}p(x\,|\,H_i)dx\geq 1-\alpha. \quad (8)$$

For solving conditional optimization problem (7), (8) we shall use the method of indeterminate Lagrange multipliers.

The Lagrange function looks like:

$$\Lambda(\delta,\lambda)=\sum_{j=1}^{S}\sum_{i=1,i\neq j}^{S}p(H_i)\int_{\Gamma_j}p(x\,|\,H_i)dx-\lambda\left[\sum_{j=1}^{S}p(H_j)\int_{\Gamma_j}p(x\,|\,H_j)dx-(1-\alpha)\right]=$$

$$=\sum_{j=1}^{S}\int_{\Gamma_j}\left[\sum_{i=1,i\neq j}^{S}p(H_i)p(x\,|\,H_i)-\lambda p(H_j)p(x\,|\,H_j)\right]dx+\lambda(1-\alpha)\Rightarrow\min_{\{\delta(x)\}}, \quad (9)$$

where λ is the Lagrange multiplier.

The Lagrange multipliers have an important economic interpretation as shadow prices of the constraints and their optimal values are very useful in sensitivity analysis [23].

As in (9) the last term is a constant, it is neglected at minimization.

The minimum in (9) is achieved by minimizing every term in it provided that in (8) the equality takes place. The minimum of integrated function by the region of integration is obtained by inclusion of those points of space of integration at which the function is negatively determined into this region, i.e.

$$\Gamma_j=\left\{x:\sum_{i=1,i\neq j}^{S}p(H_i)p(x\,|\,H_i)<\lambda p(H_j)p(x\,|\,H_j)\right\},\quad j=1,...,S, \quad (10)$$

where λ, the same scalar value for all regions, is determined so that in (8) the equality takes place.

2.2.2. Restrictions on conditional probabilities of acceptance of each true hypothesis (Task 2)

Let us determine decision rule $\delta(x)$ so that the probability of acceptance of any of tested hypotheses, if they are true, was not lower than the set level, i.e. (7) took place under the condition:

$$\int_{\Gamma_j}p(x\,|\,H_j)dx\geq 1-\alpha,\quad j=1,...,S. \quad (11)$$

The latter is the restriction on the probability of no rejection of hypotheses H_j if it is true.

Thus, in this task, it is required to minimize risk function (5) under condition (11).

The solution of task (5), (11), by using Lagrange method, has the following form:

$$\Gamma_j=\left\{x:\sum_{i=1,i\neq j}^{S}p(H_i)p(x\,|\,H_i)<\lambda_j\cdot p(x\,|\,H_j)\right\},\quad j=1,...,S, \quad (12)$$

where $\lambda_j>0,\ j=1,...,S,$ are determined so that in (11) the equality took place.

2.2.3. Restrictions on the posterior probabilities of acceptance of each true hypothesis (Task 3)

It is required to minimize average risk (5) at restrictions:

$$p(H_j)\int_{\Gamma_j} p(x\,|\,H_j)dx \geq 1-\alpha, \quad j=1,...,S. \tag{13}$$

In this case, the optimum region of acceptance of a hypothesis is:

$$\Gamma_j = \left\{x: \sum_{i=1,i\neq j}^{S} p(H_i)p(x\,|\,H_i) < \lambda_j \cdot p(H_j)p(x\,|\,H_j)\right\}, \quad j=1,...,S, \tag{14}$$

where $\lambda_j > 0, j=1,...,S,$ are determined so that in (13) the equality took place.

This solution formally will coincide with the solution of Task 2 if we introduce designation $\lambda_j^{/} = \lambda_j \cdot p(H_j)$.

From restrictions (13), it is obvious that this problem is meaningful only if $(1-\alpha)$ does not surpass a priori probabilities $p(H_j)$, $j=1,...,S$, or, otherwise, $\alpha \geq 1 - p(H_j)\int_{\Gamma_j} p(x\,|\,H_j)dx$. Therefore, for practical aims, this task is of little interest. Though the significance of this problem could increase considerably when a priori information, for any reason, is of special importance.

2.2.4. Restriction on the averaged probability of rejection of true hypotheses (Task 4)

In the previous tasks, the optimality of decision rules was defined so that the errors caused by incorrect acceptance of hypotheses were minimized at restrictions on the errors caused by incorrect rejection of hypotheses. Now we shall act on the contrary, i.e. we shall restrict the probabilities of errors caused by incorrect rejection of hypotheses and minimize the probabilities of errors caused by incorrect acceptance of hypotheses. Thus, we shall find such decision rule $\delta(x)$ for which there takes place:

$$r_\delta^{/} = \sum_{i=1}^{S} p(H_i)\cdot \rho_p(H_i,\delta) = 1 - \sum_{i=1}^{S} p(H_i)\int_{\Gamma_i} p(x\,|\,H_i)dx \Rightarrow \min_{\{\Gamma_i\}}, \tag{15}$$

at restrictions:

$$\sum_{i=1}^{S} p(H_i)\rho_f(H_i,\delta) = \sum_{i=1}^{S} p(H_i)\cdot \sum_{j=1,j\neq i}^{S}\int_{\Gamma_j} p(x\,|\,H_i)dx \leq \alpha\ . \tag{16}$$

It is obvious that the minimum in (16) is achieved at maximization of the expression:

$$G_\delta = \sum_{i=1}^{S} p(H_i)\int_{\Gamma_i} p(x\,|\,H_i)dx \Rightarrow \max_{\{\Gamma_i\}}\ . \tag{17}$$

Value G_δ is the averaged probability of acceptance of true hypotheses. We shall call it *the average power of criterion*.

Thus, the problem consists in solving task (17) under restriction (16).

Application of the Lagrange method gives:

$$\Gamma_j = \left\{x: p(H_j)p(x\,|\,H_j) > \lambda \sum_{i=1,i\neq j}^{S} p(H_i)p(x\,|\,H_i)\right\}, \quad j=1,...,S. \tag{18}$$

Coefficient $\lambda > 0$ is the same for all regions of acceptance of hypotheses, and it is determined so that in (16) the equality takes place. It is obvious that this task is inverse to Task 1 in the sense that in them opposite kinds of errors are minimized and the restrictions are also imposed on opposite types of errors. At $\lambda_{(1)} = 1/\lambda_{(4)}$, regions of acceptance of hypotheses formally coincide in both tasks. Here the indexes specify belonging to the appropriate task. Generally, $\lambda_{(1)}$ and $1/\lambda_{(4)}$, are not equal. By comparing restrictions (6) and (16), we conclude that the coincidence of regions of acceptance of hypotheses, i.e. equality $\lambda_{(1)} = 1/\lambda_{(4)}$ is possible if and only if the following takes place:

$$\overline{\Gamma}_i = \bigcup_{j=1,j\neq i}^{S} \Gamma_j\ .$$

This point will be discussed more fully in section 3.

2.2.5. Restrictions on the probabilities of rejection of each true hypothesis (Task 5)

In this case, the problem is formulated as follows. To find the decision rule for which in (17) the maximum is achieved under restrictions:

$$\int_{\Gamma_j} p(x\,|\,H_i)dx \leq \alpha, \quad i,j=1,...,S; \quad i\neq j. \tag{19}$$

In this case, application of the Lagrange method gives:

$$\Gamma_j = \left\{ x : p(H_j) \cdot p(x \mid H_j) > \sum_{i=1, i \neq j}^{S} \lambda_{ij} p(x \mid H_i) \right\}, \quad j = 1, ..., S, \qquad (20)$$

where $(S-1)$-dimensional vectors of parameters $\lambda_j = (\lambda_{1,j}, ..., \lambda_{j-1,j}, \lambda_{j+1,j}, ..., \lambda_{S,j})$, $j = 1, ..., S$, with positive components, are determined so that in (19) the equality took place.

2.2.6. Restrictions on the posteriori probabilities of rejection of each true hypothesis (Task 6)

The problem consists in maximization of averaged power of criterion (17) under the condition:

$$p(H_i) \int_{\Gamma_j} p(x \mid H_i) dx \leq \alpha, \quad i, j = 1, ..., S; \quad i \neq j. \qquad (21)$$

Lagrange solution of this task is:

$$\Gamma_j = \left\{ x : p(H_j) p(x \mid H_j) > \sum_{i=1, i \neq j}^{S} \lambda_{ij} p(H_i) p(x \mid H_i) \right\}, \quad j = 1, ..., S, \qquad (22)$$

where $\lambda_{ij} > 0$ are determined so that in (21) the equality took place.

At introduction of designations $\lambda_{ij}^{/} = \lambda_{ij} \cdot p(H_i)$, this solution formally coincides with the solution of Task 5 (22), i.e. the values λ_{ij} in (22) in principle can be chosen so that the regions of acceptance of hypotheses of Tasks 5 and 6 coincide. It is obvious that, in general, these regions differ from each other.

2.2.7. Restrictions on averaged probabilities of rejected true hypotheses (Task 7)

Let us determine decision rule $\delta(x)$ so that condition (17) was satisfied under restrictions:

$$\sum_{i=1, i \neq j}^{S} p(H_i) \int_{\Gamma_j} p(x \mid H_i) dx \leq \alpha, \quad j = 1, ..., S. \qquad (23)$$

By solving the Lagrange problem we get:

$$\Gamma_j = \left\{ x : p(H_j) p(x \mid H_j) > \lambda_j \sum_{i=1, i \neq j}^{S} p(H_i) p(x \mid H_i) \right\}, \quad j = 1, ..., S, \quad (24)$$

where coefficients $\lambda_j > 0, j = 1, ..., S,$ are determined so that in restrictions (23) the equality took place.

If we introduce the designations $\lambda_j^{/} = p(H_j) / \lambda_j$, solution (24) formally coincides with the solution of Task 2, i.e. by selection of coefficients λ_j both regions (12) and (24) could be identical, but, in general, these regions obviously differ from each other.

Analyzing the forms of regions of acceptance of hypotheses in the considered tasks, it is not difficult to be convinced that they have the form analogous to the regions defined in the generalized Neyman-Pearson criterion [17]. Though, in contradistinction to the latter, in the considered cases, the regions of acceptance of hypotheses are more complex and, as we shall see below, in general case, they are not mutually exclusive regions.

3. Properties of Hypotheses Acceptance Regions

It is known that, in classical statements of the problem of statistical hypotheses testing, their acceptance regions are not intersected, i.e. $\Gamma_i \cap \Gamma_j = \varnothing$, $i \neq j$, and the union of all regions of acceptance of hypotheses coincides with the observation space, i.e. $\bigcup_{i=1}^{S} \Gamma_i = R^n$. In the validity of these conditions, it is easy to be sure by consideration of regions of acceptance of hypotheses in classical Bayesian task of hypotheses testing (3). In particular, it is not difficult to be sure that, at $S = 2$, the hypotheses acceptance regions for classical Bayesian task (3) have the form:

$$\Gamma_1 = \{ x : p(H_2) p(x \mid H_2) < p(H_1) p(x \mid H_1) \},$$
$$\Gamma_2 = \{ x : p(H_1) p(x \mid H_1) < p(H_2) p(x \mid H_2) \}. \qquad (25)$$

It is obvious that the following conditions are satisfied: $\overline{\Gamma_1} = R^n - \Gamma_1 = \Gamma_2$ and $\overline{\Gamma_2} = R^n - \Gamma_2 = \Gamma_1$, as was shown above. These conditions break down at consideration of above-formulated conditional Bayesian task of hypotheses testing. Let us investigate this fact. From the analysis of the regions (10) and

$$\overline{\Gamma}_j = \left\{ x : \sum_{i=1, i \neq j}^{S} p(H_i) p(x \mid H_i) > \lambda \cdot p(H_j) \cdot p(x \mid H_j) \right\}, j = 1, ..., S,$$

we infer that, analogously of the case $S = 2$, here is some value λ^* for which the rejection region of hypothesis H_j and the acceptance region of any other hypothesis $H_i, i = 1, ..., S; i \neq j,$ coincide, i.e. there take place:

$$\bigcup_{i=1}^{S}\Gamma_i = R^n,\ \Gamma_i \bigcap \Gamma_j = \varnothing,\ i,j=1,...,S, i\neq j. \qquad (26)$$

In this case, on the basis of observation result x there will always be accepted one of the tested hypotheses. Though, on the basis of comparison of regions (10) and the regions of acceptance of hypotheses in unconditional Bayesian task (3), irrespective of the kind of loss function, we infer that, these regions differ from one another, i.e. the regions of acceptance of hypotheses in conditional Bayesian Task 1, at $\lambda = \lambda^*$, do not coincide with the regions of acceptance of hypotheses in the unconditional Bayesian Task.

At $\lambda > \lambda^*$, there takes place $\Gamma_j(\lambda > \lambda^*) > \Gamma_j(\lambda = \lambda^*)$. This is possible only when $\Gamma_i \bigcap \Gamma_j \neq \varnothing$, $i,j=1,...,S, i\neq j$. In this case

$$\bigcup_{i=1, i\neq j}^{S}\Gamma_i \ni \overline{\Gamma}_j,$$

i.e. rejection region of hypotheses H_j is contained in the united region of acceptance of other hypotheses. This is available only if region Γ_j of acceptance of hypothesis H_j intersects with one or more (in the limit, with all) regions of acceptance of other hypotheses. At $\lambda < \lambda^*$, there takes place $\Gamma_j(\lambda < \lambda^*) < \Gamma_j(\lambda = \lambda^*)$. This is possible only when in observation space R^n there are sub-regions which do not belong to any region $\Gamma_j(\lambda < \lambda^*)$, $j=1,...,S$. In this case there takes place

$$\bigcup_{i=1, i\neq j}^{S}\Gamma_i \in \overline{\Gamma}_j,$$

i.e. the united region of acceptance of hypotheses $\{H_1,...H_{j-1}, H_{j+1},..., H_S\}$ is contained in the rejection region of hypotheses H_j. Thus, in the observation space R^n, there are such sub-regions which do not belong to any region of acceptance of the tested hypotheses.

Here arose the situation analogous to the one considered above, i.e. at testing many hypotheses, in Task 1 it could appear impossible to make a simple decision or to make any decision when the measured value falls into the sub-regions of intersection of regions of acceptance of hypotheses (at $\lambda > \lambda^*$) or falls into the sub-regions which do not belong to any region of acceptance of hypothesis (at $\lambda < \lambda^*$) respectively. In such cases, for acceptance of any tested hypotheses, we have to use one of the methods: 1) to realize repeated observations (if it is possible) until the moment when the arithmetic mean of the observation results appears only in one of hypotheses acceptance regions and to accept the corresponding hypotheses; 2) to increase or to decrease α (to which correspond decreasing or increasing λ) until the measured value appears only in one of hypotheses acceptance regions. In limit, when $\lambda = \lambda^*$ there will be accepted without fail one hypothesis for any measured value. If λ^* for which the ratio (26) is fulfilled does not exist that means that for given x to make simple decision is impossible without additional information (see example, the case $x = (2.5, 2.5)$). Additional information can be given as new values of a priori probabilities of hypotheses or repeated observations as was mentioned above. It is not difficult to be convinced that hypotheses acceptance regions in other conditional tasks have the same properties.

4. On the Ratio of Average Risks in Conditional Bayesian Tasks

Proceeding from the essence of stated conditional Bayesian tasks, they can be grouped as follows: tasks in which the average value of probabilities of falsely rejected hypotheses is minimized, i.e. the average risk under restrictions on the probabilities of errors caused by incorrect acceptance of hypotheses (Tasks 1, 2 and 3), and the tasks in which is minimized the average probability of errors caused by incorrect acceptance of hypotheses, which is equivalent to maximization of the average power of criterion under the restrictions on the probabilities of incorrectly rejected hypotheses (Tasks 4, 5, 6 and 7). These at a glance mutually inverse tasks, as it has been shown above (see Sections 2), in the general case, are not mutually inverse, i.e. by simple transformation it is impossible to obtain another target function. Therefore, in the general case, by the achieved level of target function, it is possible to compare separately Tasks 1, 2 and 3 and separately Tasks 4, 5, 6, 7. In that specific case, for the certain values of undetermined Lagrange multipliers in the solutions of these tasks, it is possible to reason about the ratio of target functions calculated in different groups of the tasks.

Let us notice that, about the interrelation among the average risks calculated in considered conditional Bayesian tasks, we can reason only under the condition that the values of probabilities α in the restrictions of all considered tasks are identical. Let us consider the first group of the tasks. The comparison of restrictions (8) and (11) shows that the fulfillment of restrictions (11) always causes the fulfillment of conditions (8), but not on the contrary. That is, from these two restrictions, more “rigid” is condition (11). Therefore, it is natural to conclude that the average risk calculated in Task 2 is always not more than the average risk calculated in Task 1, i.e. there takes place:

$$r_{\delta^*,2} \le r_{\delta^*,1}.$$

Comparing restrictions (11) and (13) we infer, that restrictions (13) are more "rigid" than restrictions (11), because the fulfillment of conditions (13) always causes the fulfillment of condition (11), but not on the contrary. Therefore the average risk calculated in Task 2 is always not less than the average risk calculated in Task 3. Thus, for the first group of the tasks, the following ratio between the optimum values of average risks calculated in these tasks takes place:

$$r_{\delta^*,3} \le r_{\delta^*,2} \le r_{\delta^*,1}.$$

Let us compare the optimum values of average criterion powers calculated in the second group of the tasks. It is not difficult to guess that restrictions (16) are less "rigid" than restrictions (19), because the fulfillment of restrictions (19) always entails the fulfillment of restrictions (16). Therefore, the average power of criterion corresponding to restrictions (19) (Task 5) will always be not more than the similar value corresponding to restrictions (16) (Task 4), i.e. there takes place:

$$G_{\delta^*,5} \le G_{\delta^*,4}.$$

Similar reasoning for restrictions (16), (19), (21) and (23) shows that the most "rigid" are restrictions (21) (task 6), then - restrictions (23) (task 7), then - restrictions (19) (Task 5) and, at last, the weakest restriction is (16) (Task 4). Therefore, among the average powers of criterion calculated in these tasks, there is the following ratio:

$$G_{\delta^*,6} \le G_{\delta^*,7} \le G_{\delta^*,5} \le G_{\delta^*,4}.$$

As it was shown in Section 3, if the values of Lagrange multipliers in Tasks 1 and 4 are equal to λ^*, the corresponding regions of acceptance of hypotheses in these tasks coincide and the following equality takes place:

$$r_{\delta^*,1} = \sum_{i=1}^{S} p(H_i) \sum_{j=1, j\ne i}^{S} \int_{\Gamma_j} p(x \mid H_i)dx = \sum_{i=1}^{S} p(H_i)\left[1 - \int_{\Gamma_i} p(x \mid H_i)dx\right] =$$

$$= 1 - \sum_{i=1}^{S} p(H_i) \int_{\Gamma_i} p(x \mid H_i)dx = 1 - G_{\delta^*,4}. \qquad (27)$$

In accordance with the results given in Section 3, if the number of hypotheses is equal to two, the following statement is true.

Proposition 4.1. *If for Lagrange multipliers in the solutions of the stated tasks, the following equalities are satisfied: 1)* $\lambda^{(1)} = 1$; *2)* $\lambda_1^{(2)} \cdot \lambda_2^{(2)} = p(H_1) \cdot p(H_2)$; *3)* $\lambda_1^{(3)} \cdot \lambda_2^{(3)} = 1$; *4)* $\lambda^{(4)} = 1$; *5)* $\lambda_{12}^{(5)} \cdot \lambda_{21}^{(5)} = p(H_1) \cdot p(H_2)$; *6)* $\lambda_{12}^{(6)} \cdot \lambda_{21}^{(6)} = 1$; *7)* $\lambda_1^{(7)} \cdot \lambda_2^{(7)} = 1$, *the following conditions are true:*

$$r_{\delta^*,uncond} = r_{\delta^*,1} = r_{\delta^*,2} = r_{\delta^*,4} = r_{\delta^*,5} = \alpha, \qquad (28^1)$$

where $r_{\delta^*,4} = 1 - G_{\delta^*,4}$ *and* $r_{\delta^*,5} = 1 - G_{\delta^*,5}$;

$$r_{\delta^*,6} = r_{\delta^*,7} = 2 \cdot \alpha, \qquad (28^2)$$

where $r_{\delta^*,6} = 1 - G_{\delta^*,6}$ *and* $r_{\delta^*,7} = 1 - G_{\delta^*,7}$.

Here the indices in brackets specify belonging to the corresponding tasks. It is not difficult to be convinced in validity of (28) by substitution in relation (27) the suitable restrictions of the considered tasks.

5. Comparison of Unconditional and Conditional Methods

Let us consider example for a concrete probability distribution law, in particular, the normal law

$$p(x \mid H_i) = (2\pi)^{-n/2} \left|W\right|^{-1/2} \exp\left\{-\frac{1}{2}(x - a^i)^T W^{-1} (x - a^i)\right\}, \ i = 1, ..., S,$$

for experimental research of the properties of offered algorithms. In Example there are investigated the qualities of hypotheses testing by unconditional and conditional Bayesian algorithms. From Example the character of changes in coefficients λ and regions of acceptance of hypotheses with changing α under the appropriate restriction of the considered tasks, and also of the ratios among the qualities of hypotheses testing in conditional and unconditional Bayesian tasks, is evident.

Example. Tested hypotheses: $H_1: \theta_1^1 = 1, \theta_2^1 = 1$, $H_2: \theta_1^2 = 4, \theta_2^2 = 4$. A priori probabilities of the hypotheses: $p(H_1) = 0.5$, $p(H_2) = 0.5$. Covariance matrices used:

$$W_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ W_2 = \begin{pmatrix} 1 & 0.4 \\ 0.6 & 1 \end{pmatrix}, \ W_2 = \begin{pmatrix} 1 & 0.8 \\ 0.99 & 1 \end{pmatrix}.$$

On the basis of calculation results brought in Table, we infer the following. The calculated values of average risk r_δ in Tasks 1 and 2 and the average power of criterion subtracted from one $1-G_\delta$ in Tasks 4 and 7 for the values of α for which Lagrange multipliers take the values $\lambda^{(1)}=1$, $\lambda_1^{(2)}=p(H_1)=0.5$, $\lambda_2^{(2)}=p(H_2)=0.5$, $\lambda^{(4)}=1$, and $\lambda_{12}^{(7)}\cdot\lambda_{21}^{(7)}=1$, respectively, coincide with α for Tasks 1, 2 and 4 and with the value of average risk in the unconditional task and with 2α for Task 7.

In the general case, in all conditional Bayesian tasks, the interval of changing α contains three sub-intervals. If α falls in the middle subinterval the correct decision is made, and, if it falls in the left or the right subintervals, there are accepted or rejected both hypotheses. To the extreme sub-intervals correspond the values of Lagrange multipliers opposite concerning unity, i.e. less or more than one. For example, in conditional Task 1 for $x_1=1.49, x_2=1.49$ and W_1, for extreme points of subinterval α where hypothesis H_1 is accepted, i.e. for $\alpha=0.0002$ and $\alpha=0.244$, there takes place $\lambda=411.387$ and $\lambda=0.0234$, respectively, i.e. coefficient λ changes from 411.387 to 0.0234.

For all considered covariance matrices and α for which Lagrange coefficients in conditional tasks satisfy condition $\lambda^{(1)}=\lambda_1^{(2)}+\lambda_2^{(2)}=\lambda^{(4)}=\lambda_{12}^{(7)}\cdot\lambda_{21}^{(7)}=1$, comparison of calculation results of unconditional and conditional tasks confirm the validity of Proposition 4.1. In particular, there takes place $r_{\delta^*,uncond}=r_{\delta^*,1}=r_{\delta^*,2}=1-G_{\delta^*,4}=(1-G_{\delta^*,7})/2$.

At $x_1=2.5, x_2=2.5$, the middle subinterval, i.e. the subinterval of acceptance of one tested hypothesis, degenerates into an empty set and decision is not made, since, in accordance with the condition of the example, this measured value could be generated by both distributions with equal probabilities.

At $x_1=2.5, x_2=2.5$ for Tasks 1, 2, 4 and for all considered W, the thresholds of α separating the sub-regions of acceptance of both hypotheses or acceptance of neither hypothesis, coincide. In Task 7 these thresholds are equal to the suitable thresholds of the previous tasks divided by two. This is easy to explain by comparing the restrictions of Task 7 with the restrictions of Tasks 1, 2 and 4.

In Tasks 2 and 7, there takes place $\lambda_1^{(2)}=\lambda_2^{(2)}$ and $\lambda_1^{(7)}=\lambda_2^{(7)}$, because, in the appropriate restrictions, identical values of α are used.

6. Conclusion

Obtained theoretical and computed of the practical example results clearly show the advantage of the offered conditional Bayesian statements of testing many hypotheses. The introduced conditionality allows impose restrictions on the errors of one type and, under such conditions, to minimize the errors of the second type. Such opportunity is very important for correct solving many practical problems. For example, 1) air defense – the cost of incorrectly detected target and the missed one is different, and defence interests demand guaranteed detection of hostile flying vehicles; 2) identification of river water emergency pollution sources; 3) medicine production – the cost of overdosing and underdosing is not identical and the safety of patients requires guaranteed protection of prepared medicines against overdosing; 4) market investigation with the purpose of making recommendations about investments - guaranteed protection from the loss of invested credits; 5) revealing the fact of ship bending on the basis of the measurement results of special sensors; 6) the problem of sustainable development of production and so on. The investigation of the stated problems proves their uniqueness and high quality especially in specific situations when information is not sufficient for making decision with given reliability.

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Table. The results of hypotheses testing by unconditional and conditional Bayesian tasks

Covariance matrix	Measurement result	**Unconditional task**				
		Restriction level	Accepted hypothesis	Risk function	Lagrange multipliers	
W	x	α	H_i	r_{δ^*}	λ_1	λ_2
W_1	2.5, 2.5		H_2	0.01695		
W_2			H_2	0.04163		
W_3			H_2	0.06632		
W_1	2.51, 2.51		H_2	0.01695		
W_2			H_2	0.04163		
W_3			H_2	0.06632		
W_1	1.49, 1,49		H_1	0.01695		
W_2			H_1	0.04163		
W_3			H_1	0.06632		
		Conditional tasks				
		Task 1				
W_1	2.5, 2.5	≤ 0.01694 $= 0.01694$ >0.01694	Both hypotheses are accepted Both hypotheses are accepted No hypothesis is accepted	0.01695	1.00074	
W_2		≤ 0.0416 $= 0.0416$ >0.0416	Both hypotheses are accepted Both hypotheses are accepted No hypothesis is accepted	0.04166	1.00125	
W_3		≤ 0.06632 $= 0.06632$ >0.06632	Both hypotheses are accepted Both hypotheses are accepted No hypothesis is accepted	0.06632	1.00002	
W_1	2.51, 2.51	≤ 0.0163 $\alpha \in (0.0163, 0.0175)$ >0.0175	Both hypotheses are accepted H_2 No hypothesis is accepted	0.01699 ($\alpha = 0.0169$)	1.0048	
W_2		≤ 0.0406 $\alpha \in (0.04063, 0.042)$ >0.042	Both hypotheses are accepted H_2 No hypothesis is accepted	0.04179 ($\alpha = 0.04127$)	1.00634	
W_3		≤ 0.065 $\alpha \in (0.0651, 0.0676)$ >0.0676	Both hypotheses are accepted H_2 No hypothesis is accepted	0.06756 ($\alpha = 0.0651$)	1.02913	
W_1	1.49, 1.49	≤ 0.0002 $\alpha \in (0.0002, 0.244)$ >0.244	Both hypotheses are accepted H_1 No hypothesis is accepted	0.01699 ($\alpha = 0.0169$)	1.0048	

W_2		≤ 0.0018 $\alpha \in (0.0018, 0.285)$ >0.285	Both hypotheses are accepted H_1 No hypothesis is accepted	0.04176 ($\alpha = 0.0415$)	1.00516	
W_3		≤ 0.021 $\alpha \in (0.021, 0.344)$ >0.344	Both hypotheses are accepted H_1 No hypothesis is accepted	0.06664 ($\alpha = 0.066$)	1.00754	
		Task 2				
W_1	2.5, 2.5	≤ 0.01694 $= 0.01694$ >0.1694	Both hypotheses are accepted Both hypotheses are accepted No hypothesis is accepted	0.01695	0.5004	0.5004
W_2		≤ 0.04193 $= 0.04193$ >0.04193	Both hypotheses are accepted Both hypotheses are accepted No hypothesis is accepted	0.04164	0.5002	0.5002
W_3		≤ 0.06632 $= 0.06632$ >0.06632	Both hypotheses are accepted Both hypotheses are accepted No hypothesis is accepted	0.06632	0.5	0.5
W_1	2.51, 2.51	≤ 0.01636 $\alpha \in (0.01636, 0.01755)$ >0.01755	Both hypotheses are accepted H_2 No hypothesis is accepted	0.01699 ($\alpha = 0.0169$)	0.5024	0.5024
W_2		≤ 0.0409 $\alpha \in (0.0409, 0.043)$ >0.043	Both hypotheses are accepted H_2 No hypothesis is accepted	0.04167 ($\alpha = 0.0419$)	0.5007	0.5007
W_3		≤ 0.065 $\alpha \in (0.065, 0.0676)$ >0.0676	Both hypotheses are accepted H_2 No hypothesis is accepted	0.06634 ($\alpha = 0.0663$)	0.5002	0.5002
W_1	1.49, 1.49	≤ 0.00019 $\alpha \in (0.00019, 0.244)$ >0.244	Both hypotheses are accepted H_1 No hypothesis is accepted	0.01694 ($\alpha = 0.01695$)	0.4999	0.4999
W_2		≤ 0.000187 $\alpha \in (0.000187, 0.2857)$ >0.2857	Both hypotheses are accepted H_1 No hypothesis is accepted	0.04167 ($\alpha = 0.0419$)	0.5007	0.5007
W_3		≤ 0.00592 $\alpha \in (0.00592, 0.311)$ >0.311	Both hypotheses are accepted H_1 No hypothesis is accepted	0.06632 ($\alpha = 0.06632$)	0.5	0.5
		Task 4		G_{δ^*}		
W_1	2.5, 2.5	≤ 0.01694 >0.01694 $= 0.01695$	No hypothesis is accepted Both hypotheses are accepted Both hypotheses are accepted	0.9830 0.9831	1.0008 0.9997	
W_2		≤ 0.04163 >0.04163	No hypothesis is accepted Both hypotheses are accepted	0.9584 0.9584	1.0001 0.9997	
W_3		≤ 0.06632 >0.06633 $= 0.06633$	No hypothesis is accepted Both hypotheses are accepted Both hypotheses are accepted	0.9337 0.9337	1.0000 0.9998	
W_1	2.51, 2.51	≤ 0.01755 $\alpha \in (0.01637, 0.01755)$ >0.01755	No hypothesis is accepted H_2 Both hypotheses are accepted	0.9825 ($\alpha = 0.01637$)	>1 1.0609 <1	
W_2		≤ 0.0406 $\alpha \in (0.0407, 0.0426)$ >0.0426	No hypothesis is accepted H_2 Both hypotheses are accepted	0.9574 ($\alpha = 0.01637$)	>1 1.0373 <1	
W_3		≤ 0.0651 $\alpha \in (0.0651, 0.0676)$ >0.0676	No hypothesis is accepted H_2 Both hypotheses are accepted	0.9324 ($\alpha = 0.0651$)	>1 1.0291 <1	
W_1	1.49, 1.49	≤ 0.000192 $\alpha \in (0.000193, 0.2441)$ >0.2441	No hypothesis is accepted H_1 Both hypotheses are accepted	0.9829 ($\alpha = 0.01679$)	>1 1.0160 <1	
W_2		≤ 0.00188 $\alpha \in (0.00188, 0.28578)$	No hypothesis is accepted H_1	0.9582	>1 1.0052	

		>0.28578	Both hypotheses are accepted	($\alpha = 0.0415$)	<1	
W_3		≤ 0.00592 $\alpha \in (0.00593, 0.3116)$ >0.3116	No hypothesis is accepted H_1 Both hypotheses are accepted	0.9334 ($\alpha = 0.066$)	>1 1.0075 <1	
		Task 7				
W_1	2.5, 2.5	≤ 0.00847 >0.00847 $= 0.00847$	No hypothesis is accepted Both hypotheses are accepted No hypothesis is accepted	0.98305	1.0007	1.0007
W_2		≤ 0.0208 >0.0208 $= 0.0208$	No hypothesis is accepted Both hypotheses are accepted No hypothesis is accepted	0.95834	1.0013	1.0013
W_3		≤ 0.0331 >0.0331 $= 0.0331$	No hypothesis is accepted Both hypotheses are accepted Both hypotheses are accepted	0.93356	1.0028	1.0028
W_1	2.51, 2.51	≤ 0.0081 $\alpha \in (0.0082, 0.0087)$ >0.0087	No hypothesis is accepted H_2 Both hypotheses are accepted	0.98244 ($\alpha = 0.00842$)	>1 1.0109 <1	>1 1.0109 <1
W_2		≤ 0.0204 $\alpha \in (0.0204, 0.0213)$ >0.0213	No hypothesis is accepted H_2 Both hypotheses are accepted	0.95813 ($\alpha = 0.0207$)	>1 1.0091 <1	>1 1.0091 <1
W_3		≤ 0.0325 $\alpha \in (0.0326, 0.0338)$ >0.0338	No hypothesis is accepted H_2 Both hypotheses are accepted	0.93336 ($\alpha = 0.033$)	>1 1.0075 <1	>1 1.0075 <1
W_1	1.49, 1.49	≤ 0.000096 $\alpha \in (0.000097, 0.122)$ >0.122	No hypothesis is accepted H_1 Both hypotheses are accepted	0.98301 ($\alpha = 0.00845$)	>1 1.0048 <1	>1 1.0048 <1
W_2		≤ 0.00093 $\alpha \in (0.00094, 0.142)$ >0.142	No hypothesis is accepted H_1 Both hypotheses are accepted	0.95813 ($\alpha = 0.0207$)	>1 1.0091 <1	>1 1.0091 <1
W_3		≤ 0.00296 $\alpha \in (0.00297, 0.155)$ >0.155	No hypothesis is accepted H_1 Both hypotheses are accepted	0.93336 ($\alpha = 0.033$)	>1 1.0075 <1	>1 1.0075 <1

СРАВНИТЕЛЬНЫЙ АНАЛИЗ БЕЗУСЛОВНОЙ И УСЛОВНЫХ БАЙЕСОВСКИХ ПРОБЛЕМ ПРОВЕРКИ МНОГИХ ГИПОТЕЗ

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Резюме

В байесовской постановке задачи проверки многих гипотез, вместо безусловной проблемы минимизации среднего риска, обусловленного ошибками первого и второго рода, предложено решить условную оптимизационную проблему, когда ограничения наложены на ошибки одного типа и в этих условиях минимизируются ошибки второго типа. В зависимости от типа ограничений рассмотрены разные условные оптимизационные задачи. Для поставленных задач исследованы свойства областей принятия гипотез и в конце осуществленно сравнение свойств безусловной и условных байесовских методов. Результаты вычислений конкретных примеров подтверждают значимость теоретических рассуждений.

მრავალი ჰიპოთეზის შემოწმების უპირობო და პირობითი ბაიესის ამოცანების შედარებით

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რეზიუმე

ჰიპოთეზების შემოწმების ამოცანის ბაიესის დასმაში პირველი და მეორე ტიპის შეცდომებით გამოწვეული საშუალო რისკის მინიმიზაციის უპირობო ამოცანის ნაცვლად შემოთავაზებულია პირობითი ოპტიმიზაციის ამოცანის გადაწყვეტა როდესაც შეზღუდვები დადებულია ერთი ტიპის შეცდომებზე და ამ პირობებში მეორე ტიპის შეცდომები არის მინიმიზირებული. შეზღუდვების სახეებისაგან დამოკიდებულებით განხილულია პირობითი ოპტიმიზაციის სხვადასხვა ამოცანები. დასმული ამოცანებისათვის ჰიპოთეზების მიღების არეების თვისებები არის გამოკვლეული და ბოლოს უპირობო და პირობითი მეთოდების თვისებების შედარება არის განხორციელებული. გამოთვლილი მაგალითის შედეგები ადასტურებენ მიღებული თეორიული შედეგების სისწორეს.