

CONNECTIVITY FACTOR FOR INVESTMENT POLICY IMPLEMENTATION

Akhobadze Merab, Kipshidze David
Georgian Technical University

Summary

Investment objects are often represent the components of long-term strategy plan. During the realization of this plan these features are most likely to be corrected that may become as an important reason for changing the investment environment. Thus the results of the projects' particular changes are very important for the investor to provide right investment policy. The paper concerns the method for investment policy implementation on the example of urban systems planning.

Keywords: investment policy. Urban systems. Planning.

1. Introduction

Investment objects often represent the components of long-term strategy plan. During the realization of this plan these features are most likely to be corrected that may become as an important reason for changing the investment environment. Thus the results of the projects particular changes are very important for the investor to provide right investment policy.

On the example of urban systems planning is shown the method of investment policy implementation. Generally, for simplicity, consider that a region of urban system, consisting of two districts is under construction. Consider, that some objects from the set represented by the symbol f is to be constructed (or demolished) during the time period $(0, T)$

Every plan for carrying out the above-said work can be considered as a mapping $p: F \rightarrow (0, T) \times \{1, 2\}$, where $p(f) = (t_f, i_f)$ if according to the plan, the construction (destruction) of the facility f will be commenced at the instant t_f of time in the district i_f .

Let us an increasing sequence $\theta = (\theta_m)_{0 \leq m \leq n}$ of numbers correspond to each plan p ; more precisely, $\theta_0 = 0$ and $\{\theta_m | 1 \leq m \leq n\} = \{t_f | f \in F\} \cup \{t_f + \tau_f | f \in F\}$, where τ_f is the time required for the construction (destruction) of a facility f . In other words, θ is the sequence of those instants of time when according to the plan p the construction or destruction of some facilities should be commenced or completed (zero instant in addition), which is ordered by increment.

It is obvious that the parameters λ_1 and λ_2 of the satisfaction of population interests by the first and second districts are changed during the construction or destruction of new facilities. These parameters can be defined as follows. For every interest i and district j , let us denote the satisfaction parameter of the interest i in the district j and the inevitability weight of the interest i by symbols $\xi(i, j)$ and $\eta(i)$ respectively ($\sum_{i \in I} \eta(i) = 1$; I is the set of interests of population). Then, the numbers λ_1 and λ_2 can be defined as follows:

$$\lambda_j = \sum_{i \in I} \xi(i, j) \eta(i), \quad j = 1; 2.$$

We will accept the agreement that the weights $\eta(i)$ are constant in the time interval $(0, T)$, and only the works determined by the plan p influence the parameters $\xi(i, j)$ and consequently, parameters λ_j . Therefore, we can mean that in the time interval $(0, T)$, the parameters λ_1 and λ_2 can be changed only at instant θ_m ($1 \leq m \leq n$) of time. Hence, some saquence of points L_0, L_1, \dots, L_n of the plane (λ_1, λ_2) corresponds to each plan p .

As it is shown in the paper [1], if the parameters λ_1 and λ_2 remain unchanged during the certain interval of time (as in the case of intervals $[\theta_m, \theta_{m+1})$), then the numbers $x_1(t)$ and $x_2(t)$ of population in the first and respectively, second districts satisfy the following differential equations system:

$$x_i' = (\alpha_i - h_i)x_i + \frac{\lambda_i x_i (V_i - x_i)(h_1 x_1 + h_2 x_2)}{V^2 + \lambda_1 x_1 (V_1 - x_1) + \lambda_2 x_2 (V_2 - x_2)}, \quad i = 1; 2, \quad (1)$$

where α_1 and α_2 are the coefficients of overproduction of population in the districts, h_1 and h_2 are the coefficients of mobility, V_1 and V_2 - the capacities of the districts, V - the exterior capacity (we mean that the region is open for the migration and $h_i > \alpha_i$).

It may be proved that system (1) does not have limit cycles in the first quarter of the phase plane (x_1, x_2) . Indeed, let us introduce new variables: $y_i = \ln x_i, \quad i = 1; 2$.

Then system (1) takes the following form:

$$y_i' = (a_i - h_i) + F_i(y_1, y_2), \quad i=1;2, \quad (2)$$

where

$$F_i(y_1, y_2) = \frac{\lambda_i(V_i - \exp(y_i))(h_1 \exp(y_1) + h_2 \exp(y_2))}{V^2 + \lambda_1 \exp(y_1)(V_1 - \exp(y_1)) + \lambda_2 \exp(y_2)(V_2 - \exp(y_2))}, \quad i=1;2.$$

Let us consider the function

$$B(y_1, y_2) = \frac{V^2 + \lambda_1 \exp(y_1)(V_1 - \exp(y_1)) + \lambda_2 \exp(y_2)(V_2 - \exp(y_2))}{h_1 \exp(y_1) + h_2 \exp(y_2)}.$$

We have:

$$B(y_1, y_2) = \frac{\partial(BF_1)}{\partial y_1} + \frac{\partial(BF_2)}{\partial y_2} = -\lambda_1 \exp(y_1) - \lambda_2 \exp(y_2).$$

It follows that, for every $\lambda_1, \lambda_2 > 0$, the function $B(y_1, y_2)$ has one and the same sign for all points of the plane (y_1, y_2) . According to the Dulac's criterion [2], the system of differential equations $y_i' = F_i(y_1, y_2), \quad i=1;2$, does not have limit cycles. Hence, system (2) and consequently, system (1) do not have limit cycles too. One can verify that (x_1, x_2) may be a saddle of the system only if x_1 and x_2 take very large values. Consequently, according to the Poincaré-Bendixon theorem [2], every non-negative coordinated solution $(x_1(t), x_2(t))$ of our interest infinitely approaches to some non-negative coordinated constant solution of system (1) (the phase portraits of system (1) for some values of the parameters are given in Fig.1).

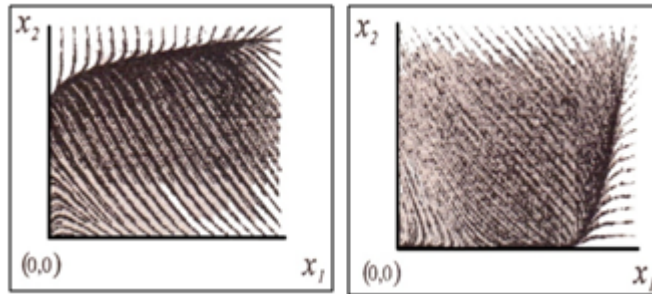


Fig. 1. The phase portraits of system (1) for $\alpha_1 = 0, 10, \quad \alpha_2 = 0, 11, \quad h_1 = 0, 30, \quad h_2 = 0, 35, \quad V_1 = 30000, \quad V_2 = 27000, \quad V = 9600$ and a) $\lambda_1 = 0, 25, \quad \lambda_2 = 1, 28$ (the point (λ_1, λ_2) lies in the sea A described below); b) $\lambda_1 = 0, 30, \quad \lambda_2 = 1, 27$ (the point (λ_1, λ_2) lies in the sea B described below);

Let us accept the following agreement: for every m , the interval $[\theta_m, \theta_{m+1})$ is long enough so that solutions of system (1) could have time to sufficiently approach the corresponding constant solutions of (1). Consequently, in every time interval $[\theta_m, \theta_{m+1})$ we can restrict ourselves to the discussion of only non-negative coordinated constant solutions of system (1). It can be verified that constant solutions of (1) are the pairs

$$(0,0), (0, \pm x_2) \text{ (if } \lambda_2 \geq \frac{4k_2}{\alpha_2 e_2^2}), (\pm x_1, 0) \text{ (if } \lambda_1 \geq \frac{4k_1}{\alpha_1 e_1^2}),$$

$$(x_1^+, x_2^+) \text{ and } (x_1^-, x_2^-) \text{ (if } A^2 - 4\left(\frac{k_1}{\lambda_1} + \frac{k_2}{\lambda_2}\right) \geq 0).$$

Here $\pm x_i = \frac{V_i}{2} \pm \frac{V}{2\alpha_i} \sqrt{\alpha_i^2 e_i^2 - \frac{4k_i}{\lambda_i}},$

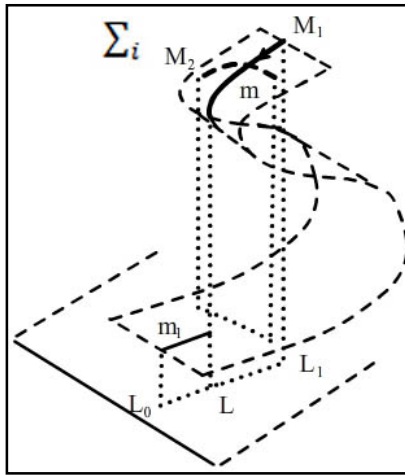
$$x_i^\pm = V_i - V(h_i - \alpha_i) \frac{A \pm \sqrt{A^2 - 4\left(\frac{k_1}{\lambda_1} + \frac{k_2}{\lambda_2}\right)}}{2\lambda_i \left(\frac{k_1}{\lambda_1} + \frac{k_2}{\lambda_2}\right)},$$

$$k_i = \alpha_i(h_i - \alpha_i), \quad A = \frac{\alpha_1 V_1 + \alpha_2 V_2}{V}$$

It is obvious that the union Σ_i of those portions of graphs $0_i, +X_i, -X_i, X_i^+, X_i^-$ of the functions $x_i = 0, x_i = +x_i(\lambda_1, \lambda_2), x_i = -x_i(\lambda_1, \lambda_2), x_i = x_i^+(\lambda_1, \lambda_2), x_i = x_i^-(\lambda_1, \lambda_2)$, which are positioned on the upper half of the space $(\lambda_1, \lambda_2, x_i)$, is the so called catastrophe surface, i.e. a surface such that $(\lambda_1, \lambda_2, x_i)$ lies in it if and only if x_i is i -th coordinate of some equilibrium state of the system for the parameters' values (λ_1, λ_2) .

During the realization of the plan p , the points M_m^1 and M_m^2 , corresponding to L_m , appear on the surfaces Σ_1 and Σ_2 .

Let us accept one more agreement: we consider only plans p such that the points L_m and L_{m+1} , for every $0 \leq m \leq n - 1$, are placed sufficiently close to each other.



Therefore, $L = \{L_m | 0 \leq m \leq n\}$ and consequently, the sets $M^1 = \{M_m^1 | 0 \leq m \leq n\}$ and $M^2 = \{M_m^2 | 0 \leq m \leq n\}$ may be considered as oriented curves. In the i -th district the catastrophe (demographic explosion) occurs if and only if the curve M^i has the discontinuity. The knowledge of the form of the surfaces Σ_1, Σ_2 and the principle of maximal delay [3] (according to which the system undergoes catastrophe if and only if it has no other way) would make it possible to answer the questions whether demographic explosions are expected in a region during the realization of a plan p (Fig. 2).

Fig. 2. a) The curve M is discontinuous and consequently, the catastrophe will occur in the district; b) the curve M is continuous and consequently, the catastrophe will not occur in the district.

As a rule it is difficult to observe the behavior of curves M^i on the surfaces Σ_i . Thus, there appears the problem of finding out the possibility of demographic explosions in a region applying only a curve, without referring to the spatial picture.

For this purpose, let us in the first quarter of the plane (λ_1, λ_2) mark out those areas, where the functions $+x_i, -x_i, x_i^+, x_i^-$ ($i = 1; 2$) are defined and non-negative. Let us also single out the curves where the surfaces $0_i, +x_i, -x_i, x_i^+, x_i^-$ intersect. It can be verified that abiding by the above we will obtain the following pictures (fig. 3):

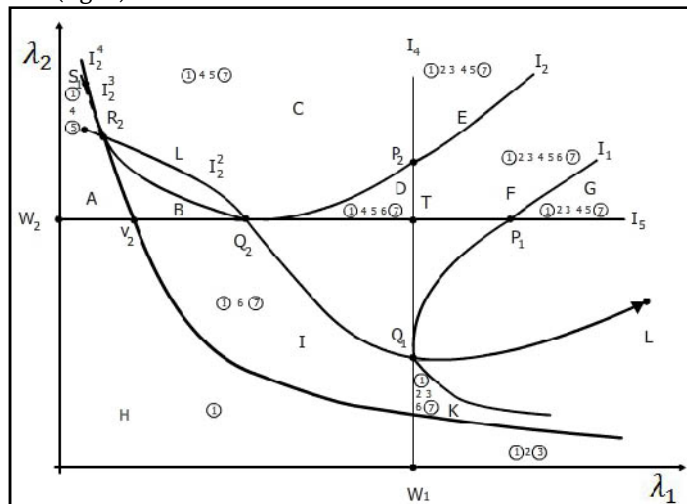


Fig. 3. The bifurcational curves of the system a) when $\alpha_1 V_1 > \alpha_2 V_2$;

The equation of the curve l_1 depicted in fig. 3 is

$$\lambda_1 = \frac{k_1 \alpha_2^2 e_2^2 \lambda_2^2}{k_1 (\alpha_2 e_2 \alpha_1 e_1 \lambda_2 - k_1)} \left(\lambda_2 \geq \frac{k_2}{\alpha_1 e_1 \alpha_2 e_2} \right);$$

the equation of the curve l_2 depicted in fig. 3 is

$$\lambda_2 = \frac{k_2 \alpha_1^2 e_1^2 \lambda_1^2}{k_1 (\alpha_1 e_1 \alpha_2 e_2 \lambda_1 - k_1)} \left(\lambda_1 \geq \frac{k_1}{\alpha_1 e_1 \alpha_2 e_2} \right);$$

the equation of the curve l_3 depicted in fig. 3 is

$$4k_2 \lambda_1 - A^2 \lambda_1 \lambda_2 + 4k_1 \lambda_2 = 0 \quad \left(\lambda_1 \geq \frac{4k_1}{A^2} \right);$$

the lines l_4 and l_5 in fig. 3 are given by the equalities

$$\lambda_1 = \frac{4k_1}{\alpha_1^2 e_1^2}; \quad \lambda_2 = \frac{4k_2}{\alpha_2^2 e_2^2}; \quad \text{where} \quad e_i = \frac{V_i}{V} \quad i = 1; 2.$$

In every area in fig. 3 there are written the equilibrium states with non-negative coordinates existing in it. Herewith, we have accepted the following notations:

$$(0,0) \equiv 1; \quad ({}^-x_1, 0) \equiv 2; \quad x_1, 0) \equiv 3; \quad (0, {}^-x_2) \equiv 4; \quad (0, {}^+x_1) \equiv 5; \quad (x_1^+, x_2^+) \equiv 6; \quad (x_1^-, x_2^-) \equiv 7;$$

the stable equilibrium states are taken in circles.

It can be verified that

$$(O_i \cap X_i^+) = \begin{cases} l_j & \text{if } a_i V_i \geq a_j V_j \\ l_i^1 \cup l_i^2 & \text{if } a_i V_i < a_j V_j \end{cases} \quad (3)$$

$$\pi(O_i \quad X_i^-) = \begin{cases} \emptyset & \text{if } a_i V_i \geq a_j V_j \\ l_j^3 \quad l_j^4 & \text{if } a_i V_i < a_j V_j \end{cases} \quad (4)$$

$$\pi(O_i \quad X_i) = \emptyset \quad (5)$$

$$\pi(O_i \quad {}^-X_i) = \emptyset \quad (6)$$

$$\pi({}^+X_i \quad X_i^-) = \begin{cases} l_i^3 \quad l_i^4 & \text{if } a_i V_i > a_j V_j \\ \emptyset & \text{if } a_i V_i \leq a_j V_j \end{cases} \quad (7)$$

$$\pi({}^-X_i \quad X_i^-) = \emptyset \quad (8)$$

$$\pi({}^+X_i \cap X_i^+) = \begin{cases} l_i^2 \quad l_i^3 & \text{if } a_i V_i > a_j V_j \\ l_i^2 & \text{if } a_i V_i \leq a_j V_j \end{cases} \quad (9)$$

$$\pi({}^-X_i \quad X_i^-) = l_i^1 \quad (10)$$

$$\pi(X_i^+ \quad X_i^-) = l_3 \quad (11)$$

$$\pi({}^+X_i \quad {}^-X_i) = l_{i+3} \quad (12)$$

where j is a number of the district different from the i -th district, $\pi(Y)$ denotes the projection of a set Y on the plane (λ_1, λ_2) .

Now we can observe the qualitative side of the population dynamics, based on a curve L and, in particular, answer the question whether demographic explosions are expected.

Let us give an example. Consider the curve L in fig. 3. It is evident that while the current point of the curve L is placed within an area, the catastrophe does not occur. It may take place only while passing boundaries. Assume that in the initial instant of time the regional system is in the equilibrium state (x_1^+, x_2^+) ; then $M_0^1 \quad X_1^+$ and $M_0^2 \quad X_2^+$. When L intersects the curve l_2^2 , then according to the maximal delay principle M_0^1 (respectively M_0^2) should continue the movement on a surface X_1 (respectively X_2) such that the projection on the plane (λ_1, λ_2) of the intersection of X_1 (respectively X_2) with X_1^+ (respectively X_2^+) contains the point L .

According to (3-12), the surface O_1 appear to be such one (respectively $+X_2$). Therefore, in the area C the system turns out in the equilibrium state $(0, +X_2)$ (existing in C according to fig. 2). Where the parameters' point is passing the point Q_2 , M^1 should remain again on O_1 or come to the surface X_1^+ , but M^2 should remain again on surface X_2^+ or change over to $-X_2$. Consequently, when point of L is going to the area I , the system passes to one of the states $(0, +x_2)$, $(0, -x_2)$, (x_1^+, x_2^+) , $(x_1^+, -x_2)$. The first and the latter states do not exist at all; the state $(0, -x_2)$ does not exist in the area I . Hence, when the parameters' point comes to I , the system is in the equilibrium state (x_1^+, x_2^+) and $M^1 = X_1^+$, $M^2 = X_2^+$. The similar arguments show that in the area L there do not exist an equilibrium state (x_1, x_2) such that $Q_1 \in \pi(X_1^+ - X_1) \cap \pi(X_2^+ - X_2)$. Hence, it can be concluded that when passing the point Q_1 , the catastrophe will occur.

In general, such straightforward answer cannot be given to the question whether the catastrophe will occur if the point (λ_1, λ_2) traces out a given curve. Therefore, it is expedient to set the problem of finding the probability of the latter event.

Hereinafter, without loss of generality, we will assume that the initial and terminal points of curves are positioned within areas.

It is easy to observe that in fact it does not matter of what type a curve L is, but it is important what sequence of areas the curve is passing. It also makes it difference which of the common vertices and edges are intersected by a curve L while coming over from one area to another. Consequently, it is natural that instead of curves L we consider objects of following form:

$$A_0 v_0 A_1 v_1 A_2 \dots A_{n-1} v_{n-1} A_n, \tag{13}$$

where n is any natural number A_i are areas indicated in fig. 3 and v_i are common vertices or edges (which are considered without "boundary" points) of area A_i and A_{i+1} .

Let us introduce some notations. Let S_A be the set of equilibrium states with non-negative coordinates existing in area A . Let us denote by S_1 (by S_2 respectively), for every $s \in S_A$, the graphs of the first component (of the second component respectively) of s in the space $(\lambda_1, \lambda_2, x_1)$ (in the space $(\lambda_1, \lambda_2, x_2)$ respectively) (for instance, $S_1 = O_1$, $S_2 = -X_2$ for $s = 4$).

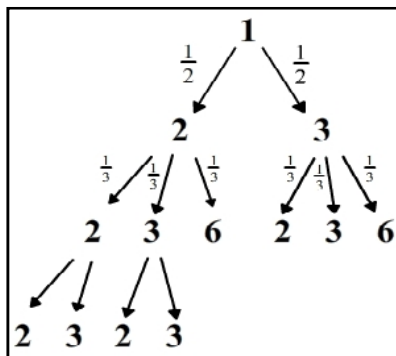
For every two neighboring areas A_i and A_{i+1} and for their common vertex or edge v , there appears the relation R_v between the sets S_{A_i} and $S_{A_{i+1}}$ that is given in the following way: for any $s \in S_{A_i}$ and $s' \in S_{A_{i+1}}$, $s R_v s'$ if and only if

$$v \in \pi(S_1 - S_1') \cap \pi(S_2 - S_2') \tag{14}$$

(if v is an edge, then we mean v instead of \in in (14)).

It is obvious that (14) is equivalent to the requirement that during the movement of the parameters' point from the area A_i , by passing v , to the area A_{i+1} , the equilibrium state s may come over to the state s' without the catastrophe [4]. The relations induced by all the vertices and edge depicted in fig. 4 are described in tables 1-13 below.

Let us correspond some tree to every route (13). For the sake of brevity we give here not quite strict, but intelligible definition of it:



- let us write " A_0 " and call it 0 th-level vertex;
 - let us write out all the elements of the S_{A_0} (call them the first-level vertices). For every $s \in S_{A_0}$, construct the arrow $A_0 - s$ and call it a 0 th-level edge of the tree.
 - if the i th-level vertices are constructed, then the $(i + 1)$ th-level ones can be constructed in the following way: if s is a i th-level vertex, $s' \in S_{A_{i+1}}$ and $s R_v s'$, then s' is a $(i + 1)$ th-level vertex and arrows $s \rightarrow s'$ is an edge of the tree, that is said to be a i th-level edge.
- For instance, the tree of the route is given in fig. 4.

Fig. 4. The tree of rout, when initial equilibrium state is 2

Let us assign some number to every i th-level edge $s \rightarrow s'$ ($i = 1, 2, \dots, n$) of the tree corresponding to rout (13); this number expresses the conditional probability that the system will be in the equilibrium state s' under the condition that the parameters' point has passed from A_i to A_{i+1} via v_i and when the parameters' point was in the area A_i , the system was in the equilibrium state s .

To find the mentioned probability, let us suppose that there exists and does not depend on i the conditional probability that the system will pass to a stable (unstable respectively) equilibrium state under the condition that the parameters' point has passed from A_i to A_{i+1} and that there exist both stable and unstable states s'' in $S_{A_{i+1}}$ with $sR_{v_i}s''$; let us denote this probability by p (q respectively). Obviously $p + q = 1$ and $p > q$. Let k_{i+1} (m_{i+1} respectively) be the number of the i th-level arrows $s \rightarrow s''$ with stable (unstable respectively) s'' . Assuming now that all possible transition $s \rightarrow s''$ of this kind are equiprobable, we conclude that the desired probability is equal to $1/k_{i+1}$ ($1/m_{i+1}$ respectively) if s' is stable (unstable respectively) and there are not unstable (stable respectively) states s'' in $S_{A_{i+1}}$ with $sR_{v_i}s''$, and it is equal to p/k_{i+1} (q/m_{i+1} respectively) if s' is stable (unstable respectively), but there are unstable equilibrium states s'' in $S_{A_{i+1}}$ with $sR_{v_i}s''$.

Let us also assign the similarly defined number to all zero-level edge. After this, it is easy to calculate the probability that the catastrophe will not occur: we have to mark out all "long" chains and then to take the sum of products of numbers assigned to edges of marked chains.

In the initial time, the number of population is often known and consequently, the current equilibrium state in the initial area of a route is known too. Hence, instead of the whole tree, we can consider only part of it.

For each $L(L_0, L_1, \dots, L_m)$ curve can be calculated probabilities, that for some realization of urban development plan what is probability of explosion to occur [4]. This allows us to evaluate the investment risks that are placed in the region.

For this, consider the region as an economic system. Suppose it is composed of n -interconnected subsystems.

Let us describe each subsystem with income $y_i(t, p)$, which it already has and with investment $I_i(t, p)$ which it is realizing. Obviously it depends on the particular development p plan.

$$I_i(t, p) = N_i(t, p) + \mu(p)(W_i(t, p) - E_i(t, p)), \quad i = 1, 2, \dots, n \quad (15)$$

$$y_i(t, p) \geq 0; \quad I_i(t, p) \geq 0.$$

Here: $N_i(t, p)$ - is an investment value of their income from the investments realized in the same subsystem;

$E_i(t, p)$ - is an investment value from i subsystem to another subsystem;

$W_i(t, p)$ - is an investment value from another subsystem to i subsystem;

$\mu(p)$ ($0 \leq \mu(p) \leq 1$)-is an incidental ratio;

if $\mu(p) = 0$, then there is no connection between subsystems at the time of the p plan realization. When

$\mu(p) = 1$, then all of the subsystems are interconnected.

If we denote $x_{ij}(t, p)$ as the investment flow from subsystem i to subsystem j , we will have the following balance equalities:

$$E_i(t, p) = \sum_{j=1, j \neq i}^n x_{ji}(t, p) \quad (16)$$

$$W_i(t, p) = \sum_{j=1, j \neq i}^n x_{ij}(t, p)$$

If we allow that each particular p plan realization investment portion of the probability distribution $a_{ij}(p)$ between subsystems are random distribution values and the investments are totally consumed then:

$$\sum_{j=1}^n a_{ij}(p) = 1; \quad i = 1, 2, \dots, n \quad (17)$$

At that kind of limitations we can imagine the investment portion dynamics as a local - stationary state sequence for each t -moment - of which entropy is defined with the equation:

$$H(X(t, p)) = - \sum_{i,j=1}^n x_{ij}(t, p) \ln \frac{x_{ij}(t, p)}{ea_{ij}(p)} \quad \max \quad (18)$$

in case of appropriate (17) and (18) limitations.

Here probabilities of the investment $a_{ij}(p)$ distribution in the subsystems are defined prior, in accordance with the method described above (see Fig.4) . For each subsystem the for the portion of the investment value we should take $x_{ij}(t, p)$, in which case the system entropy $H(X(t, p))$ is maximal. In this case the investment risk is minimal.

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აგულობის ფაქტორი საინვესტიციო პოლიტიკის წარმართვისას

მერაბ ახოვაძე, დავით ყიფშიძე
საქართველოს ტექნიკური უნივერსიტეტი

რეზიუმე

ხშირად საინვესტიციო ობიექტები წარმოადგენს მრავალწლიანი სტრატეგიული გეგმის შემადგენელ ნაწილებს. პროექტის რეალიზაციისას მოსალოდნელია მისი მცირე კორექტირება, რაც საინვესტიციო გარემოს მნიშვნელოვანი ცვლილების მიზეზი შეიძლება გახდეს. აქედან გამომდინარე, ინვესტორისათვის მეტად მნიშვნელოვანია იცოდეს, რა შედეგები შეიძლება მოყვეს პროექტის ამა თუ იმ ცვლილებას, რათა სწორად წარმართოს საინვესტიციო პოლიტიკა. ნაშრომში ნაჩვენებია საინვესტიციო პოლიტიკის წარმართვის ალგორითმი ურბანული სისტემის განაშენიანების მაგალითზე.