საქართველოს ტექნიკური უნივერსიტეტი

ხელნაწერის უფლებით

ვასილ სოხაძე

ფირფიტების გაანგარიშება პლასტიკური დეფორმაციების გათვალისწინებით

წარმოდგენილია დოქტორის აკადემიური ხარისხის მოსაპოვებლად

სადოქტორო პროგრამა „მშენებლობა”. შიფრი 0406

საქართველოს ტექნიკური უნივერსიტეტი

თბილისი, 0175, საქართველო

თვე, 2015 წელი

სამშენებლო ფაკულტეტი

ჩვენ, ქვემორე ხელისმომწერნი ვადასტურებთ, რომ გავეცანით ვასილ სოხაძის მიერ შესრულებულ სადისერტაციო ნაშრომს დასახელებით:

„ფირფიტების გაანგარიშება პლასტიკური დეფორმაციების გათვალისწინებით“ და ვაძლევთ რეკომენდაციას საქართველოს ტექნიკური უნივერსიტეტის სამშენებლო ფაკულტეტის სადისერტაციო საბჭოში მის განხილვას დოქტორის აკადემიური ხარისხის მოსაპოვებლად.

თარიღი

ხელმძღვანელი:	პროფესორი თამაზ ბაციკაძე
რეცენზენტი:	ასოც. პროფ. დავით ჯანყარაშვილი
რეცენზენტი:	პროფ. მურმან ყალაბეგაშვილი აგრარული უნივერსიტეტი
რეცენზენტი:	

საქართველოს ტექნიკური უნივერსიტეტი

2015

ავტორი: ვასილ სოხაძე

დასახელება:	„ფირფიტების გაანგარიშება პლასტიკური დეფორმაციების გათვალისწინებით“
ფაკულტეტი :	სამშენებლო ფაკულტეტი
აკადემიური ხარისხი:	დოქტორი
სხდომა ჩატარდა:	თარიღი

ინდივიდუალური პიროვნებების ან ინსტიტუტების მიერ ზემომოყვანილი დასახელების დისერტაციის გაცნობის მიზნით მოთხოვნის შემთხვევაში მისი არაკომერციული მიზნებით კოპირებისა და გავრცელების უფლება მინიჭებული აქვს საქართველოს ტექნიკურ უნივერსიტეტს.

ავტორის ხელმოწერა

ავტორი ინარჩუნებს დანარჩენ საგამომცემლო უფლებებს და არც მთლიანი ნაშრომის და არც მისი ცალკეული კომპონენტების გადაბეჭდვა ან სხვა რაიმე მეთოდით რეპროდუქცია დაუშვებელია ავტორის წერილობითი ნებართვის გარეშე.

ავტორი ირწმუნება, რომ ნაშრომში გამოყენებული საავტორო უფლებებით დაცული მასალებზე მიღებულია შესაბამისი ნებართვა (გარდა იმ მცირე ზომის ციტატებისა, რომლებიც მოითხოვენ მხოლოდ სპეციფიურ მიმართებას ლიტერატურის ციტირებაში, როგორც ეს მიღებულია სამეცნიერო ნაშრომების შესრულებისას) და ყველა მათგანზე იღებს პასუხისმგებლობას.

რეზიუმე

ნაშრომი ეძღვნება თხელკედლიანი სივცრული კონსტრუქციების გაანგარიშებას, რომელთა მთავარ შემადგენელ ელემენტს წარმოადგენს თხელი ფილები (ფირფიტები). მათი შესწავლის აქტუალობა განპირობებულია იმ მზარდი მოთხოვნებით, რომელებსაც მათ უყენებს თანამედროვე ტექნიკა და მშენებლობა. ისინი ფართოდ გამოიყენება სამრეწველო და სამოქალაქო მშენებლობაში, გემთმშენებლობაში, მანქანათმშენებლობაში, ავიამშენებლობაში აეროდრომების მშენებლობაში და სხვ. აღნიშნული სისტემების ასეთი პოპულარობა განპირობებულია, ერთის მხრივ, მათი სიმტკიცის მაღალი მაჩვენებლებით, მეორეს მხრივ კი კონსტრუქციის წონის მკვეთრად შემცირების შესაძლებლობით, რასაც გადამწყვეტი მნიშვნელობა ენიჭება საფრენი აპარატების (თვითმფრინავები, რაკეტები და სხვა) პროექტების განხორცილებისას.

თანამედროვე მანქანა-დანადგარებისა და ნაგებობების გაანგარიშება სიმტკიცეზე ხშირად მოითხოვს ინჟინრისაგან ისეთი ფაქტორების და მოვლენების გათვალისწინებას, რაც მხოლოდ პლასტიკურობის თეორიის დახმარებით არის შესაძლებელი. ამასთანავე, დრეკადი მოდელების საფუძველზე შესრულებულმა გაანგარიშებამ შეიძლება გამოიწვიოს როგორც მასალის გადახარჯვა, ასევე კონსტრუქციის წონის გაუმართლებელი ზრდა, რაც განსაკუთრებით მნიშვნელოვანია საფრენი აპარატებისადმი წაყენებული მოთხოვნების თვალსაზრისით.

დეფორმაციული თეორიის შესაძლებლობათა კრიტიკული ანალიზი გვარწმუნებს, რომ მისი გამოყენება დრეკად–პლასტიკური წონასწორობის შესასწავლად მიზანშეწონილია მხოლოდ გარკვეული შეზღუდვების პირობებში დეფორმაციისა და დატვირთვის მიმართ. კერძოდ, ზიდვის უნარის განსაზღვრის მიზნით გამართლებულია სენ-ვენან, ლევი, მიზესის თეორიით სარგებლობა, რადგან იგი განკუთვნილია იდეალურად პლასტიკურ–ხისტი ტანისათვის.

RESUME

The work is related to analysis of thin-walled spatial structures, whose main constituent element is presented by thin plates. The urgency of their study is stipulated due the increasing demands that are arisen by modern technology and construction. They are widely used in industrial and civil construction, shipbuilding, machine building, aircraft engineering, considered of aerodrome and so on. Such popularity of these systems is stipulated due, on one hand, their high strength characteristics, and on the other hand, due the possibility to reduce the weight of structure that is of crucial importance for implementation of aircraft engineering's (airplanes, missiles and so on) design.

The strength analysis of modern machinery and engineering facilities often requires from engineers consideration of such factors and phenomena that are available only by the theory of plasticity. At the same time, carried out analysis on the basis of elastic model would cause the excessive consumption of materials, as well as an unjustified increase in the weight of structure that is especially important for requirements to aircrafts.

The critical analysis of deformation theory assures us that its application for the study of elastic-plastic equilibrium is advisable only for condition of certain restrictions related to deformation and load. In particular, in order of determination of carrying capacity is justified the application of Saint-Venant, Levy, Mises theory, because it is stipulated for ideal plastic-rigid body.

შინაარსი

ნახაზები

ცხრილები

საქართველოს ტექნიკური უნივერსიტეტი

ხელნაწერის უფლებით

ვასილ სოხაძე

ფირფიტების გაანგარიშება პლასტიკური დეფორმაციების გათვალისწინებით

წარმოდგენილია დოქტორის აკადემიური ხარისხის მოსაპოვებლად

სადოქტორო პროგრამა „მშენებლობა". შიფრი 0406

საქართველოს ტექნიკური უნივერსიტეტი

თბილისი, 0175, საქართველო

თვე, 2015 წელი

სამშენებლო ფაკულტეტი

ჩვენ, ქვემორე ხელისმომწერნი ვადასტურებთ, რომ გავეცანით ვასილ სოხაძის მიერ შესრულებულ სადისერტაციო ნაშრომს დასახელებით: „ფირფიტების გაანგარიშება პლასტიკური დეფორმაციების გათვალისწინებით“ და ვაძლევთ რეკომენდაციას საქართველოს ტექნიკური უნივერსიტეტის სამშენებლო ფაკულტეტის სადისერტაციო საბჭოში მის განხილვას დოქტორის აკადემიური ხარისხის მოსაპოვებლად.

თარიღი

ხელმძღვანელი:	პროფესორი თამაზ ბაციკაძე
რეცენზენტი:	ასოც. პროფ. დავით ჯანყარაშვილი
რეცენზენტი:	პროფ. მურმან ყალაბეგაშვილი აგრარული უნივერსიტეტი
რეცენზენტი:	

საქართველოს ტექნიკური უნივერსიტეტი

2015

ავტორი: ვასილ სოხაძე

დასახელება: „ფირფიტების გაანგარიშება პლასტიკური დეფორმაციების გათვალისწინებით"

ფაკულტეტი : სამშენებლო ფაკულტეტი

აკადემიური ხარისხი: დოქტორი

სხდომა ჩატარდა: თარიღი

ავტორი ირწმუნება, რომ ნაშრომში გამოყენებული საავტორო უფლებებით დაცული მასალებზე მიღებულია შესაბამისი ნებართვა (გარდა იმ მცირე ზომის ციტატებისა, რომლებიც მოითხოვენ მხოლოდ სპეციფიურ მიმართებას ლიტერატურის ციტირებაში, როგორც ეს მიღებულია სამეცნიერო ნაშრომების შესრულებისას) და ყველა მათგანზე იღებს პასუხისმგებლობას.

რეზიუმე

ნაშრომი ეძღვნება თხელკედლიანი სივცრული კონსტრუქციების გაანგარიშებას, რომელთა მთავარ შემადგენელ ელემენტს წარმოადგენს თხელი ფილები (ფირფიტები). მათი შესწავლის აქტუალობა განპირობებულია იმ მზარდი მოთხოვნებით, რომელებსაც მათ უყენებს თანამედროვე ტექნიკა და მშენებლობა. ისინი ფართოდ გამოიყენება სამრეწველო და სამოქალაქო მშენებლობაში, გემთმშენებლობაში, მანქანათმშენებლობაში, ავიამშენებლობაში აეროდრომების მშენებლობაში და სხვ. აღნიშნული სისტემების ასეთი პოპულარობა განპირობებულია, ერთის მხრივ, მათი სიმტკიცის მაღალი მაჩვენებლებით, მეორეს მხრივ კი კონსტრუქციის წონის მკვეთრად შემცირების შესაძლებლობით, რასაც გადამწყვეტი მნიშვნელობა ენიჭება საფრენი აპარატების (თვითმფრინავები, რაკეტები და სხვა) პროექტების განხორცილებისას.

თანამედროვე მანქანა-დანადგარებისა და ნაგებობების გაანგარიშება სიმტკიცეზე ხშირად მოითხოვს ინჟინრისაგან ისეთი ფაქტორების და მოვლენების გათვალისწინებას, რაც მხოლოდ პლასტიკურობის თეორიის დახმარებით არის შესაძლებელი. ამასთანავე, დრეკადი მოდელების საფუძველზე შესრულებულმა გაანგარიშებამ შეიძლება გამოიწვიოს როგორც მასალის გადახარჯვა, ასევე კონსტრუქციის წონის გაუმართლებელი ზრდა, რაც განსაკუთრებით მნიშვნელოვანია საფრენი აპარატებისადმი წაყენებული მოთხოვნების თვალსაზრისით.

დეფორმაციული თეორიის შესაძლებლობათა კრიტიკული ანალიზი გვარწმუნებს, რომ მისი გამოყენება დრეკად–პლასტიკური წონასწორობის შესასწავლად მიზანშეწონილია მხოლოდ გარკვეული შეზღუდვების პირობებში დეფორმაციისა და დატვირთვის მიმართ. კერძოდ, ზიდვის უნარის განსაზღვრის მიზნით გამართლებულია სენ-ვენან, ლევი, მიზესის თეორიით სარგებლობა, რადგან იგი განკუთვნილია იდეალურად პლასტიკურ–ხისტი ტანისათვის.

RESUME

The work is related to analysis of thin-walled spatial structures, whose main constituent element is presented by thin plates. The urgency of their study is stipulated due the increasing demands that are arisen by modern technology and construction. They are widely used in industrial and civil construction, shipbuilding, machine building, aircraft engineering, considered of aerodrome and so on. Such popularity of these systems is stipulated due, on one hand, their high strength characteristics, and on the other hand, due the possibility to reduce the weight of structure that is of crucial importance for implementation of aircraft engineering's (airplanes, missiles and so on) design.

The strength analysis of modern machinery and engineering facilities often requires from engineers consideration of such factors and phenomena that are available only by the theory of plasticity. At the same time, carried out analysis on the basis of elastic model would cause the excessive consumption of materials, as well as an unjustified increase in the weight of structure that is especially important for requirements to aircrafts.

The critical analysis of deformation theory assures us that its application for the study of elastic-plastic equilibrium is advisable only for condition of certain restrictions related to deformation and load. In particular, in order of determination of carrying capacity is justified the application of Saint-Venant, Levy, Mises theory, because it is stipulated for ideal plastic-rigid body.

შინაარსი

zogierTi cnoba filebisa da firfitebis Teoriis Tanamedrove mdgomareobis Sesaxeb

filebisa da firfitebis Teoriis ganviTarebis etapebi

filebis da firfitebis Teoria warmoadgens myari deformadi tanis meqanikis vrcel ganStoebas, romelsac gaaCnia rTuli struqtura da misi gaangariSebis sakiTxi, rogorc mkacri sainJinro dispiclinis, daiwyo me–19 saukunis pirveli naxevridan, l. navies, o. koSis da s. puasonis Sromebis gamoqveynebiT. amJamad filebis da firfitebis Teorias eZRvneba ramodenime aTasi naSromi. gamoqveynebuli literaturis mokle anotaciebi mocemulia i. alumiaes [29], [30], o. oniaSvilis [30], g. janeliZis [31], i. nemiSis [32], p. nahdis [34] da sxva SromebSi. cnobilia, rom filebis da firfitebis praqtikaSi gamoyenebam, bevrad gauswro win gaangariSebis Teorias. erTis mxriv, inJinrebi konkretuli proeqtebis Sesrulebisas qmnidnen gamartivebul modelebs da amis safuZvelze gaangariSebebi dahyavdaT ricxviT formebze. meores mxriv, iqmneboda Txeli filebis da firfitebis zogadi wrfivi Teoria.

qarTvel mecnierebs Soris filebis TeoriaSi didi Rvawli miuZRviT n. musxeliSvils [21], S. miqelaZes [30], m. miqelaZes [36], a. kakuSaZes [37], o. oniaSvils [30] da sxvebs.

XX saukunis 20-50–ian wlebSi daiwyo filebisa da firfitebis Teoriis Seqmnis Semdegi etapi. igi viTardeba sxvadasxva, xSirad gadamkveTi mimarTulebebiT. ganixileba iseTi konstruqciuli Taviseburebebic, rogoricaa Sreebis, wiboebis, diafragmebis, konturis elementebis, naxvretebis arseboba. ganviTarda filebis Teoriis sakontaqto amocana da lokaluri zemoqmedebis gavlenis Sefaseba. ganixileba masalis ufro rTuli Tvisebebi: arawrfivi drekadoba, anizotropuloba, plastikuroba, siblante. mcire deformaciebis ganxilvidan gadavidnen did gadaadgilebebze. viTardeba geometriulad arawrfivi Teoria da mis bazaze konstruqciis yofaqceva – rogorc statikuri ise dinamikuri zemoqmedebisas. filebze Zalovan zemoqmedebasTan erTad Seiswavleba Tburic – konstruqciis Termodrekadoba da eleqtromagnituri drekadoba.

sakmaod gafarTovda maTematikuri aparatis gamoyeneba rogorc ukve arsebuli problemebis realizaciisaTvis, aseve axali problemebis uzrunvelsayofad. Teoriulis paralelurad gamoiyeneba kvlevis eqsperimentaluri gza. rTuli problemebis ricxviT Sedegebamde miyvana SesaZlebeli gaxada kompiuterebis farTod gamoyenebam ricxviTi meTodebis da diskretuli saangariSo modelebis ganviTarebis pirobebSi**.**

naxvretebisa da bzarebis mqone firfitebis da filebis simtkicis sakiTxebi

pirvelad zogadi Teoria, romelic eyrdnoboda kirhxofis hipoTezas, SemoTavazebuli iyo g. aronis mier, magram es Teoria Seicavda uzustobebs, romlebic gaaswora a. lavinma. filebis zogadi Teoriis CamoyalibebaSi garkveuli roli Seasrula i. bubnovis [5], z. galiorkinis [7,8,9], s. timoSenkos [6] monografiebma.

firfitebisa da filebis Teoria, romelic aris myari deformadi sxeulebis meqanikis nawili, erTis mxriv, sazrdoobs misi sxva dargebis ideebiTa da meTodebiT, xolo meores mxriv, ayalibebs safuZvels meqanikis axali dargebis ganviTarebisaTvis. ase moxda, rodesac Semoitanes kompleqsuri cvladis funqciis Teoriis gamoyenebis idea drekadobis TeoriaSi, romelic gamoTqva g. kolosovma [43] da ganaviTara n. musxeliSvilma [21]. es idea farTod gamoiyeneba garsebSi naxvretebTan axlos ZabvaTa koncentraciis amocanaTa gadasawyvetad. kerZod, g. savinisa da misi mowafeebis mier [16]. es

Sedegebi mTel rig SemTxvevebSi mniSvnelovania bzarebis Teoriisa, da amgvarad, rRvevis meqanikis ganviTarebisaTvis.

naxvretebis siaxloveSi Zabvebis koncentraciis Sesaxeb fundamenturi Sroma gamoaqveyna g. savinma [57]. am monografiaSi warmodgenil kvlevebs, garda Teoriuli interesisa, aqvT praqtikuli mniSvneloba.

amave problemas eZRvneba g. savinisa da v. tulCiis [16] monografia, romelic Seicavs naxvretis siaxloveSi ZabvaTa koncentraciis Teoriul da eqsperimentalur gamokvlevebs brtyeli deformaciisa da ganzogadoebuli brtyeli daZabuli mdgomareobis pirobebSi. Sedegebi warmodgenilia grafikebisa da cxrilebis saxiT.

naxvretebis siaxloveSi Zabvebis koncentracias ikvleven a. guzi da misi TanamSromlebi [45]. [46] monografiaSi ganxilulia daZabul-deformirebuli mdgomareoba mravalSriani areebisaTvis. gamokvleulia filebis drekad-plastikuri mdgomareoba saboloo CaRunvebis gaTvaliswinebiT.

[47,48] monografiaSi sxva sakiTxebTan erTad ganixileba Zabvis intensivobis koeficientebi, maT Soris Termuli ZabvebisaTvis firfitebSi, romelTac aqvT wvetiani ubnebi da sxva defeqtebi, mrudwiruli Runvis, gaWimvisa da grexis pirobebSi. gaanalizebulia ZabvaTa intensivobis koefcientebze masalis anizotropulobis, gadamWreli Zalebis da ZabvaTa tenzoris arasimetriulobis gavlena. warmodgenilia gaangariSebis Sedegebis safuZvelze agebuli grafikebi.

teqnikis dargSi firfitebisa da filebis Teoriis gamoyenebis Sesaxeb

mTel rig SromebSi gaSuqebulia TviTmfrinavebisa da gemebis Txelkedliani konstruqciebis gaanagriSeba. am SromaTa nawili ukve ganxilulia Cvens mier. aRvniSnoT kidev ramodenime.

TviTmfrinavebis samSeneblo meqanikis Sesaxeb mTeli rigi Sromebi ekuTvnis i. obrazcovs [49,50]. i. obrazcovi iyo redkolegiis Tavmjdomare mTeli rigi gamocemebisa, romlebic eZRvneboda TviTmfrinavis samSeneblo meqanikis problemebs.

aRvniSnoT kidev ramodenime Sroma, romlebic aSuqeben TviTmfrinavTmSeneblobasTan dakavSirebul sakiTxebs. es aris k. balubaxis sami statia, sadac warmodgenilia TviTmfrinavis konstruqciis fragmentebis gaangariSeba. s. kanisa da i. panovkos da a. umanskis SromebSi aris oTxi Tavi, romlebSic warmodgenilia firfitebis Runvisa da mdgradobis sakiTxebi, maTi drekadobis miRma muSaobis CaTvliT, agreTve cilindruli garsebis Teoriis elementebi. meore tomSi ganixileba TviTmfrinavis frTis zogadi simtkice da sixiste. mesame tomSi sxva sakiTxebTan erTad warmodgenilia TviTmfrinavis hermetuli kabinis simtkicis sakiTxebi da misi nawilebis drekadi rxevebi. naSromSi TviTmfrinavis konstruqciis elementebis gaangariSebis garda, mocemulia fiuzelaJis da frTis konstruqciis mzidunarianobis gansazRvra. a. umanskim Seqmna Caketili ganivkveTis mqone Txelkedliani Reros gaangariSebis Teoria, romelic agreTve gamiznulia TviTmfrinavis konstruqciebisaTvis.

s. kanis naSromic exeba TviTmfrinavTmSeneblobas, Tumca ganixilavs Teoriis da gaangariSebis zogad sakiTxebs.

a. aleqsandrovis da l. kulSinis redaqciiT gamoqveynebul or krebulSi ganixileba samfenovani panelebis gaangariSeba saaviacio konstruqciebis elementebis daproeqtebis mizniT. am krebulebis gamocema win uswrebda specialurad samfenovani panelebisadmi miZRvnil naSroms.

aRsaniSnavia agreTve sami krebuli: (v. kabanovis redaqciiT), (i. obrazcovisa da a. volmiris redaqciiT) da (v. vasilievis redaqciiT), sadac warmodgenilia TviTmfrinavis konstruqciebis gaangariSeba. gamokvleulia arawrfivi deformacia da mdgradoba. ganxilulia safreni aparatebis saimedoobis problemebi maTi gamocdis, navigaciisa da manevrirebis SemTxvevaSi.

firfitebis daZabul-deformirebuli mdgomareobis gansazRvra da mdgradobis Sefaseba maTi gemis konstruqciebSi gamoyenebis TvalsazrisiT ganTavsebulia fundamentalur naSromSi `gemis samSeneblo meqanika” [10]. firfitebis gaanagriSebas eZRvneba agreTve i. Simanskis naSromi.

gemTmSeneblobaSi gamoyenebuli samfenovani konstruqciebisadmi wayenebul moTxovnebs exeba b. proxorovisa da v. kobalevis naSromi. masSi kerZod gamokvleulia defeqtebis gaCenis mizezebi da mocemulia samfenovani konstruqciebis teqnikur-ekonomikuri Sefaseba. gansakuTrebuli specifikisaa garsebi, romlebsac iyeneben mSeneblobaSi. am konstruqciebis gamoyenebis ZiriTadi sferoa SenobaTa saxuravebi. pirveli naSromi, romelSic garsebi ganixileboda am aspeqtSi, iyo f.

diSingeris [51]. Semdegi iyo i. Staermanis naSromi [52], ufro mogvianebiT gamovida misi naSromi `Seafrebuli filebi, daproeqteba da mSenebloba”.

Ddrekadobis da plastikurobis Teoriebis daxasiaTeba

drekadi sxeulebisaTvis damaxasiaTebeli Tvisebaa wrfivi damokidebuleba Zabvebsa da deformaciebs Soris. drekadobis zRvarze es damokidebuleba xSirad irRveva. igi icvleba mrudwiruli kanoniT, magram aq SesaZlebelia saqme gvqondes or garemoebasTan: neli gantvirTvisas sxeuli an daubrundeba Tavis pirvandel mdgomareobas, an adgili eqneba narCen, anu plastikur deformacias. sazogadod, myari sxeulebis deformacia Sedgeba ori nawilisagan – drekadi da plastikuri deformaciebi. SesaZlebelia viTardebodes drekadi deformaciebi da Semdeg plastikuri (foladi), an orive deformacias erTdoulad hqondes adgili (betoni).

plastikurobis Teoria swavlobs plastikuri sxeulebis deformirebul da ZabviT mdgomareobas. is adgens plastikuri deformaciebis da Zabvebis warmoSobis zogad kanonebs.

rogorc drekadobis TeoriaSi, aqac dgeba zogadi gantolebebi, romlebic erTimeoresTan akavSirebs deformaciebs da Zabvebs (drekad-plastikuri deformaciebis Teoria), an deformaciebis siCqareebs da Zabvebs (plastkuri denadobis Teoria). plastikurobis Teoriac iyofa or nawilad: plastikurobis matematikuri Teoria da plastikurobis gamoyenebiTi Teoria (sruliad im saxiT, rogorc iyofa drekadobis Teoria).

sxeulebis deformirebul da ZabviT mdgomareobas gansazRvravs ara marto drekadi da plastikuri Tvisebebi (ZiriTadad esenia mTavari Tvisebebi), aramed mniSneloba aqvs drois faqtorsac. Cven am faqtors vexebiT imdenad, ramdenadac masTan damokidebulebaSi (misi gavleniT) icvleba sxeulis meqanikuri Tvisebebi. drekadobisa da plastikurobis Teoriis Seswavlisas, drois aseT gavlenas mxedvelobaSi ar vRebulobT, risi ugulebelyofac bevri sxvadasxva xasiaTis amocanebis ganxilvisas SeuZlebelia.

cnobilia samSeneblo meqanikis statikuri da dinamikuri amocanebi. aseTive saxis amocanebs vswavlobT drekadobisa da plastikurobis TeoriaSic. dinamikuri amocanebi imiT xasiaTdeba, rom sxeulis wertilebs aqvT aCqareba, ris gamoc datvirTvebs emateba inerciis Zalebi. aCqareba ki drozea damokidebuli da, maSasadame, drois faqtoris ugulebelyofa aq ar SeiZleba, magram aseT SemTxvevbSi sxeulis meqanikuri Tvisebebi ar icvleba da amitom aq laparakia drois sul sxva gavlenaze, romelTanac saqme gvaqvs dinamikuri amocanebis Seswavlisas.

am saerTo SeniSvnis Semdeg SegviZlia davaxasiaToT sxeulis drekadi Tu plastikuri mdgomareoba.

myari sxeulis drekadi mdgomareoba ewodeba iseT mdgomareobas, rodesac sxeulis yoveli temperaturisaTvis, droisagan damoukideblad, arsebobs urTierT-calsaxa damokidebuleba Zabvebsa da deformaciebs Soris. es damokidebuleba Cveulebriv wrfivia da gamoisaxeba hukis kanoniT.

plastikuri mdgomareoba ewodeba myari sxeulis iseT mdgomareobas, rodesac mocemuli temperaturisaTvis kavSiri Zabvebsa da deformaciebs Soris drois yovel mocemul momentSi xdeba calsaxa, Tu cnobilia yvela misi winaTmyofi ZabviTi da deformirebuli mdgomareoba da temperaturis Sesabamisi mniSvnelobani, winaaRmdeg SemTxvevaSi es kavSiri ganusazRvrelia: [28] [55].

Ddrekadobisa da plastikurobis Teoriis ganviTarebis mokle mimoxilva

drekadobis Teoriis, rogorc calke mecnieruli disciplinis Camoyalibeba iwyeba me-19 saukunis pirveli meoTxedidan. drekadobis TeoriaSi pirvel Sromad iTvleba frangi mecnieris navies memuari, sadac mocemuli iyo drekadobis Teoriis wonasworobis zogadi diferencialuri gantolebani. imave periods ekuTvnis puasonis, koSis da ostrogradskis Sromebi. 1822 wels koSim miiRo drekadobis Teoriis ZiriTadi damokidebulebis da cnebebis umravlesoba. aRsaniSnavia, rom pirvelad koSim Semoitana mocemul wertilSi Zabvis cneba. puasonma Tavisi memuari (pirveli memuari) warudgina parizis akademias 1828 wels. is ixilavda drekadobis Teoriis mraval sakiTxs, romlebic ganxiluli iyo koSisa da mas navies mier da mas Camoyalibebuli hqonda drekadobis zogadi Teoria.

1829-1832 wlebSi ostrogradskim gamoaqveyna ramdenime Sroma drekadobis dinamikur TeoriaSi. amave mimarTulebiT muSaobda puasonic. amnairad naviem, koSim, puasonma da ostrogradskim safuZveli Cauyares matematikuri drekadobis Teorias.

1837 wels grinma energiis Senaxvis principis gamoyenebis safuZvelze sabolood daadgina, rom drekadobis damoukidebeli mudmivebis ricxvi aris 21. grinma aq Semoitana drekadi potencialis cneba, romlis arsebobis damtkiceba mogvca kelvinma 1855 wels.

cnobilia, rom hukma 1660w. aRmoaCina kanoni, romelic mis saxels atarebs, xolo iungma 1807 wels SemoiRo drekadobis modulis cneba. erTi mxriv am faqtebis gamoyenebam da meore mxriv im garemoebam, rom drekadobis Teoriis safuZvlebi ukve mocemuli iyo, saSualeba misca stokss (1845 w) ganesazRvra izotropuli sxeulebis winaRobis ZiriTadi saxeebi. aq man Semoitana moculobiTi kumSvis da Zvris modulis cnebebi.

me-19 saukunis meore da mesame meoTxeds ekuTvnis lamesa da klapeironis Sromebi, romlebic muSaobdnen peterburgSi. pirvelad maT mier iyo gamoyenebuli mwkrivebad daSlis meTodi drekadobis Teoriis amocanebis amosaxsnelad. cnobilia agreTve lames amocana, romelic sqelkedliani WurWlebis gaangariSebas exeba. am mimarTulebiT rus mecniers gadolins ekuTvnis udidesi mniSvnelobis Sromebi, romlebic man gamoaqveyna 1852-1854 wlebSi. gadolinma Seqmna saartilerio iaraRebis simtkiceze gaangariSebis Teoria. Tu navies, koSis, puasonis, ostrogradskis da sxvebis Sromebis safuZvelze ganviTarda maTematikuri drekadobis Teoria, lames, klapeironis, kirhxofis, klebSis, gadolinisa da bevri sxva mecnieris gamokvlevebiT viTardeboda gamoyenebiTi drekadobis Teoria.

saerTod, drekadobis Teoriis ganviTarebis saqmeSi gansakuTrebuli Rvawli miuZRvis sen-venans. man mogvca grexisa da Runvis amocanebis zusti amoxsna (1855 w), Camoayaliba principi, romelic mis saxels atarebs. treskas (1868 w) eqsperimentuli gamokvlevebis safuZvelze sen-venanma pirvelma Teoriulad daadgina plastikurobis piroba. sen-venans ekuTvnis drekadobis Teoriis sxva bevri sakiTxis Seswavla. 1850 wels kirhxofma daamuSava Txeli filebis gaangariSebis Teoria. manve 1859 w. mogvca grexisa da Runvis gansakuTrebuli SemTxvevebis amoxsna.

1872 w. betim drekadobis TeoriaSi pirvelad gamoiyena e.w. amoxsnis meTodi gansakuTrebuli wertilebiT. naxevarsivrceSi Zabvebis ganawilebis da Seyursuli Zalis mier `adgilobrivi SeSfoTebis” Teoria mogvca businekma 1878-1879 wlebSi. gercma (1882 w.) gamoikvlia ori sxeulis erTmaneTze dawolis problema. Tu zemoT dasaxelebuli da bevri sxva mecnieris naSromebi umeteswilad ekuTvnoda wminda drekadobis Teorias, amave dros paralelurad viTardeboda masalaTa gamZleoba, romlis safuZvelis Camyrelia d.i. Juravski. Juravskis, f.s. iasinskis, v.l. kirpiCevis, rankinis da sxvebis Sromebis safuZvelze Camoyalibda masalaTa gamZleoba, rogorc calke mecnieruli disciplina.

gansakuTrebiT unda ganvixiloT drekadobis Teoriis e.w. brtyeli amocanebi (organzomilebiani sistemis SemTxveva).

brtyeli amocanebis SeswavlaSi didi Rvawli miuZRvis moris levis. drekadobis TeoriaSi cnobilia gantoleba, romelic mis saxels atarebs.

SedarebiT ufro zogadi Teoria mocemuli iyo ribieris (1888 w) da failonis (1903 w.) mier. drekadobis Teoriis brtyeli amocanebis amoxsna SesaZlebelia kompleqsuri cvladis funqciebis gamoyenebiT. am mimarTulebiT cnobilia klebSis, liavis da failonis (1903 w.) Sromebi. gansakuTrebiT aRsaniSnavia g.v. kolosovis Sroma (1909 w.), romelSiac mocemulia brtyeli amocanebis amoxsnis meTodi analitikuri funqciebis Teoriis gamoyenebiT.

gansakuTrebiT didi mniSvneloba aqvs n. i. musxeliSvilis Sromebs, romelmac kompleqsuri cvladis funqciis Teoriis gamoyenebiT gaafarTova drekadobis Teoriis gamoyenebis sfero.

me-19 saukunis ukanaskneli meoTxedidan saerTod drekadobis Teoriis bevr sakiTxSi wamyvani roli ruseTis mecnierebma daikaves. fundamentalur warmatebas miaRwies Semdegma mecnierebma: n. s. golovinma, v. l. kirpiCevma, g. v. kolosovma,

i. g. bubnovma, b. g. galiorkinma, a. n. krilovma, n. a. beleliubskim, l. s. leibenzonma, a. n. dinnikma, c. v. papkoviCma, n. m. beliaevma. da sxva [28].

bubnovma daamuSava jaWvuri ZabvebiT Txeli filebis gaangariSebis Teoria, man pirvelma daayena sakiTxi, ritcis e.w. variaciuli da energetikuli meTodebisagan gansxvavebiT sxva, ufro moxerxebuli meTodis gamoyenebisa drekadobis Teoriis amocanebis amosaxsnelad. es meTodi daamuSava galiorkinma, romelic (galiorkinis variaciuli meTodi), mTeli msoflios bevri mecnieris kvlevis sagani iyo da aris. galiorkins ekuTvnis fundamentaluri Sromebi filebis gaangariSebis sakiTxSi. drekadobis Teoriis amocanebis (maT Soris brtyeli amocanebis) amoxsnis zogadi meTodebis damuSaveba da sxva. gemis samSeneblo meqanikis dargSi klasikuri Sromebis avtorebia krilovi, papkoviCi, bubnovi. drekadobis Teoriis ganviTarebasi didi Rvawli miuZRvis s. p. timoSenkos.

XX saukunis 20 wlebis Semdeg damuSavda brtyeli amocanebis amoxsnis zogadi meTodebi kompleqsuri cvladis funqciebis gamoyenebiT (musxeliSvili da misi skola); variaciuli meTodebi (galiorkini, leibenzoni, l. kantoroviCi, timoSenko, g. i. janeliZe, o.d. oniaSvili, a. m. kakuSaZe, da sxva); Txelkedliani Reroebis gaangariSebis Teoria (v. z. vlasovi, a. a. umanski, a. r. aladurovi, g. i. janeliZe da sxva), anizotropuli sxeulebis drekadobis Teoria (s. g. lexnicki, n. g. Cencovi da sxva), sqeli da Txeli filebis gaangariSebis meTodebi (galiorkini, papkoviCi, a. o. lurie, l. g. maRnaraZe, i. a. Simanski. a. s. kalmanoviCi, s. a. gerSgoriani da sxva); drekad fuZeze mdebare filebis gaangariSebis Teoria (n. m. gersevanovi, m. i. gorbunov-posadovi, goCkini da sxvebi).

sabWoTa kavSiris sazRvarsgareTeli mecnierebidan, SegviZlia davasaxeloT ritci, trefci, henki, liavi, neimani, veberi, fepli da bevri sxva.

me-19 saukunis meore naxevridan drekadobis Teoriis gverdiT viTardebida plastikurobis Teoriac. gaangariSebis zogadi meTodebi mogvces mxolod sabWoTa kavSiris mecnierebma. s.a. xristianoviCis, s. l. sobolevis, a. a. iliuSinis, v. v. sokolovskis, l. m. kaCanovis, v. i. novotorcevis, s. s. goluSkeviCis, a. a. gvozdevis da sxvebis Sromebis safuZvelze Seiqmna plastikurobis Teoria iseTive saxiT, rogoric drekadobis Teoriaa da amoixsna mravali amocana, romelTa amoxsna mxolod drekad mdgomareobaSi iyo cnobili. iliuSinis mier damuSavebuli iqna plastikurobis amocanebis amoxsnis sqemebi drekadobis Teoriis meTodebis gamoyenebiT. amiT moxda drekadobisa da plastikurobis Teoriis daaxloeba. aseTive daaxloeba warmoebs samSeneblo meqanikis sxva dargebisa drekadobis TeoriasTan. am mxriv aRsaniSnavia b. n. JemoCkinis da m. g. filonenko-borodiCis gamokvlevebi (drekad naxevarsivrceSi Tu naxevarsibrtyeze mdebare filebis da koWebis gaangariSeba), gvozdevis da a. a. rJanicinis Sromebi (plastikurobis Teoriis da samSeneblo meqanikis meTodebis Serwyma) da sxva [26].

Sedegebi da maTi gansja

Tavi 1 deformaciebsa da Zabvebs Soris damokidebulebebi

deformaciebsa da Zabvebs Soris damokidebulebebi aRwers myari sxeulis drekad da plastikur deformaciebs. SemdegSi droisa da temperaturis zemoqmedebas mxedvelobaSi ar miviRebT, magram mxedvelobaSi miviRebT ganmtkicebas. garda amisa davuSvebT, rom myari masala izotropulia da bauSingeris efeqti SeiZleba ugulebelvyoT [55].

unda aRiniSnos, rom kuTxuri deformaciis (γ_{xy}, γ_{yz}, γ_{zx}) tenzoris mdgenelebis mniSvnelobebi tolia da aris Sesabamisi fardobiTi kuTxuri deformaciebis mniSvnelobaTa naxevrebi.

1.1. deformaciebsa da Zabvebs Soris damokidebulebani drekadobis Teoriis mixedviT. prandtl-reisis gantolebebi. levi-mizesisi gantolebebi.

Ddeformaciebsa da Zabvebs Soris damokidebulebani drekadobis Teoriis mixedviT.

izotropuli myari masalisaTvis Tanafardobani Zabvebsa da deformaciebs Soris drekadobis TeoriaSi warmodgeba Semdegi saxiT [23]:

$$
\begin{cases}
E \cdot e_x = \sigma_x - \nu\left(\sigma_y + \sigma_z\right) \\
E \cdot e_y = \sigma_y - \nu\left(\sigma_z + \sigma_x\right) \\
E \cdot e_z = \sigma_z - \nu\left(\sigma_x + \sigma_y\right) \\
2G\gamma_{yz} = \tau_{yz} \\
2G\gamma_{zx} = \tau_{zx} \\
2G\gamma_{xy} = \tau_{xy}
\end{cases}
\qquad (1.1.1)
$$

sadac: E – iungis modulia;

ν - puasonis koeficientia;

G – Zvris moduli.

mocemul sistemaSi mosaxerxebelia ganvasxvavoT formisa da moculobis cvlilebasTan dakavSirebuli deformaciebi, Tu σ_m -hidrostatikuri wnevaa, e_m –

Sesabamisi moculobiTi deformacia, maSin drekadobis mudmivebis gamoyenebiT (1.1.1) sistema SeiZleba warmovadginoT Semdegi saxiT:

$$\begin{cases} e_x = \dfrac{1}{2G}\left(\sigma_x - \sigma_m\right) + \dfrac{\left(1-2\nu\right)}{E}\sigma_m \\ e_y = \dfrac{1}{2G}\left(\sigma_y - \sigma_m\right) + \dfrac{\left(1-2\nu\right)}{E}\sigma_m \\ e_z = \dfrac{1}{2G}\left(\sigma_z - \sigma_m\right) + \dfrac{\left(1-2\nu\right)}{E}\sigma_m \\ \gamma_{yz} = \dfrac{\tau_{yz}}{2G} \\ \gamma_{zx} = \dfrac{\tau_{zx}}{2G} \\ \gamma_{xy} = \dfrac{\tau_{xy}}{2G} \end{cases} \tag{1.1.2}$$

sadac $3\sigma_m = \sigma_x + \sigma_y + \sigma_z$ da $e_m = e_x + e_y + e_z = \dfrac{\sigma_m}{K}$ $K = \dfrac{E}{3\left(1-2\nu\right)}$;

$\left(\sigma_x - \sigma_m\right)$ da sxva - Semcirebuli Zabvebia, anu Zabvebis deviatoris komponentebi, romlebsac avRniSnavT $\left(\sigma'_x\right)$-iT, maSin miviRebT ormag indeqsacias da sruli Tanafardoba Zabvebs da deformaciebs Soris SeiZleba Caiweros rogorc:

$$\left.\begin{aligned} e_{ij} &= \frac{\sigma'_{ij}}{2G} + \frac{\left(1-2\nu\right)}{E}\delta_{ij}\sigma_m \\ \sigma_m &= \frac{1}{3}\sigma_{ii} \end{aligned}\right\} \tag{2.2.3}$$

kronekeris simbolo (delta-simbolo) $\delta_{ij} = 1$ roca i=j da $\delta_{ij} = 0$ roca $i \neq j$.

prandtl-reisis gantolebebi.

pirvelad idealuri drekad-plastikuri myari sxeulebis brtyeli deformaciebisas Zabva-deformaciis Tanafardobebi warmodgenili iyo 1924w. prandtlis mier, xolo 1930w. reisma warmoadgina igive gantolebebi zogadi saxiT. prandtlis gantolebebi aris levi-mizesis gantolebis ganzogadeba, romelic mocemulia qvemoT (1.1.10).

reiss daSvebuli hqonda, rom plastikuri deformaciis nazrdebi (romlebic gantolebaSi aRniSn. P zeda indeqsiT) drois nebismier momentSi proporciulia erTdroulad moqmedi deviatoruli da mxebi Zabvebisa:

$$\left.\frac{d\varepsilon_x^p}{\sigma_x'}=\frac{d\varepsilon_y^p}{\sigma_y'}=\frac{d\varepsilon_z^p}{\sigma_z'}=\frac{d\gamma_{yz}^p}{\tau_{yz}}=\frac{d\gamma_{zx}^p}{\tau_{zx}}=\frac{d\gamma_{xy}^p}{\tau_{xy}}=d\lambda\right\} \qquad (1.1.4)$$

an

$$d\varepsilon_{ij}^p=\sigma_{ij}'d\lambda$$

sadac $d\lambda$ - proporciulobis dadebiTi konstantaa, romelic zogjer Seicvleba deformaciasTan erTad.

gantolebebidan Cans, rom deformaciis mcire nazrdi damokidebulia deviatoris komponentebis mimdinare mniSvnelobebze da ara misi gamomwvevi Zabvebis nazrdze. plastikuri deformaciis nazrdisa da Zabvebis mTavari RerZebi erTmaneTs emTxveva. gantolebebi warmodgenas iZleva plastikuri deformaciis nazrdze x; y; z RerZebis mimarTulebiT da ar iZleva uSualo informaciebs maT absolutur sidideze.

sruli deformaciis nazrdi aris jami drekadi deformaciis (romelsac axla $d\varepsilon$-s nacvlad avRniSnavT $d\varepsilon^e$) da plastikuri deformaciis nazrdebisa. amgvarad (1.1.3) da (1.1.4) gantolebebidan miviRebT:

$$d\varepsilon_{ij}=d\varepsilon_{ij}^p+d\varepsilon_{ij}^e=\sigma_{ij}'d\lambda+\frac{d\sigma_{ij}'}{2G}+\frac{(1-2\nu)}{E}\delta_{ij}d\sigma_m \qquad (1.1.5)$$

ramdenadac plastikuri deformacia ar iwvevs sxeulis moculobis cvlilebas, gamoviyenebT ra mTavar normalur deformaciebs SeiZleba CavweroT piroba:

$$d\varepsilon_1^p + d\varepsilon_2^p + d\varepsilon_3^p = d\varepsilon_x^p + d\varepsilon_y^p + d\varepsilon_z^p = 0 \qquad (1.1.6)$$

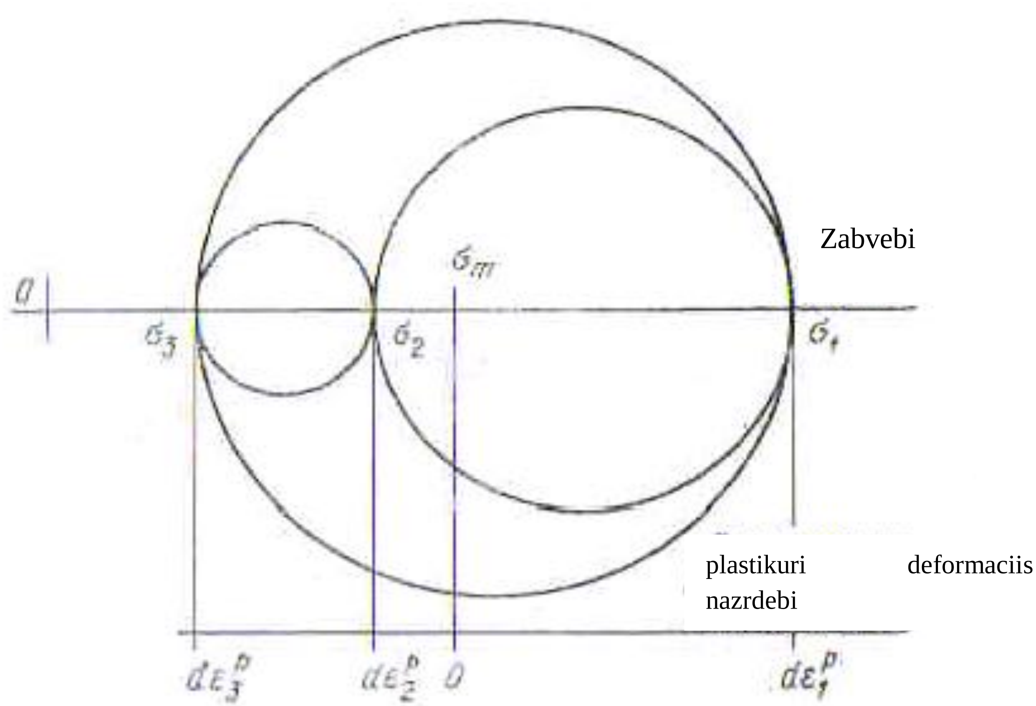
Aan

$$d\varepsilon_{ii}^p = 0$$

(1.1.4) gantolebidan mTavari ZabvebisaTvis miviRebT:

$$\frac{d\varepsilon_1^p - d\varepsilon_2^p}{\sigma_1 - \sigma_2} = \frac{d\varepsilon_2^p - d\varepsilon_3^p}{\sigma_2 - \sigma_3} = \frac{d\varepsilon_3^p - d\varepsilon_1^p}{\sigma_3 - \sigma_1} = d\lambda \qquad (2.1.7)$$

zemoT aRniSnul gantoleba (1.1.7) amtkicebs, rom moris wreebi ZabvebisaTvis da plastikuri deformaciis nazrdebisaTvis msgavsia. (nax. 1.1.1)



nax. 1.1.1 Mmoris wreebi Zabvebisa da Pplastikur deformaciaTa nazrdebisaTvis

(1.1.4) formulas Tu CavwerT normaluri Zabvebis aRniSvnebis gamoyenebiT, miviRebT:

$$d\varepsilon_x^p = \frac{2}{3} d\lambda \left[\sigma_x - \frac{1}{2}\left(\sigma_y + \sigma_z\right) \right]$$

tipis sam gantolebas.

xolo (1.1.5) gantoleba daiSleba sam gantolebad:

$$d\varepsilon_x = \frac{2}{3} d\lambda \left[\sigma_x - \frac{1}{2}\left(\sigma_y + \sigma_z\right) \right] + \left[d\sigma_x - \nu\left(d\sigma_y + d\sigma_z\right) \right] \Big/ E$$
$$d\varepsilon_y = \frac{2}{3} d\lambda \left[\sigma_y - \frac{1}{2}\left(\sigma_x + \sigma_z\right) \right] + \left[d\sigma_y - \nu\left(d\sigma_x + d\sigma_z\right) \right] \Big/ E$$
$$d\varepsilon_z = \frac{2}{3} d\lambda \left[\sigma_z - \frac{1}{2}\left(\sigma_x + \sigma_y\right) \right] + \left[d\sigma_z - \nu\left(d\sigma_x + d\sigma_y\right) \right] \Big/ E$$

da sam Semdegi saxis gantolebad:

$$\begin{aligned} d\gamma_{yz} &= \tau_{yz} d\lambda + d\tau_{yz} \big/ 2G \\ d\gamma_{xz} &= \tau_{xz} d\lambda + d\tau_{xz} \big/ 2G \\ d\gamma_{xy} &= \tau_{xy} d\lambda + d\tau_{xy} \big/ 2G \end{aligned} \qquad (1.1.8)$$

da bolos, (1.1.5)-is gantolebis Ggamokvlevidan SeiZleba aRiniSnos, rom deformaciis sruli nazrdebis gamosaxulebidan SeiZleba gamovyoT moculobiTi deformaciisa da deformaciis deviatoris nazrdebi. mizesis plastikurobis pirobis gaTvaliswinebiT prandtl-reisis gantolebebi SeiZleba CaviweroT Semdeg saxiT:

$$\left.\begin{aligned} d\varepsilon'_{ij} &= \sigma'_{ij} d\lambda + d\sigma'_{ij}/2G \\ d\varepsilon_{ii} &= \frac{(1-2\nu)}{E} d\sigma_{ii} \\ \sigma'_{ij}\sigma'_{ij} &= 2k^2 \end{aligned}\right\} \qquad (2.1..9)$$

sadac k denadobis zRvaria sufTa Zvrisas.

xSirad zemoT _ moyvanili gantolebebi drekad-plastikuri Mmyari sxeulebisaTvis praqtikulad Znelad gamosayenebelia da amitom mcirea maTi gamoyenebis SemTxvevebi. amitom gamartivebis mizniT mniSvnelovani sasruli plastikuri deformaciebis dros drekadoba SeiZleba ugulebelvyoT. Mmasala am SemTxvevaSi ganixileba rogorc idealurad xist-plastikuri. Aam SemTxvevaSi denadobis zRvarze naklebi ZabvebisaTvis deformireba ar xdeba, xolo sruli da plastikuri Ddeformaciebis nazrdebi emTxveva erTmaneTs. aseTi SemTxvevisaTvis damokidebulebebi Zabvebsa da deformaciebs Soris warmoadgines levim da mizesma.

levi-mizesis gantolebebi

vwerdiT ra Tanafardobebs Zabvebsa da deformaciebs Soris, Cven movlenaTa istoriul ganviTarebas ar mivyvebodiT. swored exla gveCveneba yvelaze ufro logikurad, ganvixiloT levi-mizesis gantolebebi, rogorc prandtl-reisis gantolebis kerZo SemTxveva. Tumca pirveli vinc ivarauda, rom deformaciaTa nazrdebis mTavari RerZebi ZabvaTa mTavar RerZebs emTxveva, iyo sen-venani (1870w.)

Tanafardoba deformaciebis nazrdebsa da ZabvaTa deviatoris komponentebs Soris zogadi saxiT pirvelad ganixiles levim (1871w.) da misgan damoukideblad mizesma (1913w.) axla es gantolebebi levi-mizesis saxels atareben, da isini SeiZleba Caiweros ase:

$$\frac{d\varepsilon_x}{\sigma'_x} = \frac{d\varepsilon_y}{\sigma'_y} = \frac{d\varepsilon_z}{\sigma'_z} = \frac{d\gamma_{yz}}{\tau_{yz}} = \frac{d\gamma_{zx}}{\tau_{zx}} = \frac{d\gamma_{xy}}{\tau_{xy}} = d\lambda \qquad (1.1.10)$$

zeda indeqsi p (1.1.4)-dan SeiZleba gamovtovoT, radganac axla sruli da plastikuri deformaciebis nazrdebi erTi da igivea.

sruli Zabvebis aRniSvnebis gamoyenebiT Cawerili levi-mizesis Tanafardoba Seicavs:

$$d\varepsilon_x = \frac{2}{3}d\lambda\left[\sigma_x - \frac{1}{2}\left(\sigma_y + \sigma_z\right)\right]$$
$$d\varepsilon_y = \frac{2}{3}d\lambda\left[\sigma_y - \frac{1}{2}\left(\sigma_x + \sigma_z\right)\right]$$
$$d\varepsilon_z = \frac{2}{3}d\lambda\left[\sigma_z - \frac{1}{2}\left(\sigma_x + \sigma_y\right)\right]$$

sam gantolebas da

$$\begin{aligned} d\gamma_x &= \tau_{yz}d\lambda \\ d\gamma_y &= \tau_{xz}d\lambda \\ d\gamma_z &= \tau_{xy}d\lambda \end{aligned} \qquad (1.1.11)$$

sam gantolebas.

vinaidan levi-mizesis gantolebebi drekad deformaciebs ar iTvaliswinebs, drekad narCen Zabvebze informaciis misaRebad maTi gamoyeneba ar SeiZleba. Aam SemTxvevaSi saWiroa visargebloT prandtl-risis ufro rTuli gantolebebiT.

1.2. masalis ganmtkiceba

Tuki masala eqvemdebareba civ damuSavebas, maSin damuSavebis Semdeg is ufro mtkice xdeba, e.i. deformaciisas izrdeba misi winaaRmdegoba formis Semdgomi cvlilebisadmi, amitom varaudoben rom ganmtkicebis xarisxi damokidebulia plastikuri deformaciis dros Sesrulebul muSaobaze da ara deformaciis gzebze. Aamas ewodeba plastikuri deformaciis muSaobis eqvivalenti. sxva sityvebiT rom vTqvaT, Semdgomi deformirebisadmi winaaRmdegoba damokidebulia Tavdapirvelad ganmtkicebul mdgomareobaSi myof sxeulze Sesrulebul plastikuri deformaciis muSaobaze. winaaRmdegobis es cvlileba fasdeba plastikurobis pirobiT.

dadgenilia, rom mizesis plastikurobis piroba aris gacilebiT martivi Tanafardoba, romelic kargad akmayofilebs eqsperimentul monacemebs winaswari deformaciis xarisxis gauTvaliswineblad. Aam pirobiT plastikurobis wertilTa saboloo geometriuli adgili ar aris damokidebuli hidrostatikur wnevaze. mTavari Zabvebis (σ_1,σ_2,σ_3) gamoyenebiT es SeiZleba ase CavweroT:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=6k^2$$

sadac k aris denadobis Zabva Zvrisas, da is damokidebulia Tavdapirvel deformaciaze.

hilma (1950w.) isargebla izotropuli ganmtkicebis wesiT, romlis mixedviT ivaraudeba, rom plastikuri deformaciis procesSi plastikurobis zedapiri farTovdeba, magram Tavdapirveli forma da mdebareoba hidrostatikuri wneviT gansazRvruli xazis mimarT ar icvleba. 1955w. pragerma SemogvTavaza ganmtkicebis wesi: plastikurobis zedapiri inarCunebs Tavis zomebs, magram gadaadgildeba ZabvaTa sivrceSi deformaciis nazrdis mimarTulebiT. Ees meore wesi pirvelisagan gansxvavebiT iTvaliswinebs bauSingeris efeqts, magram is gacilebiT rTulia maTematikuri TvalsazrisiT. Aamitom dasaSvebia plastikurobis piroba CavweroT Semdegi saxiT:

$$\bar{\sigma}=\sqrt{\frac{1}{2}\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)\right\}^2}=$$
$$=\sqrt{\frac{3}{2}\left(\sigma_1'^2+\sigma_2'^2+\sigma_3'^2\right)} \qquad (1.2.1)$$

sadac $\bar{\sigma}$ aris ZabvaTa intensivoba an eqvivalenturi Zabva.

ricxviTi koeficienti SerCeulia ise, rom martivi gaWimvisaTvis $\bar{\sigma}=Y$. Y denadobis zRvaria sufTa gaWimvisas Zabvis intensivoba $\bar{\sigma}$ zemoT SemoTavazebuli winadadebiT aris plastikuri deformaciis sruli muSaobis funqcia:

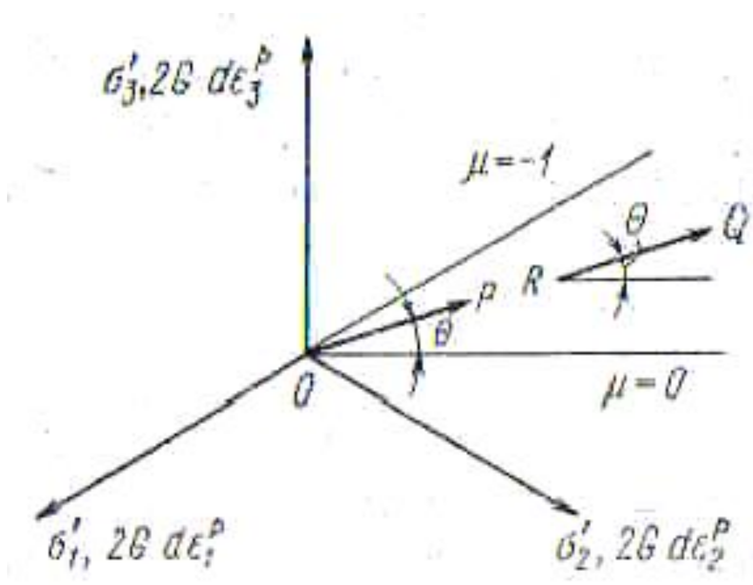
$$\bar{\sigma} = F(W_p) \qquad (1.2.2)$$

es toloba samarTliania mxolod idealuri liTonisaTvis, roca deformacia izotropulia da bauSingeris efeqti gamoricxulia. Aamis garda, plastikurobis pirobiT ivaraudeba, rom hidrostatikuri wneva ar iwvevs plastikur deformaciebs, amitom moculobis TandaTanobiT cvlilebas adgili ara aqvs. es mtkiceba eqsperimentuli monacemebiT. formis cvlilebaze daxarjuli muSaobis nazrdi erTeul moculobaSi tolia

$$dW_p = \sigma_1' d\varepsilon_1^p + \sigma_2' d\varepsilon_2^p + \sigma_3' d\varepsilon_3^p \qquad (1.2.3)$$

radganac $d\varepsilon_1^p + d\varepsilon_2^p + d\varepsilon_3^p = 0$ narCeni deformaciis nazrdi SeiZleba warmovadginoT veqtoriT π sibrtyeSi. Tu Zabvis ganzomilebisaTvis SemovitanT 2G faqtors, narCeni deformaciis nazrdis veqtori, SeiZleba gamoisaxos igive diagramaze, romelzec Zabvis deviatoris veqtoria gamosaxuli. vinaidan ivaraudeba, rom plastikuri deformaciis nazrdis mTavari RerZebi emTxveva Zabvebis mTavar RerZebs, (nax. 1.2.1)-ze Zabvis $\overrightarrow{OP}$ veqtori paraleluria plastikuri deformaciis nazrdis $\overrightarrow{RQ}$ veqtoris

da $dW_p = \dfrac{\overrightarrow{OP} \cdot \overrightarrow{RQ}}{2G}$



nax. 1.2.1

$$\left.\begin{array}{l} |OP| = \sqrt{\left(\sigma_1^{'2} + \sigma_2^{'2} + \sigma_3^{'2}\right)} = \sqrt{\frac{2}{3}}\bar{\sigma} \\ |RQ| = 2G\sqrt{d\varepsilon_1^{p2} + d\varepsilon_2^{p2} + d\varepsilon_3^{p2}} = 2G\sqrt{\frac{3}{2}}3\overline{d\varepsilon^{p}} \end{array}\right\} \qquad (1.2.4)$$

narCeni deformaciis nazrdis mdgenelebis invariantuli funqcia $d\bar{\varepsilon}^{p}$ iseve, rogorc ZabvaTa deviatoris komponenti σ, SeiZleba ase Caiweros:

$$d\bar{\varepsilon}^{p} = \sqrt{\frac{2}{9}\left[\left(d\varepsilon_1^{p} - d\varepsilon_2^{p}\right)^2 + \left(d\varepsilon_2^{p} - d\varepsilon_3^{p}\right)^2 + \left(d\varepsilon_3^{p} - d\varepsilon_1^{p}\right)^2\right]} \qquad (1.2.5)$$

ase rom

$$dW_p = \bar{\sigma} d\bar{\varepsilon}^{p} \qquad (1.2.6)$$

axla ganmtkicebis hipoTeza (1.2.2) SeiZleba ase Caiweros:

$$\bar{\sigma} = F\left(\int \bar{\sigma} d\bar{\varepsilon}^{p}\right)$$

aqedan gamomdinareobs, rom $\bar{\sigma}$ mxolod $d\bar{\varepsilon}^{p}$-is funqciaa, sadac intervals deformirebis procesSi viRebT. amitom

$$\bar{\sigma} = H\int d\bar{\varepsilon}^{p} \qquad (1.2.7)$$

Cveulebriv am gamosaxulebiT sargebloba mosaxerxebelia, vidre (1.2.3)-iT. (1.2.3) da (1.2.7)-is amoxsniT SeiZleba miviRoT gansxvavebuli Sedegebi anizotropulobis da bauSingeris efeqtis gamo. Tu Cven saqme gvaqvs masalasTan, romelic Zalian axloa xist-plastikurTan, maSin (1.2.5) toloba SeiZleba ase Caiweros:

$$d\bar{\varepsilon} = \sqrt{\frac{2}{9}\left[\left(d\varepsilon_1 - d\varepsilon_2\right)^2 + \left(d\varepsilon_2 - d\varepsilon_3\right)^2 + \left(d\varepsilon_3 - d\varepsilon_1\right)^2\right]} \qquad (1.2.8)$$

radgan plastikuri deformaciis nazrdi emTxveva mTliani deformaciis nazrds. Tu momdevno deformaciis nazrdis mTavari RerZebi ar Semobrundeba deformaciis qveS myofi elementis mimarT, maSin nazrdebis fardoba iqneba mudmivi:

$$\frac{d\varepsilon_2}{d\varepsilon_1} = x; \ \frac{d\varepsilon_3}{d\varepsilon_1} = y$$

gvaqvs: $d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$, x+y+1=0 an y=-(1+x)

ase rom $d\bar{\varepsilon}^{2} = \frac{2}{3}d\varepsilon_1^2\left[1 + x^2 + y^2\right]$ an $d\bar{\varepsilon} = \frac{2}{\sqrt{3}}d\varepsilon_1\left[1 + x + x^2\right]^{\frac{1}{2}}$

integrebis Semdeg miviRebT:

$$\bar{\varepsilon} = \frac{2}{\sqrt{3}}\left[1 + x + x^2\right]^{\frac{1}{2}}\varepsilon_1 \qquad (1.2.9)$$

amitom (1.2.7) Caiwereba Semdegi saxiT:

$$\bar{\sigma} = H\left(\bar{\varepsilon}\right) \qquad (1.210)$$

xolo ukumSvelobis gantoleba Semdegnairad:

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$$

vsargeblobT ra (1.2.7), (1.2.9), (1.2.2) da (1.2.5) formulebiT, sruli damokidebulebebi Zabvebsa da deformaciebs Soris SeiZleba Caiweros Semdegi saxiT:

$$\left.\begin{aligned} d\varepsilon'_{ij} &= \frac{3}{2}\sigma'_{ij}\frac{d\bar{\varepsilon}^{p}}{\bar{\sigma}} + \frac{d\sigma'_{ij}}{2G} \\ d\varepsilon_{ii} &= \frac{(1-2\nu)}{E}d\sigma_{ii} \\ \sigma'_{ij}\sigma'_{ij} &= 2k^{2} \end{aligned}\right\} \qquad (1.2.11)$$

(1.2.11) gamosaxuleba Seicavs iseT gantolebebs, rogoricaa

$$\begin{aligned} d\varepsilon_{x} &= \left[\sigma_{x} - \frac{1}{2}\left(\sigma_{y} + \sigma_{z}\right)\right]\frac{d\bar{\varepsilon}^{p}}{\bar{\sigma}} + \frac{1}{E}\left[d\sigma_{x} - \nu\left(d\sigma_{y} + d\sigma_{z}\right)\right] \\ d\gamma_{yz} &= \frac{3}{2}\tau_{yz}\frac{d\bar{\varepsilon}^{p}}{\bar{\sigma}} + \frac{d\tau_{yz}}{2G} \end{aligned} \qquad (1.2.12)$$

ufro zogadi formiT (1.2.11) gamosaxuleba ganixila hilma (1950w).

Tu saerTod ugulvebelvyobT ganmtkicebas da davuSvebT, rom denadobis Zabva masalisaTvis aris raime mudmivi sidide Y maSin narCeni deformaciis nazrdebis gamosaxuleba (1.1.4) Caiwereba Semdegi saxiT:

$$d\varepsilon_{ij}^{p} = \frac{3}{2}\sigma'_{ij}\frac{d\bar{\varepsilon}^{p}}{Y} \qquad (1.2.13)$$

(1.2.12)-dan gamomdinare $\bar{\sigma} = Y$. (1.2.13) gamosaxuleba pirvelad SemogvTavaza hilma. Ees ubralod aris prandtl-reisis gantoleba, Cawerili ufro mosaxerxebeli formiT.

roca SesaZlebelia drekadi deformaciebis ugulebelyofa, Zabvebsa da deformaciebs Soris damokidebulebebs levi-mizesis gantolebiT aRwerili masalisaTvis eqnebaT saxe:

$$d\varepsilon_{ij} = \frac{3}{2}\sigma'_{ij}\frac{d\bar{\varepsilon}^{p}}{\bar{\sigma}} \qquad (1.2.14)$$

es gantoleba Sedgeba Semdegi sami gantolebisagan:

$$d\varepsilon_x = \left[\sigma_x - \frac{1}{2}\left(\sigma_y + \sigma_z\right)\right]\frac{d\bar{\varepsilon}}{\bar{\sigma}}$$
$$d\varepsilon_y = \left[\sigma_y - \frac{1}{2}\left(\sigma_x + \sigma_z\right)\right]\frac{d\bar{\varepsilon}}{\bar{\sigma}}$$
$$d\varepsilon_z = \left[\sigma_z - \frac{1}{2}\left(\sigma_x + \sigma_y\right)\right]\frac{d\bar{\varepsilon}}{\bar{\sigma}}$$

da Semdegi sami gantolebisagan:

$$d\gamma_{yz} = \frac{3}{2}\tau_{yz}\frac{d\bar{\varepsilon}}{\bar{\sigma}}$$
$$d\gamma_{xz} = \frac{3}{2}\tau_{xz}\frac{d\bar{\varepsilon}}{\bar{\sigma}} \qquad (1.2.15)$$
$$d\gamma_{xy} = \frac{3}{2}\tau_{xy}\frac{d\bar{\varepsilon}}{\bar{\sigma}}$$

1.3. zogadi deformaciuli Teoria. henkis gantoleba.

aRmoCnda, rom plastikurobis Teoriis yvela problemas aqvs diferencialuri xasiaTi. Ghenkis gantolebebi (1924w.) aris mcdeloba gavafarTovoT drekadobis saerTo Teoria plastikurobis Teoriamde. maTSi mtkicdeba, rom

$$\varepsilon^{p}_{ij} = \varphi \cdot \sigma'_{ij} \qquad (1.3.1)$$

e.i sruli plastikuri deformaciis mdgenelebi, reiesis gantolebisagan gansxvavebiT, proporciulia Zabvebis deviatoris komponentebisa, henkis gantolebebi SeiZleba Caiweros Semdegi saxiT:

$$
\left.\begin{aligned}
\varepsilon'_{ij} &= \left(\varphi + \frac{1}{2G}\right)\sigma'_{ij} \\
\varepsilon_{ii} &= \frac{(1-2\nu)}{E}\sigma_{ii}
\end{aligned}\right\} \qquad (1.3.2)
$$

sadac φ skalaruli sididea, dadebiTi _ datvirTvisas da nulis toli gantvirTvisas. (1.3.2) formulidan gamodis, rom Tu mocemulia Zabvis sidide raime wertilSi, maSinve SesaZlebelia ganisazRvros mTliani deformacia. naTelia, rom es asea mxolod maSin, roca Zabva-deformaciis damokidebuleba rCeba mudmivi. (gansakuTrebuli SemTxvevebis dros). aseT SemTxvevebSi henkis gantolebebi gamomdinareobs reisis gantolebebis integrebidan. aq Zalian samarTliania hilis SeniSvna (1950w), romelmac aRwera, rom `Zalian iolia aCveno, rom henkis gantoleba mouxerxebelia idealuri plastikuri liTonebis qcevis aRwerisaTvis. vivaraudoT, rom gansazRvruli plastikuri deformaciebis Semdeg elementi nawilobriv an mTlianad ganitvirTeba, xolo Semdeg isev daitvirTeba sxva daZabuli mdgomareobiT plastikurobis imave zedapirze. Tu Zabvis maxasiaTebeli wertili ekuTvnis plastikurobis wertilTa geometriul adgils, SesaZlebelia moxdes mxolod drekadi deformaciebi, xolo saerTo plastikuri deformacia ar icvleba. (1.3.1) gantolebis Tanaxmad ki narCeni deformaciebis Sefardebebi sruliad gansxvavebulia, radganac Seicvala daZabuli mdgomareoba es niSnavs, rom TviTon plastikuri deformacia Seicvala gantvirTvisa da xelaxali datvirTvis dros, rac absurdia”.

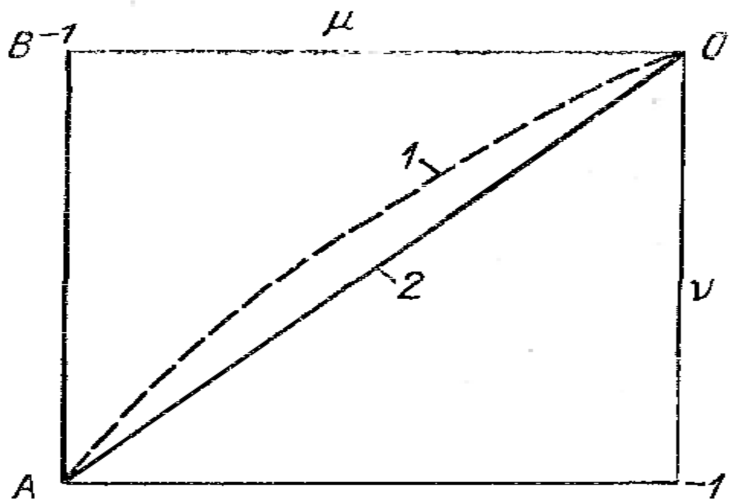
Ppirveli eqsperimentebi Zabva-deformaciis Tanafardobebis gamosakvlevad Caatara lodem (1926w) man Semoitana ori parametri:

$$
\mu = \frac{(\sigma_3 - \sigma_1) - (\sigma_2 - \sigma_3)}{\sigma_1 - \sigma_2} \qquad (1.3.3)
$$

$$
\nu = \frac{\left(d\varepsilon_3^p - d\varepsilon_1^p\right) - \left(d\varepsilon_2^p - d\varepsilon_3^p\right)}{d\varepsilon_1^p - d\varepsilon_2^p} \qquad (1.3.4)
$$

advilad SesamCnevia, rom Tu Teoriulad miRebuli gantoleba (1.1.7) sworia, maSin μ unda iyos ν-s toli (aq ar unda avurioT lodes parametri ν puasonis koeficientSi).

Tu $\mu=-1$, $\sigma_2=\sigma_3$, maSin daZabuli mdgomareoba Seesabameba erTRerZa gaWimvas (σ_1-σ_2) hidrostatikuri wneviT - σ_2. Tu $\mu=0$, $\sigma_3=(\sigma_1+\sigma_2)/2$, maSin is Seesabameba sufTa Zvris mdgomareobas $[(\sigma_1-\sigma_2)/2,\ (\sigma_2-\sigma_1)/2]$ hidrostatikuri wneviT $(\sigma_1+\sigma_2)/2$. sufTa Zvrisa da erTRerZa gaWimvis pirobiT gansazRvruli wertilebi, romlebSic $\mu=\nu$, gvaZleven wrfes OA. (nax. 1.3.1).



nax. 1.3.1 eqsperimentuli (1) da Teoriuli (2) mrudebi levi-lodes koeficientis cvlilebisaTvis

gamokvlevis meTodi martivdeba, Tu eqsperimentSi Zabvebis Tanafardobebi darCeba mudmivi deformaciis mTeli drois manZilze. es amcirebs deformaciis drekad mdgenels minimumamde da winaswar sustad deformirebuli masalebisaTvis is SeiZleba ukuvagdoT. aseT SemTxvevaSi ν SeiZleba gamovsaxoT sruli deformaciebiT

$$\nu=\frac{(\varepsilon_3-\varepsilon_1)-(\varepsilon_2-\varepsilon_3)}{\varepsilon_1-\varepsilon_2} \qquad (1.3.5)$$

yvela cda lodes parametrebis gansazRvrisaTvis Sesrulda brtyeli deformirebis pirobebSi. lode (1926w) Tavis eqsperimentebSi zemoqmedebda Txelkedlian rkinis, spilenZis da nikelis milebze erTdroulad gaWimviTa da Signidan (wneviT). yvela cdis dros Tanafardoba RerZul da mxeb Zabvebs Soris rCeboda TiTqmis mudmivi. miuxedavad mniSvnelovani gabnevis, (anizotropulobis gamo). Sedegebma gviCvena gadaxra xazidan. sistematuri gadaxrebi iyo aRniSnuli teiloris da qvinis (1931w) mier, romlebic zemoqmedebdnen aluminis, spilenZis da rbili foladis Txelkedlian milebze erTdrouli gaWimviTa da grexviT. maT daadgines milebis anizotropulobis dasaSvebi zRvari. am cdebSi maqsimaluri datvirTva rCeboda mudmivi, xolo mgrexavi momenti izrdeboda. Aaqedan gamomdinare, (Zabvebis Tanafardoba ar iyo mudmivi) (1.3.5) formulis gamoyeneba ar SeiZleboda.

Ppuxma (1953w) aCvena, rom SeuZlebelia darwmunebuli viyoT Txelkedliani milebis absolutur izotropulobaSi. Aam siZnelis gadasalaxad pilma eqsperimentulad gamoikvlia damokidebuleba Zabvebsa da deformaciebs Soris dagraduirebuli zoliT, romelic iZleva SesaZleblobas mivsdioT masalis anizotropulobis cvlilebas. praqtikaSi es idea gamoiyenes handim da grinma (1954w). maT miiRes levi-mizesis gantolebis damadasturebeli Sedegebi.

hohenemzeris (1931w) da morisonis, Sefherdis (1950w) eqsperimentebSi gamoyenebuli iyo Txelkedliani mili, romelzec zemoqmedebdnen mgrexavi da gamWimavi Zalebis sxvadasxva kombinaciebiT. Aam cdebma aCvena, rom drekadi da plastikuri deformaciebi iyo erTi rigis, e.i. daadastures prantdl-reisis gantolebebis samarTlianoba, magram es piroba araa yovelTvis samarTliani lodes cvladebis gansazRvrisaTvis [55].

1.4. plastikuri potenciali da denadobis Teoriis gantolebebi.

Aaqamde Cven vixilavdiT mxolod mizesis plastikurobis pirobas da levi-mizesis gantolebebs denadobis TeoriisTvis, ramdenadac isini mtkicdeba eqsperimentuli monacemebiT. Aaxla ganvixilavT denadobis zRvars da plastikur denadobas ufro farTo saxiT, visargeblebT ra plastikuri potencialis cnebiT. SemoTavazebulia hipoTeza, rom plastikuri potenciali g(σ_{ij}) Zabvebis skalaruli funqciaa, romlis kerZo warmoebuli σ_{ij}-iT SesaZleblobas gvaZlevs miviRoT plastikuri deformaciis nazrdebis tenzoris komponentebisaTvis gantolebebi [55].

$$d\varepsilon^{p}{}_{ij} = \frac{\partial g(\sigma_{ij})}{\partial \sigma_{ij}} d\lambda'$$

an

$$\frac{d\varepsilon^{p}{}_{ij}}{\partial g / \partial \sigma_{ij}} = \ldots = d\lambda' \qquad (1.4.1)$$

sadac $d\lambda'$ dadebiTi mudmivaa ($d\lambda'$ uaryofiTi rom yofiliyo, es niSnavda imas, rom uaryofiTi deformacia dakavSirebulia dadebiT ZabvasTan, rac SeuZlebelia). es gantolebebi gadaiqceva levi-mizesis gantolebad, Tu g-s adgilas CavsvamT mizesis plastikurobis funqcias *f*. amgvarad

$$g(\sigma_{ij}) = f(\sigma_{ij}) = \sum(\sigma_1 - \sigma_2)^2$$

maSin

$$\frac{df}{d\sigma_1} = 2(\sigma_1 - \sigma_2) - 2(\sigma_3 - \sigma_1) = 4\sigma_1 - 2\sigma_2 - 2\sigma_3 = 6(\sigma_1 - \sigma_m)$$

sadac

$$\sigma_m = \frac{1}{3}\sum \sigma_1$$

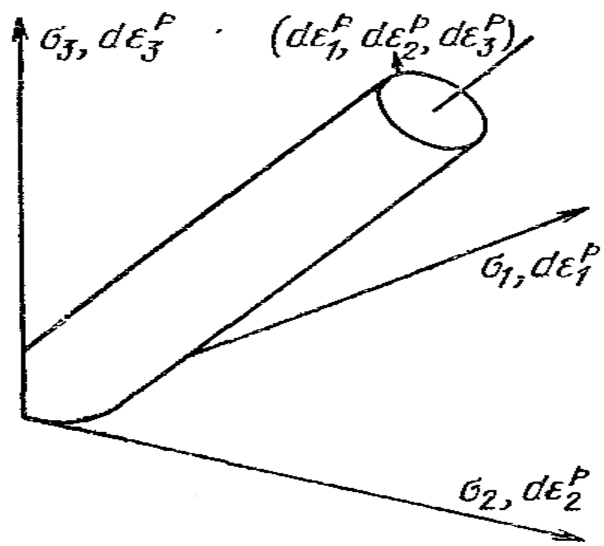
amrigad (1.4.1.)-dan gveqneba:

$$d\varepsilon_1^p = \frac{\partial g}{\partial \sigma_{ij}} d\lambda' = \frac{\partial f}{\partial \sigma_1} d\lambda' = 6(\sigma_1 - \sigma_m) d\lambda' = 6\sigma_1' d\lambda'$$

aqedan

$$\frac{d\varepsilon_1^p}{\sigma_1'} = \ldots = d\lambda \qquad (1.4.2)$$

sadac $d\lambda$ - proporciulobis sxva dadebiTi mudmivaa.



nax. 1.4.1

Ffunqcia $g(\sigma_{ij})$, romliTac SeiZleba SevcvaloT plastikurobis piroba da plastikuri potenciali, aucilebelia SeirCes ise, rom is iyos simetriuli Zabvis samive invariatorisTvis, e.i. ise, rom is ar iyos damokidebuli kordinatebis sistemis SerCevaze an mTavar Zabvebze. Ffunqcia iTvleba simetriulad, Tu masSi mTavari Zabvebidan TiToeuli Sedis erTnairi ,,woniT".

Tu σ_1 RerZze (nax. 1.4.1) plastikuri deformaciis mTavar nazrds aRvniSnavT $d\varepsilon^p{}_1$-iT da ganvixilavT σ_2 da σ_3 RerZebis mimarTebaSi, $d\varepsilon^p{}_2$ da $d\varepsilon^p{}_3$-s, romlebic agreTve arian $(\sigma_1,\sigma_2,\sigma_3)$ Zabvebis moqmedebiT gamowveuli plastikuri deformaciis nazrdebis veqtoris komponentebi, maSin mosaxerxebelia es veqtori movaTavsoT plastikurobis cilindrze mdebare wertilSi. Aaxla deformaciis nazrdis veqtoris mimarTuleba iseTivea, rogorc g(σ_{ij}) zedapiris gare normalis (σ_1, σ_2, σ_3) wertilSi. Ees aixsneba imiT, rom (σ_1, σ_2, σ_3) wertilSi cilindris zedapiris normalis mimmarTveli kosinusebis Tanafardobebi Cveulebrivi dekartuli meTodebiT ganisazRvreba.

$$\frac{\partial g}{\partial \sigma_1}:\frac{\partial g}{\partial \sigma_2}:\frac{\partial g}{\partial \sigma_3}$$

Eesaa denadobis Teoriis gantolebebis plastikuri potencialis ganxilvis Sedegebis mixedviT warmodgenis geometriuli xerxi.

A amrigad,

$$d\varepsilon_1^p : d\varepsilon_2^p : d\varepsilon_3^p = \frac{\partial g}{\partial \sigma_1}:\frac{\partial g}{\partial \sigma_2}:\frac{\partial g}{\partial \sigma_3} \qquad (1.4.3)$$

Tumca ukumSvelobis varaudidan gamomdinare, g unda iyos iseTi, rom

$$d\varepsilon_1^p + d\varepsilon_2^p + d\varepsilon_3^p = \frac{\partial g}{\partial \sigma_1}+\frac{\partial g}{\partial \sigma_2}+\frac{\partial g}{\partial \sigma_3} = 0$$

Mmizesis plastikurobis piroba akmayofilebs am moTxovnebs. Aaxla iqneba naCvenebi, rom varauds plastikuri deformaciis nazrdebis veqtoris perpendikularobaze plastikurobis zedapirisadmi, mivyavarT Zabvebisa da deformaciis nazrdebis sxva TanafardobasTan_treskas plastikurobis pirobasTan. E es niSnavs, rom arsebobs mxolod eqvsi sxvadasxva mimarTulebis normali, romlebic Seesabameba prizmis gverdiTi waxnagebis eqvs farTeuls.

Mmxolod maqsimaluri mxebi Zabva, romelsac aqvs udidesi absoluturi sidide, gamoiyeneba mocemuli (σ_1, σ_2, σ_3) ganawilebisaTvis, magaliTad (σ_1- σ_3); maSin g(σ_{ij})= (σ_1- σ_3), viyenebT ra (1.4.2)-s, miviRebT:

$$d\varepsilon_1^p : d\varepsilon_2^p : d\varepsilon_3^p = \frac{\partial(\sigma_1-\sigma_3)}{\partial \sigma_1}:\frac{\partial(\sigma_1-\sigma_3)}{\partial \sigma_2}:\frac{\partial(\sigma_1-\sigma_3)}{\partial \sigma_3} = 1:0:-1 \qquad (1.4.4)$$

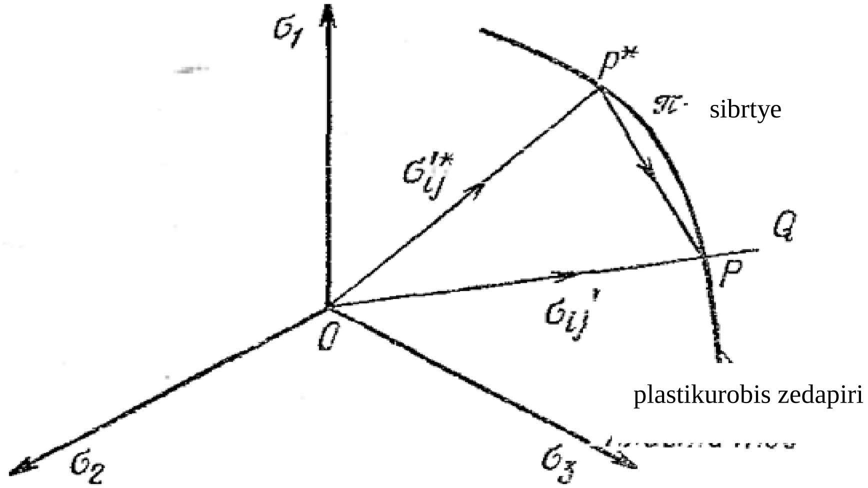
es erTaderTi mdgomareobaa, romelic SesaZlebelia plastikuri deformirebisaTvis mxolod σ_1 da σ_3-is sibrtyeSi; plastikuri deformaciebis nazrdebi sididiT tolia da urTierTsapirispirod arian mimarTulni. treskas plastikurobis piroba da misi Sesabamisi denadobis gantolebebi ganixileba koiteris (1953w), prageris (1955w) da blendis (1956w) SromebSi.

1.5. disipaciis energiis maqsimumis principi.

xistplastikuri masalis erTeul moculobaSi gabneuli energiis nazrdi, rodesac mTavari Zabvebi $\sigma_1, \sigma_2, \sigma_3$ anu σ_i erTmaneTis tolia,

$$\delta w = \sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2 + \sigma_3 d\varepsilon_3 = \sigma_i d\varepsilon_i$$

sadac $d\varepsilon_1$, $d\varepsilon_2$, $d\varepsilon_3$ - plastikuri deformaciebis mTavari nazrdebia. sxvanairad, $\delta w - \overrightarrow{OP}$ Zabvis deviatoris veqtoris skalaruli namravlia ($d\varepsilon_1$, $d\varepsilon_2$, $d\varepsilon_3$) deformaciis nazrdis veqtor $\overrightarrow{PQ}$-ze. Ees ukanaskneli $\overrightarrow{PQ}$ veqtori naCvenebia nax. 1.5.1-ze da warmoadgens plastikurobis zedapiris normals P wertilSi.



Nnax.2.1.5.1

Nnax. 1.5.1. plastikuri deformaciis nazrdi ($\overrightarrow{PQ}$ - denadobis zedapiris normalia P wertilSi).

Zabvebis sferuli tenzoris komponentebi ar asruleben muSaobas da plastikuri deformaciis muSaobis disciplis ganxilvisas mxedvelobaSi miiReba mxolod Zabvis deviatoris komponentebi. Yyvelaferi es SeiZleba ganvixiloT π - sibrtyeze (ixNnax. 1.5.1) maSin $\delta w = \sigma_i^{'} \cdot d\varepsilon_i$. axla ganvixiloT

$$\delta w^{*} = \sigma_1^{*'} d\varepsilon_1 + \sigma_2^{*'} d\varepsilon_2 + \sigma_3^{*'} d\varepsilon_3 = \sigma_i^{*'} d\varepsilon_i$$

sadac $\overrightarrow{OP}^{*} = \left(\sigma_1^{*'}, \sigma_2^{*'}, \sigma_3^{*'}\right)$ da aseve akmayofilebs plastikurobis pirobas anu P^{*} wertili imyofeba plastikurobis zedapirze. maSin

$$\delta w - \delta w^{*} = \overrightarrow{OP} \cdot \overrightarrow{PQ} - \overrightarrow{OP}^{*} \cdot \overrightarrow{PQ} = \left(\sigma_i^{'} - \sigma_i^{*'}\right) d\varepsilon_i \quad (1.5.1)$$

ufro zogad gamosaxulebaSi d_v usasrulod mcire moculobisTvis, romlis farTeulebSic moqmedeben Zabvebi σ_{ij} da iwveven (1.5.1) gamosaxulebis Sesabamis deformaciis $d\varepsilon_{ij}$ nazrdebs da $d\varepsilon_{ij}^{*}$ Zabvebs, Cven SegviZlia muSaobaTa nazrdebis sxvaoba gamovTvaloT formuliT:

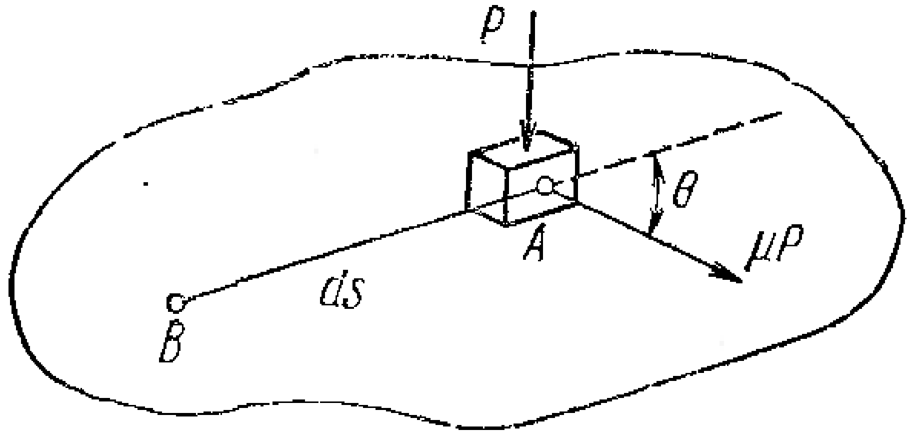
$$\left(\sigma_{ij}^{'} - \sigma_{ij}^{*'}\right) d\varepsilon_{ij} dV \qquad (1.5.2)$$

deformaciebis nazrdebi, SevcvaloT deformaciis siCqareebiT, xolo plastikuri deformaciis muSaoba-simZlavriT. aRniSvnebSi Sesabamisi cvlilebebis Setanis Semdeg gvaqvs:

$$\dot{W}-\dot{W}^{*}=\int_{v}\left(\sigma_{ij}^{'}-\sigma_{ij}^{*'}\right)\dot{\varepsilon}_{ij}\,dV \qquad (1.5.3)$$

formulaSi wertilebiT aRniSnulia deformaciis siCqare da simZlavre. Cazneqili (kordinatTa saTaveSi) plastikurobis zedapirisTvis (1.5.3) SeiZleba Caiweros am saxiT:

$$\int_{v}\left(\sigma_{ij}^{'}-\sigma_{ij}^{*'}\right)\dot{\varepsilon}_{ij}\,dV\geq 0 \qquad (1.5.4)$$



nax. 1.5.2. disipaciis maqsimumis principis ilustracia

(2.1.5.4)-iT idealuri xistplastikuri sxeulis formacvlileba an deformireba xdeba energiis maqsimaluri xarjviT. swored es aris muSaobis maqsimaluri disipaciis principi, mas aqvs farTo gamoyeneba ara marto plastikuri deformirebis dros.

am princips aqvs saintereso kavSiri meqanikur xaxunTan. horizontalur sibrtyeze (nax.1.5.2) xaxunis Zala tolia μP, sadac P - sxeulis simZimis Zalaa. μ - xaxunis koeficienti. Tu sxeuli moZraobs A wertilidan B wertilSi ds gziT, maSin xaxunis Zalis gadasalaxad daxarjuli muSaoba

$$W=\overrightarrow{\mu P}\times\overrightarrow{ds}=\mu P ds\cos\theta$$

W – maqsimaluria, rodesac $\theta=0$, anu xaxunis Zala moqmedebs moZraobis sawinaaRmdegoDmimarTulebiT, sadac P – isea mimarTuli, rom zrdis Sesrulebul muSaobas.

1.6. anizotropuli masalis plastikuri denadobis gantolebebi.

iseve rogorc izotropuli masalisaTvis, iTvleba, rom $f(\sigma_{ij})$ aris plastikuri potenciali. Tu vipoviT kerZo warmoebuls $f(\sigma_{ij})$ funqciis σ_{ij}-iT, miviRebT deformaciis nazrdebs:

$$2f(\sigma_{ij}) = F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + \\ + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2$$

$\dfrac{\partial f}{\partial \sigma_x} = G(\sigma_x - \sigma_z) + H(\sigma_x - \sigma_y)$ aqedan. gamomdinare,

$$\frac{d\varepsilon_x^p}{G(\sigma_x - \sigma_z) + H(\sigma_x - \sigma_y)} = d\lambda$$

analogiuri gamosaxulebebi miiReba deformaciis sxva nazrdebisaTvis:

$$\begin{aligned} d\varepsilon_x^p &= d\lambda\left[H(\sigma_x - \sigma_y) + G(\sigma_x - \sigma_z)\right]; \quad d\gamma_{yz}^p = d\lambda L\tau_{yz} \\ d\varepsilon_y^p &= d\lambda\left[F(\sigma_y - \sigma_z) + H(\sigma_y - \sigma_x)\right]; \quad d\gamma_{zx}^p = d\lambda M\tau_{zx} \\ d\varepsilon_z^p &= d\lambda\left[G(\sigma_z - \sigma_x) + F(\sigma_z - \sigma_y)\right]; \quad d\gamma_{xy}^p = d\lambda N\tau_{xy} \end{aligned} \qquad (1.6.1)$$

Ees gamosaxulebebi akmayofilebs pirobas:

$$d\varepsilon_x^p + d\varepsilon_y^p + d\varepsilon_z^p = 0$$

roca drekadi deformaciis nazrdebi mcirea plastikuri deformaciis nazrdebTan SedarebiT, zeda indeqsi - P (1.6.1) gamosaxulebebSi SeiZleba ukuvagdoT, rac niSnavs rom Cven gvaqvs saqme levi-mizesis idealur xistplastikur sxeulTan. Tu (x,y) sibrtyidan amoWril nimuSs gavWimavT x RerZis mimarTulebiT, romelic ganxilulia anizotropis RerZad, maSin deformaciebis nazrdebis Tanafardobebi ase Caiwereba:

$$d\varepsilon_x^p : d\varepsilon_y^p : d\varepsilon_z^p = (G+H):(-H):(-G)$$

sisqiTi deformaciis fardoba ganiv deformaciasTan aRiniSneba r-iT, $r_x = d\varepsilon_y^p \big/ d\varepsilon_z^p = H/G$ indeqsi x aRniSnavs, rom nimuSi amoWrilia x RerZis gaswvriv. y RerZis gaswvriv amoWrili nimuSisaTvis gveqneba:

$$d\varepsilon_x^p : d\varepsilon_y^p : d\varepsilon_z^p = (-H):(F+H):(-F)$$

da

$$r_y = d\varepsilon_x^p \big/ d\varepsilon_z^p = H/F$$

aqamde Cven ganvixilavdiT datvirTvebs anizotropiis RerZis mimarTulebiT. imisaTvis rom miviRoT anizotropiis saWiro parametrebi furclis sibrtyeSi, aucilebelia CavataroT gamocda gaWimvaze sul cota kidev erTi mimarTulebiT amoWril nimuSebze.

Tu anizotropuli masalis furcelze vixilavT (x,y) furclis sibrtyis marTobuli Zalebis moqmedebas, maSin τ_{yz} da τ_{zy} iqneba nulis toli. vivaraudoT, rom gasaWimi nimuSi amoWrilia x RerZis mimarT raime kuTxiT; maSin wonasworobis pirobebidan

$$\sigma_x = \sigma \cdot \cos^2\alpha \quad \sigma_y = \sigma \cdot \sin^2\alpha \quad \tau_{xy} = \sigma \cdot \sin\alpha \cdot \cos\alpha$$

sadac σ - aris denadobis Zabva gaWimvisas. CavsvaT es (1.6.1) formulebSi. miviRebT:

$$d\varepsilon_x^p = \left[(G+H)\cos^2\alpha - H\sin^2\alpha\right]\sigma d\lambda$$
$$d\varepsilon_y^p = \left[(F+H)\sin^2\alpha - H\cos^2\alpha\right]\sigma d\lambda$$
$$d\varepsilon_z^p = -\left[F\sin^2\alpha + G\cos^2\alpha\right]\sigma d\lambda$$
$$d\gamma_{xy}^p = \left[N\sin\alpha \cdot \cos\alpha\right]\sigma \cdot d\lambda$$

(1.6.2)

ganvixilavT ra mcire deformaciebis geometrias, mivdivarT daskvnamde, rom ganivi deformaciebis nazrdi $d\varepsilon^p_{\alpha+\frac{\pi}{2}}$ ganisazRvreba gamosaxulebiT:

$$d\varepsilon^p_{\alpha+\frac{\pi}{2}} = d\varepsilon^p_x \sin^2\alpha + d\varepsilon^p_y \cos^2\alpha - 2d\gamma^p_{xy} \sin\alpha \cdot \cos\alpha$$

amitom

$$r_\alpha = \frac{d\varepsilon^p_{\alpha+(\pi/2)}}{d\varepsilon^p_z} = \frac{d\varepsilon^p_x \sin^2\alpha + d\varepsilon^p_y \cos^2\alpha - 2d\gamma^p_{xy} \sin\alpha \cdot \cos\alpha}{d\varepsilon^p_z}$$

$$r_\alpha = \frac{H + (2N - F - G - 4H)\sin^2\alpha \cdot \cos^2\alpha}{F\sin^2\alpha + G\cos^2\alpha} \tag{1.6.3}$$

furclovan liTonSi glinvis mimarTuleba rogorc wesi warmoadgens anizotropiis RerZs. Aamitom x RerZs irCeven am mimarTulebiT. Aaqedan gamomdinare, zemoT dawerili denadobis gantolebebi miiReben Semdeg saxes:

$$\left.\begin{aligned} r_x &= r_0 = \frac{H}{G} \\ r_y &= r_{90} = \frac{H}{F} \\ r_{45} &= \frac{2N - (F+G)}{2(F+G)} \end{aligned}\right\} \tag{1.6.4}$$

an

$$\frac{N}{G} = \left(r_{45} + \frac{1}{2}\right)\left(1 + \frac{r_0}{r_{90}}\right)$$

sadac r_0; r_{45}, r_{90} arian r-is mniSvnelobebi gaglinvis mimarTulebiT da 45° da 90° -iani kuTxiT mis mimarT.

am gamosaxulebebis gamoyvanisas ivaraudeboda, rom anizotropiis parametrebis Tanafardoba rCeboda ucvleli gamokvlevis mTel periodSi. anizotropulobis parametrebi ucvleli rCeba aluminisTvis, (daamtkices klingerma da zaqsma 1948 w.); Zlieri WimvisaTvis gaTvaliswinebuli foladebisaTvis, titanisaTvis (bramli, melori - 1966) magram averim da bekofenma 1965w. daadgines, rom magniumis zogierTi SenadnobisTvis r-is sidide arsebiTadaa damokidebuli deformaciis xarisxze. anizotropuli masalebis denadobis zRvari gamoikvlies lim da bekofenma (1966w); babelma, itmanma da makiverma (1966 w), mehanma (1961 w). r – sidides liTonis furclebisaTvis Cveulebriv gansazRvraven, roca grZivi deformacia >5%, maSin aris safuZveli vivaraudoT, rom drekadi deformaciebi mcirea da amis gamo moculoba rCeba ucvleli. grZiv da ganiv deformaciebs zomaven, xolo sisqiT deformaciebs (gazomvis sirTulis gamo Txeli furclisaTvis) gamoTvlian:

$$r = \frac{\ln\left(\frac{w_0}{w}\right)}{\ln\left(\frac{t_0}{t}\right)} = \frac{\ln\left(\frac{w_0}{w}\right)}{\ln\left(\frac{w\lambda}{w_0\lambda_0}\right)} \qquad (1.6.5)$$

sadac ω, t da ℓ aris siganis, sisqis da sigrZis mimdinare mniSvnelobebi „0" - indeqsi miekuTvneba sawyis parametrebs. r-is gazomva dawvrilebiT ganixila atkinsonma (1967w).

1.7. damokidebulebani Zabvebsa da deformaciebs Soris ganmtkicebadi Aanizotropuli masalebisaTvis.

roca masala ganicdis plastikur deformacias, anizotropiis mdgomareoba icvleba. magram vivaraudoT, rom anizotropuli Tvisebebis cvlileba gamocdis dawyebis Semdeg mcirea. aseT SemTxvevaSi mosalodnelia Teoriuli da eqsperimentuli monacemebis Tanxvedra didi sawyisi anizotropiis mqone masalebisaTvis. Ees gvqonda mxedvelobaSi, roca vvaraudobdiT, rom gamocdamde izotropuli masala, izotropuli rCeba plastikuri deformaciis drosac. cnobilia, rom es daSveba ar aris mkacrad argumentirebuli, magram, rogorc gamocdileba gviCvenebs, mraval SemTxvevaSi aseTi miaxloeba dasaSvebia.

Tu anizotropuli mdgomareoba ar icvleba, maSin denadobis Zabvebi liTonis ganmtkicebisas unda izrdebodes mkacrad proporciulad. aqedan gamodis, rom anizotropiis parametrebi unda Semcirdes mkacri proporciiT. maSin parametrebis Tanafardoba darCeba mudmivi; cdisas zomaven swored Tanafardobebs da ara absolutur mniSvnelobebs calkeuli parametrebisa. zemoT mivuTiTeT, rom deformaciebis Tanafardoba zogierTi masalisaTvis SeiZleba darCes ucvleli, xolo zogisTvis – ara, amitom saWiroa gamoviCinoT sifrTxile anizotropiis Teoriis gamoyenebisas.

hilma 1950w. SemogvTavaza axali formula ekvivalenturi Zabvis gansazRvrisaTvis:

$$\bar{\sigma}=\sqrt{\frac{3}{2}}\left[\frac{F\left(\sigma_y-\sigma_z\right)^2+G\left(\sigma_z-\sigma_x\right)^2+H\left(\sigma_x-\sigma_y\right)^2+2L\tau_{yz}^2+2M\tau_{zx}^2+2N\tau_{xy}^2}{F+G+H}\right]^{\frac{1}{2}} \quad (1.7.1)$$

aqedan naTelia, rom ganixilaven anizotropiis parametrebis fardobebs da ara maT absolutur mniSvnelobebs. Tu anizotropia SeiZleba ugulebelvyoT, xolo datvirTva mihyveba mTavar RerZebs es gamosaxuleba gadaiqceva (1.2.1) gamosaxulebad.

jeqsonis, smitis da lankfordis kvaldakval (1948w), hilma 1950w. SemogvTavaza izotropulobis Teoriis analogiuri Teoria, sadac $\bar{\sigma}$ aris plastikuri deformaciis muSaobis funqcia. Pplastikuri deformaciis muSaobis nazrdi erTeul moculobaze xist-plastikuri sxeulisaTvis Seadgens:

$$dw = \sigma_{ij} d\varepsilon_{ij} = \sigma_{ij} \frac{\partial f}{\partial \sigma_{ij}} d\lambda \qquad (1.7.2)$$

isargebla ra $2f(\sigma_{ij}) = F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y) + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = 1$ gamosaxulebiT da eileris TeoremiT erTgvarovani funqciebis Sesaxeb,sokolnikovma (1941w) SemogvTavaza Semdegi varianti:

$$dw = \sigma_{ij} \frac{\partial f}{\partial \sigma_{ij}} d\lambda = 2 f d\lambda = d\lambda \qquad (1.7.3)$$

(1.7.3) gamosaxulebidan xist-plastikuri sxeulisaTvis Cven miviRebT Semdeg gantolebebs:

$$\begin{aligned} Gd\varepsilon_y - Hd\varepsilon_z &= (FG + GH + HF)(\sigma_y - \sigma_z) d\lambda \\ Hd\varepsilon_z - Fd\varepsilon_x &= (FG + GH + HF)(\sigma_z - \sigma_x) d\lambda \\ Fd\varepsilon_x - Gd\varepsilon_y &= (FG + GH + HF)(\sigma_x - \sigma_y) d\lambda \end{aligned}$$

$$dw = \overline{\sigma} d\overline{\varepsilon} = d\lambda$$

am gamosaxulebebis gaTvaliswinebiT, deformaciis nazrdebis intensivoba SeiZleba Semdegnairad ganisazRvros:

$$\overline{d\varepsilon} = \frac{d\lambda}{\overline{\sigma}} \sqrt{\frac{3}{2}} [F + G + H]^{1/2} \left[F \left(\frac{Gd\varepsilon_y - Hd\varepsilon_z}{FG + GH + HF} \right)^2 + \ldots + \frac{2d\gamma_{yz}^2}{L} + \ldots \right]^{1/2} \qquad (1.7.4)$$

furclovani masalisaTvis brtyeli daZabuli mdgomareobis pirobebSi z RerZis mimarT simetriiT

$$r = \frac{H}{G} = \frac{H}{F}$$

Dda gantolebebi (1.7.1) da (1.7.3) martivdeba:

$$\overline{\sigma} = \sqrt{\frac{3}{2}}\left[\frac{\sigma_x^2 + \sigma_y^2 + r\left(\sigma_x - \sigma_y\right)^2}{2+r}\right]^{1/2} \qquad (1.7.5)$$

$$\overline{d\varepsilon} = \sqrt{\frac{2}{3}}\left[\frac{2+r}{\left(1+2r\right)^2}\left\{\left(d\varepsilon_y - rd\varepsilon_z\right)^2 + \left(d\varepsilon_x - rd\varepsilon_z\right)^2 + r\left(d\varepsilon_z - d\varepsilon_y\right)^2\right\}\right]^{1/2} \qquad (1.7.6)$$

Ees gantolebebi aucilebelia normaluri anizotropiis gavlenis gansazRvrisaTvis furclovani daStampvis pirobebSi.

1.8 plastikurobis Teoriis zogierTi problemis Sesaxeb.

im masalebis gamoyeneba konstruqciebSi, romlebic xasiaTdeba fiziko-meqanikuri Tvisebebis cvlilebis farTo speqtriT temperaturuli zemoqmedebis Sedegad, masalaTa Tvisebebze drois faqtoris gavleniT, stuqturuli cvlilebebiT datvirTvis qveS, moiTxovs plastikurobis sakiTxebis kardinalur Seswavlas rogorc Teoriul, ise eqsperimentalur aspeqtebiT.

tradiciuli masalebis _ liTonebis gamoyenebas, dRes masiurad daemata plastmasebi da kompozituri masalebi plastmasebis matricebze.

konstruqciuli plastmasebi da kompozitebi farTod gamoiyeneba samoqalaqo, satransporto da samrewvelo mSeneblobaSi, saaviacio da gemTmSeneblobaSi.

amgvarad, am masalaTa fiziko-meqanikuri Tvisebebis Seswavla Tanamedrove mecnierebis aqtualuri sakiTxia. plastmasebi Tavisi fiziko-meqanikuri TvisebebiT ganekuTvnebian anizotropul masalebs. damzadebis teqnologia sagrZnobel gavlenas axdens maT Tvisebebze da anizotropiaze farTo gagebiT.

masalaTa Tvisebebis aRwera dRes xorcieldeba ramdenime TeoriiT. mag. molekuluri Zalebis TeoriiTa an dislokaciuri TeoriiT, an kidev kristaluri gisosebis defeqtebis TeoriiT da sxv.

arsebobs mTeli rigi faqtorebisa, romelnic arsebiT gavlenas axdenen im gantolebebze, romelTa saSualebiTac aRweren masalaTa daZabul-deformirebul mdgomareobas.

calkeuli amocanebisaTvis zogierTi faqtori gadamwyvetia da mxedvelobaSi pirvel rigSi maT vRebulobT. aseT faqtorebs ganekuTvneba temperaturuli velebi (stacionaruli da cvladi), masalaTa siblante, deformaciis cvalebadobis siCqare, zemaRali wnevebis gavlena, araerTgvarovneba da sxva. sainteresoa, rom anzotropia SeiZleba iyos ,,Tandayolili", gamowveuli damzadebis teqnologiis TaviseburebebiT. es uksnaskneli SeiZleba iyos masalis anizotropia an konstruqciuli anizotropia.

anizotropia SeiZleba warmoiqmnasDkonstruqciis damzadebis procesSi (glinva, tvifvra, gaWimva maRali temperaturis pirobebSi da sxva.)

praqtikuli miznebisaTvis gamoiyenebaQplastikurobis sxvadasxva Teoria; rogori Teoria gamoiyeneba konkretulad, damokidebulia masalaze da im miznebze, romelebic dgas mkvlevaris winaSe. SemovifargloT ararelogiuri plastikurobis TeoriebiT, anu drois faqtori mxedvelobaSi ar miviRoT.

am aspeqtSi ganixileba ori fundamenturi amocana.PKKD

1. deformaciebis gansazRvra determinirebuli gare datvirTvebisas (statikis da dinamikis amocanebi).

Aa) ganixileba narCeni deformaciebi,roca gare datvirTvebi moxsnilia mTlianad an nawilobriv.

B b) meqanikur TvisebaTa cvalebadobis Seswavla plastikuri deformaciebis Sedegad; deformaciebis velis dadgena ganmeorebiTi datvirTvisas (histerezisi).

g) mrRvevi tvirTebis gansazRvra globaluri an lokaluri rRvevis SemTxvevaSi.

amocanebis gadawyvetisas aucilebelia masalis gantkicebis kanonebis Seswavla, e.i miRebuli unda iyos mxedvelobaSi datvirTva-gantvirTvis wina _ istoria.

2. fundamenturi amocana plastikurobis Teoriisa, rac gansazRvruli pirobiTobiT analogiuria hidrodinamikis amocanebisa: absoluturad myari sxeulis moZraobis determinacia da misi nawilakebis gadaadgilebis siCqare-aCqarebaTa gansazRvra. Aaq SeiZleba miRebul iqnes mxedvelobaSi datvirTva-gantvirTvis wina-istoria, an ar iqnes miRebuli mxedvlobaSi).

plastikurobis Teriis ori ganStoebidan pirveli ganixilavs funqcionalur damokidebulebebs Zabvebsa da deformaciebs Soris (deformaciuli Teoria). xolo meore ganixilavs funqcionalur kavSirs Zabvebsa da deformaciebis nazrdebs Soris, e.i. diferencialur doneze nebismieri sizustiT, sadac es procesi ganixileba drois faqtorTan uSualo kavSirSi. rogorc kerZo SemTxveva, warmogvidgeba damokidebulebani deformaciebis siCqareebsa da Zabvebs Soris. aq plastikuri deformacia ganixileba rogorc masalis denadobis procesi (denadobis Teoria).

inJiner-mkvlevarTa Soris metad didi popularobiT sargeblobs mcire drekad-plastikuri deformaciebis Teoria.

am ukanasknels safuZvlad udevs muSa hipoTezebi:

1. moculobiTi deformacia saSualo normaluri Zabvebis pirdapiroporciulia. proporciulobis koeficienti igivea, rac masalis muSaobisas drekad farglebSi.
2. deformaciis tenzoris deviatorsa da Zabvebis tenzoris deviators Soris arsebobs wrfivi damokidebuleba.

ZabvaTa intensivoba aris deformaciis intensivobis funqcia da ar aris damokidebuli droze.

pirveli hipoTezis Tanaxmad saSualo fardobiTi moculobiTi deformacia

$$\varepsilon_0 = \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{3} = 0 \qquad (1.8.1)$$

Hhukis moculobiTi kanonidan

$$\varepsilon_0 = \frac{\sigma_x + \sigma_y + \sigma_z}{3K} \qquad (1.8.2)$$

sadac $K=\frac{E}{3(1-2\nu)}$ drekadobis moculobiTi modulia; pirobidan $\varepsilon_0=0$ gamomdinareobs, rom ν=0,5. swored maSin gvaqvs K→∞.

meore hipoTezidan gamomdinareobs

$$\frac{\varepsilon_x-\varepsilon_0}{\sigma_x-\sigma_0}=\frac{\varepsilon_y-\varepsilon_0}{\sigma_y-\sigma_0}=\ldots=\frac{\gamma_{yz}}{2\tau_{yz}}=\ldots=\varphi \qquad (1.8.3)$$

sadac $\varphi=\frac{1}{2G}$ mudmivaa, ε_0 da σ_0 - Sesabamisi saSualoebi. es niSnavs, rom ZabvaTa da deformaciaTa wrewirebi msagavsia. mraval amocanas plastikurobis TeoriaSi ara aqvs zusti amonaxseni da amitom didi mniSvneloba eniWeba maTA amoxsnas miaxloebiTi meTodebiT.

am meTodebidan yvelaze popularulad unda CavTvaloT variaciuli meTodebi.

gavrcelebulia agreTve aseTi amocanebis amoxsna “drekad amoxsnaTa” meTodiT, romlis realizacia xorcieldeba TandaTanobiTi miaxloebebis gziT.

sainteresoa bergeris mier SemoTavazebuli drekad-plastikuri daZabul-deformirebuli mdgomareobis gaangariSebani cvlad parametrTa meTodiT. hukis ganzogadebuli kanoni am dros formalurad Caiwereba tradiciuli saxiT.

$$\varepsilon_x=\frac{1}{E^*}\left[\sigma_x-\nu^*\left(\sigma_y+\sigma_z\right)+\varepsilon(T)\right]$$

……………………………………………

Aaq E^*, ν^* drekadobis cvladi parametrebia

$$E^*=\frac{\frac{\sigma_i}{\varepsilon_i}}{1+\frac{1-2\nu}{3E}\cdot\frac{\sigma_i}{\varepsilon_i}} \qquad \nu^*=\frac{\frac{1}{2}-\frac{1-2\nu}{3E}\cdot\frac{\sigma_i}{\varepsilon_i}}{1+\frac{1-2\nu}{3E}\cdot\frac{\sigma_i}{\varepsilon_i}} \qquad (1.8.4)$$

$$G^*=\frac{E^*}{2(1+\nu^*)}$$

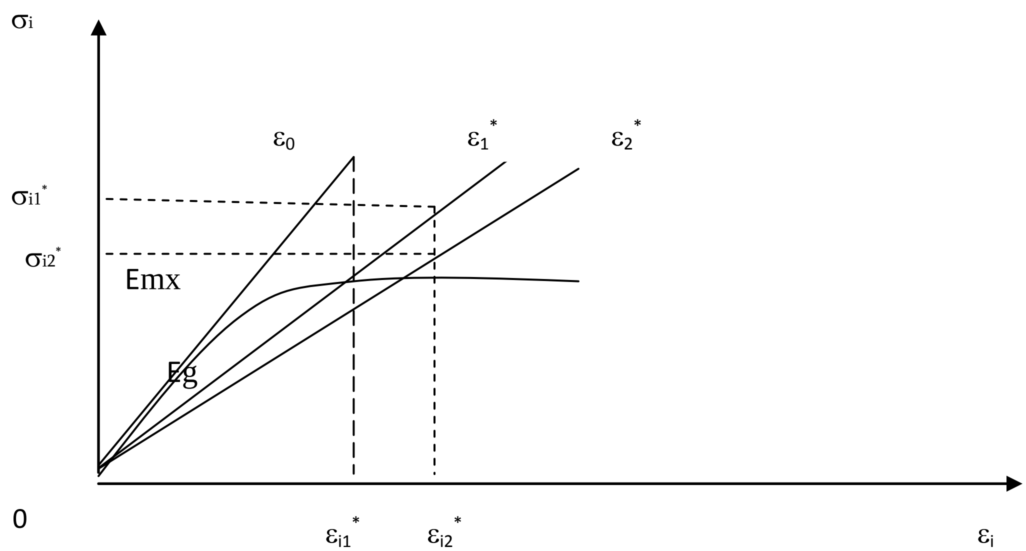
kerZo SemTxvevisTvis (ukumSvadi sxeulisaTvis) $\nu = \frac{1}{2}$ da $\nu^* = \frac{1}{2}$ $E^* = \frac{\sigma_i}{\varepsilon_i}$.

e.i. drekadobis cvladi moduli warmoadgens gamkveTi (gamWoli) drekadobis moduls.

aq σ_i da ε_i _ aris ZabvaTa da deformaciaTa intensivoba. wertilis siaxloveSi. jer wydeba (pirvel miaxloebaSi) drekadi amocana Zabvisa da deformaciaTa tenzorebis dadgeniT.

am gziT amocanis amoxsna kargad aqvs warmodgenili iliuSins.

Mmeore miaxloebaSi Semogaqvs koreqtura $3G^*$ parametrebisTvis, rogorc σ^*_{i1} da ε^*_{i1} Sefardeba, romelic aiReba deformirebis diagramidan (nax. 1.8.1).



Nnax.1.8.1

$$3G^*_{i1} = \frac{\sigma^*_{i1}}{\varepsilon^*_{i1}}$$

σ_{i1} da ε_{i1} parametrebis saSualebiT ganisazRvreba agreTve E^* da ν^* parametrebi, romelTa mniSvnelobebi, cxadia, sxvadasxva iqneba myari sxeulis nebismier wertilSi.

amgvarad warmoiqmna ZabvaTa gansazRvris amocana `kvazi- araerTgvarovani” myari tanisaTvis. σ_i da ε_i koordinatTa sistemaSi daZabul - deformirebuli mdgomareoba wertilSi ganisazRvreba 2 wertilSi diagramaze, romelic ganlagebulia sxiv 0-1’-ze, romlis daxris kuTxis tangensi $3G_1^*$ - is proporciulia.

mesame miaxloebaSi 3G parametrs vRebulobT intensivobaTa SefardebiT σ_{i2} da ε_{i2} wertilebSi.

$$3G_{i2}^* = \frac{\sigma_{i2}^*}{\varepsilon_{i2}^*}$$

σ_{i2} da ε_{i2} sidideTa saSualebiT ganisazRvreba E_2^* da μ_2^* $\sigma_{X3} \cdots \tau_{ZX3}, \varepsilon_{X3} \cdots$ γ_{zx3} sidideebi da a.S.

gaangariSeba grZeldeba manmade, sanam miaxloebaNn-ur da n+ 1 safexurzeF ar dauaxlovdeba erTmaneTs saWiro sizustiT. Nrogorc wesi, amocanis amoxsna damakmayofilebeli sizustiT miiReba mesame- meoTxe miaxloebis Semdeg.

ganvixiloT metad efeqturi variaciuli meTodi,romelic farTod gamoiyeneba msoflio praqtikaSi rTuli amocanebis amoxsnis drosac.

Kklapeironis Tanaxmad

$$A = \int_V \left(\varepsilon_x \sigma_x + \varepsilon_y \sigma_y + \varepsilon_z \sigma_z + \gamma_{xy} \cdot \tau_{xy} + \gamma_{yz} \cdot \tau_{yz} + \gamma_{zx} \cdot \tau_{zx}\right) dV \quad (1.8.5)$$

sadac A - gare Zalebis mier Sesrulebuli muSaobaa; $\int_V \Pi \cdot dV$ -potenciuri energia

wonasworobis WeSmariti forma xasiaTdeba mTliani energiis variaciuli eqstremumiT $\delta Э=0$, sadac

$$Э = \int_V \Pi \cdot dV - A \qquad (1.8.6)$$

Π SeiZleba gamoisaxos Semdegnairadac:

$$\Pi = \frac{9}{2} K \cdot \varepsilon_0^2 + \int_0^{e_i} \sigma_i d\varepsilon_i \qquad (1.8.7)$$

sadac $K = \frac{E}{3(1-2\nu)}$ moculobiTi drekadobis modulia; Tu gamoyenebulia daZabuli mdgomareobis SesaZlo cvalebadobis principi, rac mdgomareobs SemdegSi: gare Zalebis nazrdebis jami maTi modebis wertilebis gadaadgilebaTa mimarTulebiT udris damatebiTi muSaobis nazrds:

$$\int_{v}\left(U_x\delta_x + U_y\delta_y + U_z\delta_z\right)dv + \int_{s}\left(U_x dx_\nu + U_y dy_\nu + U_z dz_\nu\right)dS = \delta R \qquad (1.8.8)$$

R

sadac V - myari tanis moculobaa; S - zedapiri igive tanisa, U_x, U_y, U_z gadaadgilebani tanis nawilze; x_ν, y_ν, z_ν, - gare Zalebi tanis danarCen zedapirze; δ_x, δ_y, δ_z - gare Zalebis variacia, R - damatebiTi muSaoba.

$\delta\overline{R} = 0$ – variaciaa damatebiTi muSaobisa mTeli tanisaTvis

$$\overline{R} = \int_{v} R dV$$

SesaZlo variacia damatebiTi muSaobisa dadebiTia, rac niSnavs, rom yvela SesaZlo daZabuli mdgomareobidan mxolod is aris WeSmariti, roca $\overline{R}$ Rebulobs minimums:

kastilianos cnobili Teorema, romelic gamoiyeneba drekadobis TeoriaSi, SeiZleba gamoyenebuli iqnas im SemTxvevisTvisac, roca damokidebuleba Zabvebsa da deformaciebs Soris arawrfivia.

avRniSnoT P_i -Ti Seyursuli Zalebi, modebuli myar tanze, α_i, β_i, γ_i - saTanado mimmarTveli kosinusebi; U_{xi}, U_{yi}, U_{zi} - P_i PZalis modebis wertilebis gadaadgilebebi RerZebis gaswvriv.

variaciuli gantolebidan, roca erT-erTi P_i Zala Rebulobs umcires nazrds δP_i, vRebulobT:

$$\left(U_{xi}\alpha_i + U_{yi}\beta_i + U_{zi}\gamma_i\right)\delta P_i = \delta\overline{R} \qquad (1.8.9)$$

Aaqedan

$$\delta_i = \frac{\delta \overline{R}}{\delta P_i}$$

aq - δ_i gadaadgilebaa P_i Zalis mimarTulebiT.

e.i damatebiTi muSaobis kerZo warmoebuli P_i ZaliT udris am Zalis Sesabamis gadaadgilebas δ_i.

anizotropuli filebis gaangariSeba ZiriTadSi eyrdnoba iliuSinis drekad – plastikuri Teorias.

sokolovski, daeurdno ra henkis mier miRebul Tanafardobebs drekad-plastikuri daZabul-deformirebuli mdgomareobisTvis, ixilavs Tavisuflad dayrdnobil wriul filas da Rebulobs Sedegebs, amave dros grigorievi da sxv. ixilaven wriul filebs xistad dayrdnobili konturiT.

drekad – plastikuri amoxsnebi sworkuTxovani filebis mcire CaRunvebisTvis, roca filaze moqmedebs Tanabrad ganawilebuli datvirTva, miRebuli iqna linis da xu-s mier. maT gamoiyenes analogia plastikur deformaciebsa da datvirTvis SemTxveveSi.

engma da lopecma Caatares eqsperimentebi filis analogiur modelze, romelic Sesrulebuli iyo xisti gadamkveTi Reroebis sistemiT, da dakavSirebuli iyo erTmaneTTan kvanZebSi zambarebiT.

amocanis amoxsnisaTvis drekad-plastikur stadiaSi saWiroa:

1. normaluri da mxebi Zabvebis intensivobis codna

$$\sigma_i = \sqrt{3 I_2(D_\sigma)} \quad \tau_i = \sqrt{I_2(D_\sigma)} \qquad (1.8.10)$$

$$\sigma_i = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} = \sqrt{(\sigma_1 - \sigma_2)^2 + \sigma_1\sigma_2}$$

$$\varepsilon_i = \frac{2}{3}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + \varepsilon_1\varepsilon_2}$$

Eeqstremalur parametrebSi.

2. funqcionaluri damokidebulebani Zabvebsa da deformaciebs Soris.

$$\sigma_x = \frac{2}{3} \cdot \frac{\sigma_i}{\varepsilon_i}\left(2\varepsilon_x - \varepsilon_y\right)$$

$$\sigma_x = \frac{2}{3} \cdot \frac{\sigma_i}{\varepsilon_i}\left(2\varepsilon_y - \varepsilon_x\right) \qquad (1.8.11)$$

$$\tau_{xy} = \frac{1}{3}\frac{\sigma_i}{\varepsilon_i}\gamma_{xy}$$

3. plastikurobis energetikuli kriteriumi xuber- mizesis mixedviT:

$$\sigma_i = \sigma_\tau$$

praqtikaSi xSiria iseTi amocanebi, romelTaTvisac ar SeiZleba gamoyenebul iqnas drekad-plastikurobis Teoria. amis mizezi imaSi mdgomareobs, rom plastikurobis Sedegad bevri masala arsebiTad anizotropuli xdeba. garda amisa dRes farTod dainerga da inergeba anizotropuli (Tavisi bunebiT) masalebi. amgvarad, dRis wesrigSi dadga anizotropuli masalebisgan damzadebuli konstruqciebis gaangariSeba drekad-plastikuri daZabul-deformirebuli mdgomareobisaTvis.

ganvixiloT denadobis pirobebi orTotropuli masalebisaTvis. es piroba drekadi simetriis SemTxvevisaTvis miiRebs Semdeg saxes:

$$-\frac{1}{2}\left[a_{12}\left(\sigma_x - \sigma_y\right)^2 + a_{23}\left(\sigma_y - \sigma_z\right)^2 + a_{13}\left(\sigma_z - \sigma_z\right)^2\right] + \\ + \frac{1}{2}\left(a_{44}\tau_{xy}^2 + a_{55}\tau_{xz}^2 + a_{66}\tau_{yz}^2\right) = u \qquad (1.8.12)$$

sadac u aris deformaciis xvedriTi energia;

$$a_{12} = -\frac{\mu_{12}}{E_1} = -\frac{\mu_{21}}{E_2}; \quad a_{23} = -\frac{\mu_{23}}{E_2} = -\frac{\mu_{32}}{E_3}; \; a_{13} = -\frac{\mu_{13}}{E_1} = -\frac{\mu_{31}}{E_3}$$

$$a_{44} = \left(G_{23}\right)^{-1} \quad a_{55} = \left(G_{13}\right)^{-1} \quad a_{66} = \left(G_{12}\right)^{-1}$$

$$\mu_{12}=\frac{1}{2}\left(1+\frac{E_1}{E_2}-\frac{E_1}{E_3}\right)\quad \mu_{21}=\frac{1}{2}\left(1+\frac{E_2}{E_1}-\frac{E_2}{E_3}\right)$$

$$\mu_{13}=\frac{1}{2}\left(1+\frac{E_1}{E_3}-\frac{E_1}{E_2}\right)\quad \mu_{31}=\frac{1}{2}\left(1+\frac{E_3}{E_1}-\frac{E_3}{E_2}\right)\quad (1.8.13)$$

$$\mu_{23}=\frac{1}{2}\left(1+\frac{E_2}{E_3}-\frac{E_2}{E_1}\right)\quad \mu_{32}=\frac{1}{2}\left(1+\frac{E_3}{E_2}-\frac{E_3}{E_1}\right)$$

cxadia rom $\mu_{12}+\mu_{13}=1;\ \mu_{21}+\mu_{23}=1;\ \mu_{31}+\mu_{32}=1$. xvedriTi potenciuri energiis nazrdis ganxilvas mivyavarT gamosaxulebamde

$$\sigma_x d\varepsilon_x+\sigma_y d\varepsilon_y+\tau_{xy}d\gamma_{xy}=\tilde{\sigma}_i\tilde{\varepsilon}_i \qquad (1.8.14)$$

brtyeli amocanisaTvis daZabul-deformirebuli mdgomareoba plastikuri potencialis arsebobisas warmovidginoT Semdegi saxiT

$$\begin{aligned} d\varepsilon_x &= d\tilde{\varphi}\left(C_{11}\sigma_x+C_{12}\sigma_y+C_{13}\tau_{xy}\right)\\ d\varepsilon_y &= d\tilde{\varphi}\left(C_{12}\sigma_x+C_{22}\sigma_y+C_{23}\tau_{xy}\right)\\ d\gamma_{xy} &= d\tilde{\varphi}\left(C_{13}\sigma_x+C_{23}\sigma_y+C_{33}\tau_{xy}\right) \end{aligned} \qquad (1.8.15)$$

sadac C_{ij}(i,j=1,2,3) anizotropiis mudmivebia xy koordinatTa sistemaSi. $d\bar{\varphi}$ - proporciulobis koeficienti.

plastikurad orTotropuli masalebisaTvis $C_{13}=C_{23}=0$. vRebulobT apriorulad, rom

$$d\tilde{\varphi}=\mu\frac{d\tilde{\varepsilon}_i}{\sigma_i} \qquad (1.8.16)$$

sadac σ_i Zabvis intensivobaa, $d\tilde{\varepsilon}_i$ - deformaciis nazrdis intensivobaa anizotropuli masalebisaTvis; μ - anizotropiis koeficientia.

$$d\varepsilon_x = \mu \frac{d\tilde{\varepsilon}_i}{\sigma_i}\left(C_{11}\sigma_x + C_{12}\sigma_y\right)$$

$$d\varepsilon_y = \mu \frac{d\tilde{\varepsilon}_i}{\sigma_i}\left(C_{12}\sigma_x + C_{22}\sigma_y\right) \qquad (1.8.17)$$

$$d\gamma_{xy} = \mu \frac{d\tilde{\varepsilon}_i}{\sigma_i} C_{33}\tau_{xy}$$

sadac $\sigma_i = \sqrt{\mu\left(C_{11}\sigma_x^2 + 2C_{12}\sigma_x\sigma_y + C_{22}\sigma_y^2 + C_{33}\tau_{xy}^2\right)}$

da

$$d\tilde{\varepsilon}_i = \sqrt{\frac{1}{\mu}\left[\frac{1}{g}\left(C_{22}\varepsilon_x^2 - 2C_{12}\varepsilon_x\varepsilon_y + C_{11}\varepsilon_y^2 + \frac{1}{C_{33}}\gamma_{xy}^2\right)\right]}$$

Aanizotropiis mudmivebi ganisazRvreba Semdegi gamosaxulebiT:

$$C_{11} = 1 + \frac{1}{R_x}; \quad C_{12} = -1; \quad C_{22} = 1 + \frac{1}{R_y} \qquad (1.8.18)$$

$$C_{33} = \frac{2}{R_{xy}} \qquad \mu = \frac{3}{2\left(1 + \frac{1}{R_x} + \frac{1}{R_y}\right)} \qquad g = C_{11}C_{22} - C_{12}^2$$

R_x, R_y, R_{xy} - anizotropiis maxasiaTeblebia, x, y RerZebis da xoy sibrtyisaTvis:

$$R_x = \frac{H}{G} \quad R_y = \frac{H}{F}, \quad R_{xy} = \frac{H}{N} \qquad (1.8.19)$$

HH, G, F, N−anizotropiis eqserimentuli parametrebia, romelnic Sedian denadobis formulebSi:

$$(G+H)\sigma_x^2 - 2H\sigma_x\sigma_y + (H+F)\sigma_y^2 + 2N\tau_{xy}^2 = 0 \qquad (1.8.20)$$

Zabvebsa da deformaciebs Soris adgili aqvs Semdeg funqcionalur kavSirebs:

$$\sigma_x = \frac{1}{\mu g} \cdot \frac{\tilde{\sigma}_i}{\varepsilon_i}\left(C_{22}\varepsilon_x - C_{12}\varepsilon_y\right)$$

$$\sigma_y = \frac{1}{\mu g} \cdot \frac{\tilde{\sigma}_i}{\varepsilon_i}\left(C_{11}\varepsilon_y - C_{12}\varepsilon_x\right) \qquad (1.8.21)$$

$$\tau_{xy} = \frac{1}{\mu g C_{33}} \cdot \frac{\tilde{\sigma}_i}{\varepsilon_i}\gamma_{xy}$$

Tu am formulebs SevadarebT denadobis pirobebs (mileikovskis mixedviT) SegviZlia davweroT

$$H = \frac{\mu_{12}}{\sigma_{xp}^2}, G = \frac{\mu_{13}}{\sigma_{xp}^2};\ F = \frac{\mu_{23} \cdot E_1}{E_2 \cdot \sigma_{xp}^2};$$

$$N = \frac{E_1}{2C_{11}\sigma_{xp}},\ R_x = \frac{\mu_{12}}{\mu_{13}},\ R_y = \frac{\mu_{21}}{\mu_{23}} \qquad (1.8.22)$$

$$R_{xy} = \frac{2C_{11}\mu_{12}}{E_1}$$

Aaxla ukve SegviZlia davweroT anizotropiis mudmivebis mniSvnelobaTa gamosaxulebani

$$C_{11} = 1 + R_x^{-1}\mu_{12}^{-1};\ C_{12} = -1;\ C_{22} = 1 + R_y^{-1}\mu_{21}^{-1} \qquad (1.8.23)$$

$$C_{33} = 2 \cdot R_{xy}^{-1} = -E_1 C_{11}^{-1} \mu_{12}^{-1};\ g = C_{11} \cdot C_{22} - C_{12}^2 = \frac{1 - \mu_{12}\mu_{21}}{\mu_{12}\mu_{21}}$$

$$\mu = \frac{3 \cdot \mu_{12}\mu_{21}}{2(\mu_{12} + \mu_{21} \cdot \mu_{13})}$$

intensivobaTa gamosaxulebebi Caiwereba Semdegi saxiT

$$\sigma_{i0}^2 = \frac{3 \cdot \mu_{22}\mu_{21}}{2(\mu_{12} + \mu_{21} \cdot \mu_{13})}\left(\mu_{12}^{-1}\sigma_x^2 + 2\sigma_x\sigma_y + \mu_{21}^{-1}\sigma_y^2 + E_1 G_{11}^{-1}\mu_{12}^{-1}\tau_{xy}^2\right) \quad (1.8.24)$$

$$\varepsilon_{i0}^2 = \frac{2}{3} \cdot \frac{\mu_{12} + \mu_{21} \cdot \mu_{13}}{1 - \mu_{12}\mu_{21}}\left(\mu_{12}^{-1}\varepsilon_x^2 + 2\varepsilon_x\varepsilon_y + \mu_{21}^{-1}\varepsilon_y^2\right) + \frac{2C_{11}(\mu_{12} + \mu_{21} \cdot \mu_{13})}{3E_1\mu_{21}}\gamma_{xy}^2$$

Aaqedan SeiZleba davweroT:

$$\sigma_x = \frac{2}{9} \cdot \frac{(\mu_{12} + \mu_{21}\mu_{13})}{(1 - \mu_{12}\mu_{21})}\left(\mu_{21}^{-1}\varepsilon_x + \varepsilon_y\right) \cdot \frac{\sigma_{i0}}{\varepsilon_{i0}}$$

$$\sigma_y = \frac{2}{3} \cdot \frac{(\mu_{12} + \mu_{21}\mu_{13})}{(1 - \mu_{12}\mu_{21})}\left(\mu_{21}^{-1}\varepsilon_y + \varepsilon_x\right) \cdot \frac{\sigma_{i0}}{\varepsilon_{i0}} \quad (1.8.25)$$

$$\tau_{xy} = \frac{2}{3} \cdot \frac{C_{11}(\mu_{12} + \mu_{21}\mu_{13})}{3E_1\mu_{21}} \cdot \frac{\sigma_{i0}}{\varepsilon_{i0}}\gamma_{xy}$$

Nnormalur ZabvaTa intensivoba

$$\bar{\sigma}_i = \sqrt{\mu\left(C_{11}\sigma_1^2 + 2C_{12}\sigma_1\sigma_2 + C_{22}\sigma_2^2\right)}$$

(1.8.26)

$$\varepsilon_i = \sqrt{(\mu \cdot g)^{-1}\left(C_{11}\varepsilon_1^2 + 2C_{12}\varepsilon_1\varepsilon_2 + C_{22}\varepsilon_2^2\right)}$$

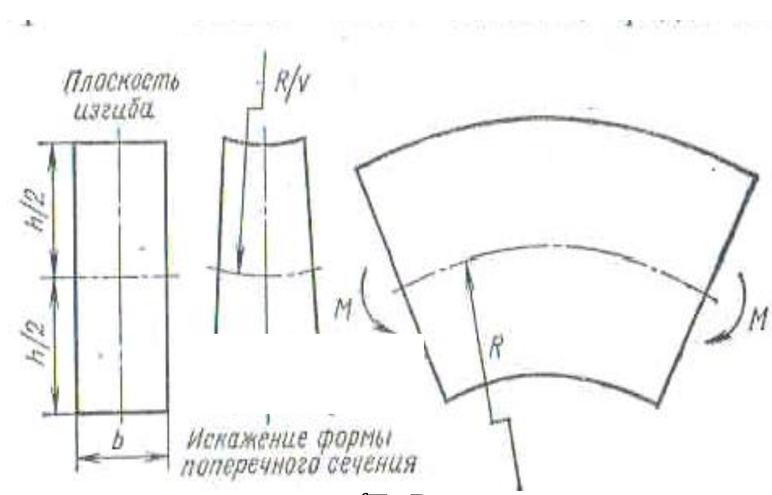
miRebuli formulebi faqtiurad pasuxobs yvela amocanas, denadobis pirobebis saTanado kriteriumTan SesabamisobaSi.

Tavi 2. koWebis, filebis da rgolebis drekad plastikuri Runvis elementaruli analizi

2.1. plastikuri Runvis Teoria

koWis plastikuri Runva erTi SexedviT maTematikur analizs advilad unda eqvemdebarebodes. sinamdvileSi zusti amoxsna jer kidev araa cnobili da yvela arsebuli Teoria iseT daSvebebzea dafuZnebuli, romlebic Sedegebs sxvadasxva saxiT azusteben. es miaxloebiTi Teoriebi Zalian sasargebloa, Tuki maT gakeTebuli daSvebebis farglebSi da eqsperimentalurad Semowmebul kerZo SemTxvevebisaTvis gamoviyenebT [55].

is sirTuleebi, romlebic plastikuri Runvis analizis dros gvxvdeba, SegviZlia SevafasoT marTkuTxa ganivi kveTis iseTi swori koWis ganxilviT, romelsac ganivi kveTis simaRle da sigane erTi rigisa aqvT. Tu aseTi koWi ganicdis sufTa Runvas (nax.2.1.1) e.i. Runvis momenti ucvlelia, maSin ganivi Zalebi ar gvaqvs da denadobis zRvari verc erT wertilSi ver miiRweva da Runvamde brtyeli ganivi kveTebi, Runvis merec rCebian brtyelebad.



sufTa Runvas

Tu R aris Runvis sibrtyeSi moTavsebuli neitraluri RerZis simrudis radiusi, maSin simrude perpendikularul sibrtyeSi iqneba ν/R, sadac ν - puasonis koeficientia. Runvis gagrZelebisas Tavdapirvelad plastikurad deformirdebian boWkoebi, romlebic yvelaze metad arian daSorebuli koWis simetriis RerZidan, Semdeg ki plastikuri deformaciebi TandaTan vrceldeba misi Sua nawilisaken, magram deformirebas kvlavindeburad umetes _ wilad misi centraluri drekadi birTvi gansazRvravs. Tumca, axl puasonis koeficienti (ν) ganivi kveTis mixedviT icvleba; drekadi deformaciebisaTvis is daaxloebiT 0,3-is toli xdeba, xolo plastikuri deformaciebisaTvis 0,5-is toli. drekad da plastikur deformaciebs Soris sazRvarze deformaciis aucilebeli uwyvetobis misaRebad unda SevinarCunoT zogierTi ganivi Zabva. am sirTulis dasaZlevad zogierTi avtori varaudobs, rom gamosakvlevi masala ukumSvadia, e.i. ν=0,5 drekadi da plastikuri deformaciebis dros.

maTematikurad swori analizi ioldeba, magram realur masalasTan mimarTebaSi es yvelaferi ganivi Zabvebis ugulebelyofis tolfasia, (plastikuri Runvis dros ganivi kveTis simrudis Sesaxeb literaturis mimoxilva, mocemulia horoqsis da jonsonis 1967w. statiaSi).

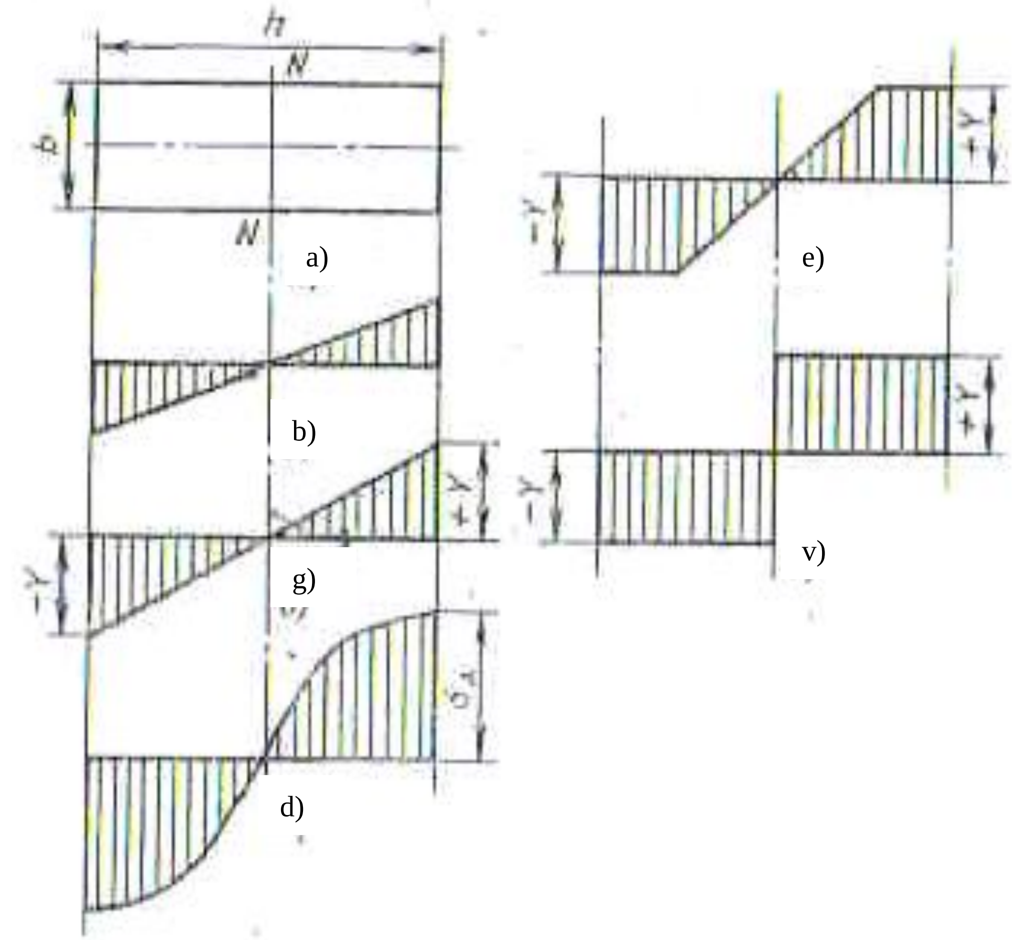
koWebis drekadi ganivi Runvis dros isevea navaraudevi, rogorc wminda Runvis dros, brtyeli kveTebis hipoTezis WeSmariteba. e.i. Runvamde brtyeli kveTebi Runvis merec brtyelebad rCeba im pirobiT, rom mxebi Zabvebi normalur ZabvebTan SedarebiT mcirea. es asea, Tu koWis sigrZe misi ganivi kveTis zomebTan didia. zustad aseve varaudoben plastikuri Runvis drosac, es damatebiTi varaudia ganivi Zabvebis ugulebelyofis zemoT naxsenebi pirobisaTvis.

Runvis elementaruli Teoriis mixedviT, drekadi deformaciebisas ZabvaTa ganawileba iseT koWSi, romelsac marTkuTxa ganivi kveTa aqvs da M momentis zemoqmedebiT ganicdis sufTa Runvas, aris wrfivi nax.2.1.1 maqismaluri Zabvebi aRmoCndeba neitraluri RerZidan $\pm h/2$ manZilze da ganisazRvreba gamosaxulebiT

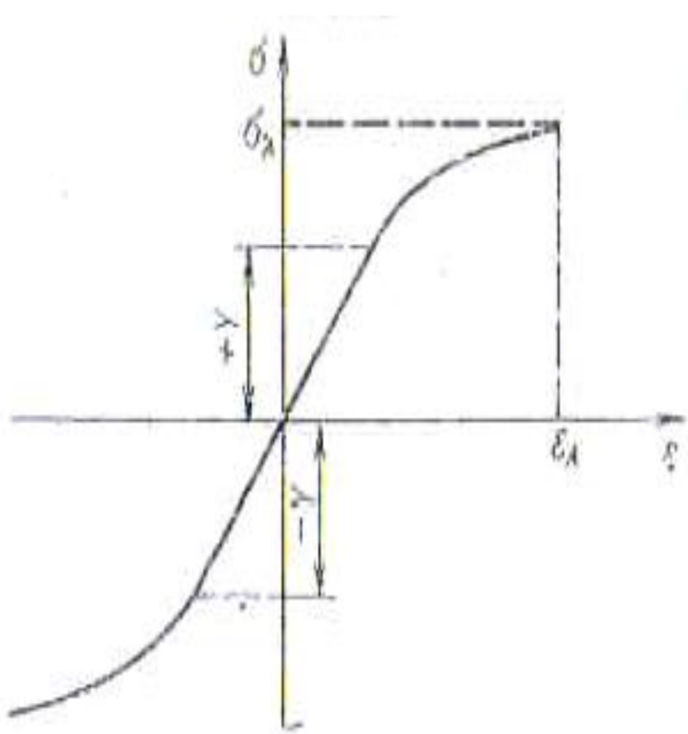
$$\sigma_{max} = \frac{M}{I}\frac{h}{2}$$

sadac I aris koWis ganivi kveTis farTobis inerciis momenti (simkvrivis cvalebadoba) neitraluri NN – is mimarT. davuSvaT, koWi iseTi masalisaganaa, romlis Zabva-deformaciis mrudebi, sufTa kumSvis da sufTa gaWimvisas erTnairia, (nax. 21.3). M – is (mRunavi momenti) zrdisas ZabvaTa ganawileba wrfivi rCeba iqamde sanam maqsimaluri Zabvebi ar gautoldeba denadobis zRvars. σ_{max}=Y (nax. 2.1.2.g). M – is (mRunavi momenti) Semdgomi gazrdisas ZabvaTa ganawileba RerZidan daSorebul boWkoebSi arawrfivi xedba. fardobiTi deformacia nietraluri RerZidan y manZilze, mcire deformaciebisaTvis y/R – is tolia, sadac R aris gaRunuli koWis neitraluri boWkos (Sua nawili) simrudis radiusi. mocemuli M – isaTvis R mudmivia (amasTanave Zalian didia) kveTisaTvis, maSasadame deformacia y – is pirdapirproporciulia, im SemTxvevaSi Tu kveTebi Tavdapirveladac brtyelia da rCeba brtyeli.

Tu Sua nawilidan yvelaze ufro, anu $y = \pm\frac{h}{2}$ manZiliT daSorebuli boWkoebis deformacia aris ε_A - maSin Sesabamisi Zabva σ_A SeiZleba iyos miRebuli uSualod Zabva-deformaciis diagramidan. am SemTxvevaSi ganiv kveTSi ZabvaTa ganawileba gamogvdis Zabva-deformaciis diagramis gadaxatvisas, Zabvis RerZuli an nulovani xazidan (nax. 21.2 d), Tanac erTi nawili dadebiTia (gamWimavi Zabvebi), xolo meore uaryofiTia (mkumSavi Zabvebi). σ/ε mrudis dasawyisi mdebareobs neitralur RerZze.



nax. 2.1.2. Zabvis ganawileba marTkuTxa kveTis mqone koWSi



nax. 2.1.3 Zabva-deformaciis mrudi

dadebiTi da uaryofiTi farTobebis toloba ganapirobebs ganiv kveTaSi moqmed normalur ZabvaTa tolqmedis nulTan tolobas, manam sanam kveTa simetriulia NN RerZis mimarT. Sida mRunavi momenti

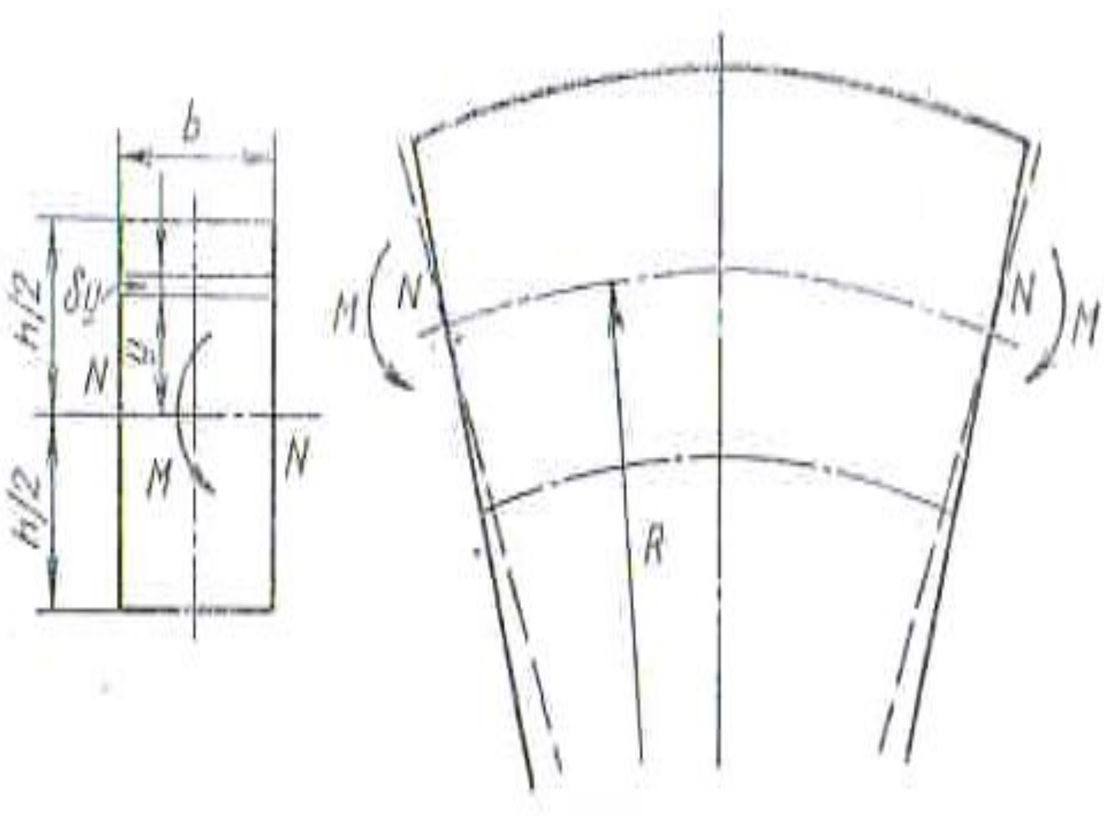
$$M = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \sigma b y dy$$

2.2 Zabvebisa da deformaciebis arawrfivi damokidebulebisas M mRunavi momentis da CaRunvebis gamosaTvleli gamosaxulebebi

davuSvaT, Zabva damokidebulia deformaciaze Semdegnairad:

$$\sigma = E \cdot e + F \cdot e^{n}$$

da aseve, Tu miviCnevT, rom $e = \frac{y}{R}$, da gamoviyenebT aRniSvnebs (nax. 2.2.1) gveqneba:



nax. 2.2.1

$$M = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \left(\frac{Eby^2}{R} + \frac{Fby^{n+1}}{R^n} \right) dy = \frac{E}{R} I_1 + \frac{F}{R^n} I_n$$

sadac

$$I_1 = \int_{-\frac{h}{2}}^{+\frac{h}{2}} by^2 dy \quad \text{da} \quad I_n = \int_{-\frac{h}{2}}^{+\frac{h}{2}} by^{n+1} dy$$

I_1 - aris kveTis inerciis momenti, I_n – aris integrali, romelic warmoiSveba Zabvebsa da deformaciebs Soris arawrfivi damokidebulebebis dros.

I_n – is gamosaxulebas xSirad iyeneben masalaTa gamZleobaSi. Zabva-deformaciebs Soris $\sigma = E \cdot e + F \cdot e^n$ da damokidebulebaSi Tu vivaraudebT E=0, miviRebT:

$$\frac{M}{I_n} = \frac{F}{R^n} = \frac{\sigma}{y^n} \qquad (2.2.1)$$

es im gantolebis ufro zogadi saxea, romelsac inJinrebi Cveulebriv iyeneben drekadi Runvis gamoTvlisas, anu roca n=1. mudmivi b–s dros I_n–is mniSvneloba Semdegia:

$$I_n = \frac{bh^{n+2}}{2^{n+1}(n+2)}$$

Runvis elementarul arawrfiv TeoriaSi saintereso midgoma iyo SemoTavazebuli sen-venanis mier da gadmocemuli iyo timoSenkos wignSi (1953w.). iseTi masalisagan damzadebuli swori koWebisaTvis, romelTac gaWimvisa da kumSvisas Zabva-deformaciis gansxvavebuli mrudebi aqvT. rogorc Runvis elementarul TeoriaSi, dy/dx mcirea da radganac R didia, amitom $\frac{dy}{dx}$ SegviZlia isev ugulebelvyoT simrudis gamoTvlisas.

amrigad,

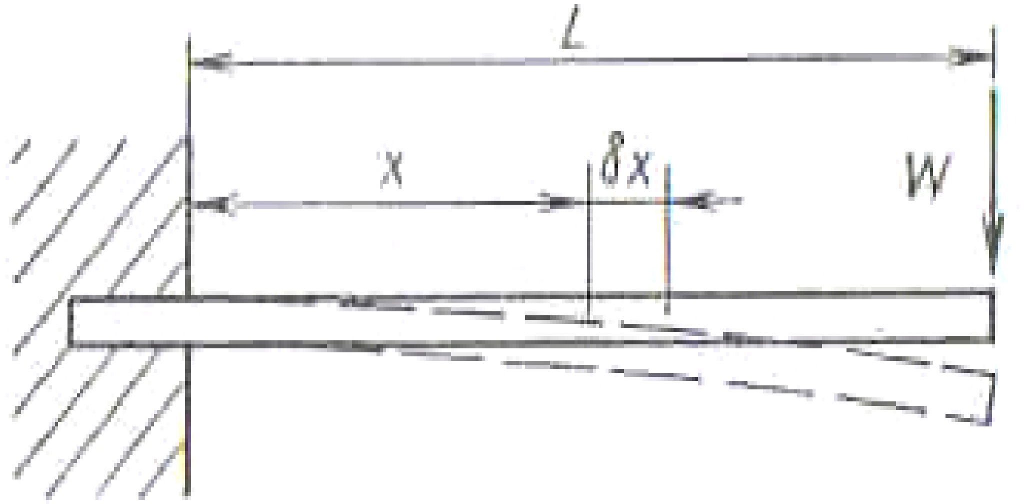
$$\frac{1}{R} = \frac{d^2y/dx^2}{\left[1+(dy/dx)^2\right]^{3/2}}$$

koWis CaRunvis gansazRvrisaTvis gantoleba SeiZleba miviRoT, Tu 1/R gamosaxulebis mniSvnelobas CavsvamT (2.2.1) gantolebaSi.

amrigad

$$\frac{d^2y}{dx^2}=\left(\frac{M}{FI_n}\right)^{1/n}$$

L sigrZis mqone erTi boloTi Camagrebuli konsoluri koWisaTvis, romelsac Tavisufal boloze aqvs modebuli W datvirTva (nax. 2.2.2.)



nax. 2.2.2 P ZaliT datvirTuli konsoluri koWi

$$M=W(L-x)$$

da

$$\frac{dy}{dx}=\frac{-C(L-x)^{1/n+1}}{(1+1/n)}+B$$

dy/dx=0 roca x=0. amitom integrebis mudmiva $B=CL^{1/(n+1)}(1/n+1)$ sadac

$C=(W/FI_n)^{1/n}$. kidev erTxel integrebiT miviRebT

$$y=\frac{c(L-x)^{1/n+2}}{(1/n+1)(1/n+2)}+B\cdot x+D$$

roca x=0, y=0, $D=-CL^{1/n+2}/(1/n+1)(1/n+2)$ amgvarad,

$$y=\frac{CL^{1/n+2}}{(1/n+1)(1/n+2)}\left[\left(1-\frac{x}{L}\right)^{1/n+2}+\frac{x}{L}\left(\frac{1}{n}+2\right)-1\right]$$

δ_w maqsimaluri CaRunva P datvirTvis moqmedebiT

$$\delta_w=\frac{CL^{1/n+2}}{(1/n+1)(1/n+2)}(1/n+1)=W^{1/n}\cdot f$$

drekadi Runvis SemTxvevaSi n=1 $\delta w=\frac{WL^3}{3FI}$

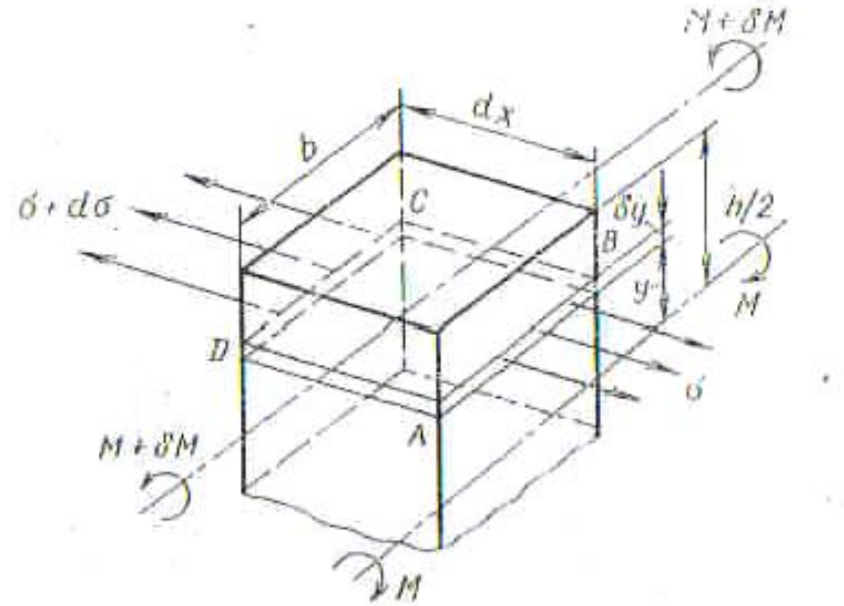
es magaliTi msgavsi tipis problemisadmi midgomis ilustraciaa.

y, $\frac{dy}{dx}$, I_n da δ mniSvnelobebi sxvadsxva rTuli SemTxvevebisaTvis mocemulia filipsis wignSi (1956w).

Tu cali mxriT Camagrebuli koWis meore boloze modebulia W-sgan gansxvavebuli W_1 datvirTva maSin $\delta W_1=W_1^{\frac{1}{n}}\cdot f$. CaRunva erTdroulad moqmedi W da W_1 datvirTvebis dros $\delta=(W+W_1)^{\frac{1}{n}}\cdot f$. aRvniSnoT, rom $\delta=\delta_{W1}+\delta_W$; anu $(W+W_1)^{\frac{1}{n}}\neq W^{\frac{1}{n}}+W_1^{\frac{1}{n}}$ manamde, sanam $n\neq1$. aqedan gamomdinare, superpoziciis principi samarTliani iqneba mxolod roca n=1.

2.3. mxebi Zabvebis ganawileba. idealuri masalis koWis Runva

Zabvis deformaciaze damokidebulebis gamoyenebis erT-erTi martivi magaliTia, marTkuTxa koWSi ganivi Runvisas mxebi Zabvebis gadanawilebis gamovlena. (nax. 2.3.1)-is mixedviT, sadac naCvenebia (nax. 2.2.2)-ze moyvanili koWis δ_x nawili, mxebi Zabva ABCD sibrtyeSi masalaTa gamZleobis elementaruli wesebis mixedviT tolia.



nax. 2.3.1 koWis elementis wonasworoba ganivi CaRunvisas

$$\tau \cdot b\delta x = \int_{y}^{\frac{h}{2}} b \cdot \frac{\partial \sigma}{\partial x} \delta x dy \qquad (2.3.1)$$

(2.3.1) gantolebidan gamomdinare $M = \frac{\sigma I_n}{y^n}$ amitom

$$\frac{\partial M}{\partial x} = \frac{I_n d\sigma}{y^n dx},$$

magram $\frac{\partial M}{\partial x} = -W$; $\frac{\partial \sigma}{\partial x} = \frac{-Wy^n}{I_n}$

maSasadame

$$\tau = \int_{y}^{\frac{h}{2}} \frac{-W}{I_n} y^n dy = \frac{-W\left[\left(\frac{h}{2}\right)^{n+1} - y^{n+1}\right]}{I_n(n+1)}$$

mocemuli masalis koWis neitralur RerZze maqsimaluri mxebi Zabvis Sefardeba wrfivi drekadobis mqone masalis koWis neitralur RerZze maqsimalur mxeb ZabvasTan tolia

$$\frac{2(n+2)}{3(n+1)}.$$

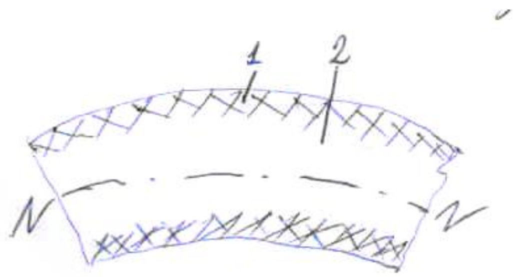
idealuri drekad-plastikuri masalis sworkuTxa koWis Suidan yvelaze ufro daSorebul boWkoebSi denadobis Zabvis warmoSobisaTvis aucilebeli M_E momenti:

$$M_E = bh^2 Y / 6 \qquad (2.3.2)$$

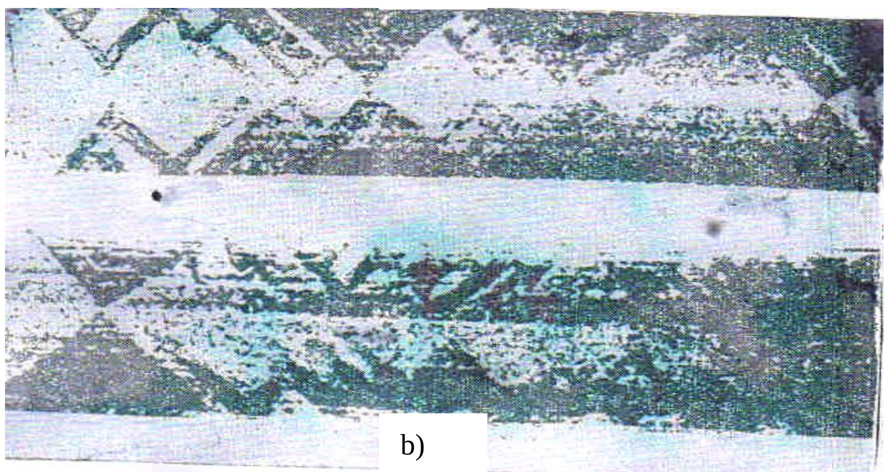
roca modebuli momenti $M > M_E$, kveTSi Zabvebis ganawileba iseTia, rogorc nax. 2.1.2.e-ze koWis simrudis radiusis Semcirebisas plastikurad deformirebuli Sris sisqe izrdeba da drekad da plastikur deformaciebs Soris sazRvari uaxlovdeba neitralur RerZs. roca es sazRvari neitraluri RerZidan y manZilzea, mRunavi momenti ganisazRvreba Semdegi gamosaxulebidan:

$$M = Yb\left(3h^2 - 4y^2\right)/12 \qquad (2.3.3)$$

masalis denadoba, romlisagac damzadebulia koWi Tu am koWis sigane da sisqe Tanazomadia – SemTxvevaa, romelic axlosaa brtyel deformaciasTan da romelsac axlavs muqi xazebis gaCena zedapirTan 45^0 kuTxiT. es xazebi gansazRvraven koWis plastikurad deformirebul nawils. muq zolebs Soris moCans zedapiris naTeli nawili, romelic Seesabameba drekad mdgomareobas. zolebis pikebi gansazRvraven plastikuri da drekadi deformaciebis sazRvris mdebareobas. mRunavi momentis gazrdisas es sazRvari uaxlovdeba neitralur RerZs. drekadi are neitraluri RerZis siaxloves swrafad mcirdeba (nax. 2.3.2 a da b).



a)



b)

nax. 2.3.2 rbili foladis firfitaSi ZalTa wyvilis modebis Sedegad warmoqmnili plastikuri deformaciebi 1. – srialis (cocvadobis) xazi zedapirisadmi ±45° kuTxiT 2. – drekadi birTvi

M-is sididis miuxedavad, yovelTvis gvaqvs garkveuli drekadi birTvi, romlis gamoc SeiZleba warmoiqmnas ZabvaTa Zalian didi gradientebi. rac ufro mcirea es drekadi birTvi, miT ufro mcire iqneba cdomileba, Tu vivaraudebT, rom Runvis dasrulebisas koWs plastikuri deformaciebi mTeli kveTis gaswvriv aqvs, Tu ar CavTvliT koWis simrudis radiuss, romelic gamogviva arsebulze gacilebiT mcire. nax. 2.1.2. v-ze naCvenebia ZabvaTa gadanawileba datvirTvis am zRvruli SemTxvevisaTvis.

rac ufro axlosaa daZabuli mdgomareoba am SemTxvevasTan, miT ufro naklebi xdeba koWis simrudis radiusi da miT ufro naklebad gamodgeba pirveladi daSvebebi saanalizod. am ukidures SemTxvevaSi neitralur RerZze yovelTvis Cndeba moulodneli naxtomi +Y-dan -Y-mde e.i. ZabvaTa gaxleCva. magram M_p mTels kveTaSi plastikuri deformaciebis misaRebad aucilebeli momenti, SeiZleba ganvsazRvroT (2.3.3)-dan. Tu CavTvliT, rom y=0.

$$M_p = \frac{bh^2 Y}{4} \qquad (2.3.2)$$

(2.3.2) gantoleba aseve SegviZlia amovxsnaT, Tu miviCnevT, rom koWi idealuri xist-plastikuri masalisagan aris damzadebuli. radganac xist-plastikur masalaSi drekadi deformaciebi nulis tolia, koWis raime deformacia an CaRunva manamde ver gaCndeba, sanam momenti ar gaxdeba imdenad didi, rom plastikuri deformaciebi mTels kveTaSi gavrceldes. drekadi birTvis nebismier dros arseboba gulisxmobs srul sixistes.

rodesac mTels kveTaSi gaCndeba plastikuri deformaciebi, koWi gare datvirTvebis mimarT winaaRmdegobis unars dakargavs, anu gaagrZelebs Runvas momentis zrdis gareSe. mRunavi momenti M_p erTnaxevarjer metia M_E –ze. es mowmobs imaze, rom Runvisas drekad nawilebs aqvs simtkicis mniSvnelovani maragi, maTi plastikur mdgomareobaSi gadayvanis SesaZleblobebis wyalobiT. es daskvna SeiZleba iqnes gamoyenebuli saangariSo sqemebis arCevisas, radganac is gacilebiT ufro nakleb sirTuleebs iwvevs drekadi analizis dros gamoyenebul meTodebTan SedarebiT. es meTodi gamoiyeneba koWebis, fermebis da rgolebis datvirTvis zogierTi martivi da gavrcelebuli SemTxvevis kvlevisaTvis.

2.4. formis gavlena. iribi (asimetriuli) plastikuri Runva

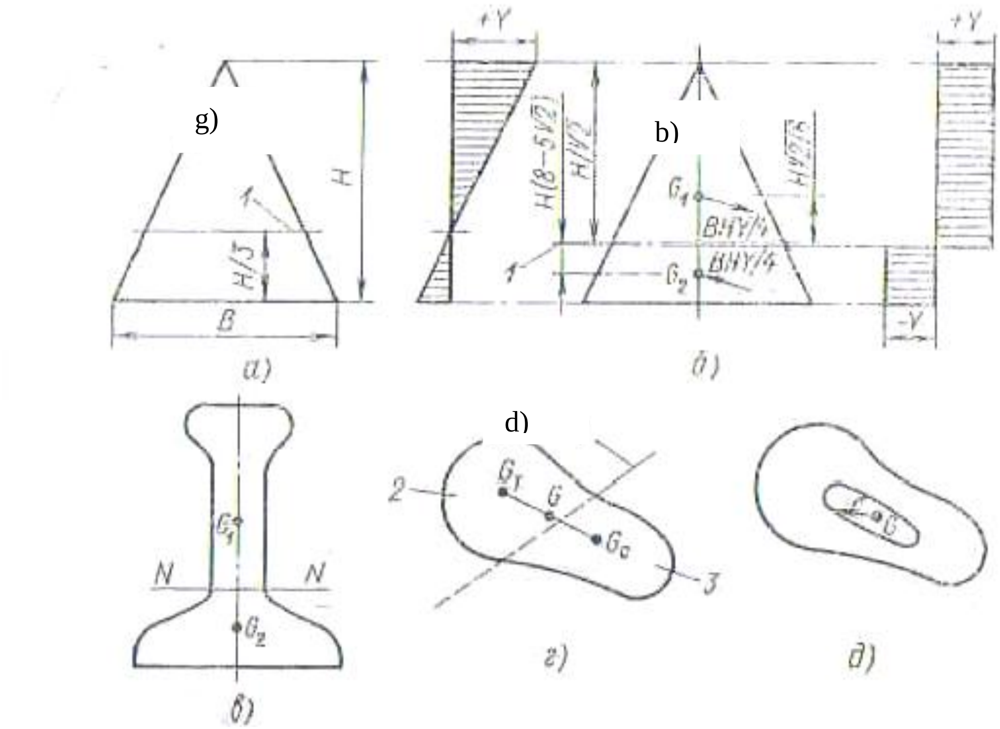
idealuri drekadplastikuri masalisagan damzadebuli marTkuTxa kveTis koWisaTvis $\frac{M_p}{M_E} = 1{,}5$. mTels kveTaSi plastikuri deformaciebis warmosaqmnelad saWiro mRunavi momentis zemoT moyvanil Sefardebas imave kveTaSi drekadi deformaciebis dros, denadobis zRvars axladmiRweuli ganapira boWkoebis gamoklebiT, arsebul mRunav momentTan, formis koeficienti ewodeba.

advili saCvenebelia, rom *a* radiusis wriuli kveTis koWisaTvis

$$M_p = \frac{4}{3}a^3Y \quad \text{da} \quad M_E = \frac{\pi}{4}a^3Y$$

am SemTxvevaSi formis koeficientia $\frac{16}{3\pi} \approx 1{,}7$, naTelia, rom wriuli kveTis koWis unari gauZlos mTeli kveTis plastikur deformirebis Sesabamis zRvrul datvirTvas, daaxloebiT 41%-iT aris arasakmarisad Sefasebuli, Tuki denadobis zRvari dgeba mxolod mocemuli kveTis simZimis Zalis centridan yvelaze metad dacilebul boWkoebSi.

arasimetriuli ganivi kveTis mqone koWebisaTvis pirvel rigSi neitraluri sibrtyis povnaa saWiro. sufTa wrfivi drekadi Runvisas neitraluri sibrtye kveTis centrze gaivlis, magram ganiv kveTaSi plastikuri deformaciebis warmoqmnisas is ukve aRar emTxveva drekadi deformirebisaTvis gansazRvrul sibrtyes. neitraluri sibrtyis mdebareoba icvleba koWSi plastikuri Sris siRrmis zrdasTan erTad. magram neitraluri sibrtyis mdebareoba plastikuri deformaciebiT ukve mTlianad moculi koWis kveTaSi advili dasadgenia Tu iseve, rogorc drekadi Runvisas, ZalTa wonasworobis pirobas ganvixilavT. mxolod mRunavi momentiT datvirTuli koWisaTvios, neitraluri sibrtyis cal mxares arsebul gamWimav ZalaTa jami toli unda iyos meore mxares arsebuli mkumSav ZalaTa jamisa. radganac Runvis normaluri Zabva mudmivia kveTaSi da tolia Y, wonasworobis pirobidan gamomdinareobs, rom neitraluri sibrtye saWiroa ise iyos ganlagebuli, rom mis zeviT da mis qvemoT mdebare kveTis nawilebis farTobebi iyos toli.



nax. 2.4.1 asimetriuli ganivi kveTis mqone koWis Runva

magaliTisaTvis ganvixiloT koWi, romlis kveTa H simaRlisa da B fuZis mqone tolferda samkuTxedia. am SemTxvevaSi Runvisas nebismieri neitraluri sibrtye fuZis paraleluri iqneba (nax. 2.4.1 a,b). sufTa drekadi Runvisas neitraluri RerZi fuZidan H/3 manZiliT maRlaa. advili saCvenebelia, $M_E = \frac{YBH^2}{24}$, roca samkuTxedis wveroSi Runvis Zabvas denadobis zRvari axali miRweuli aqvs.

mTlianad plastikuri Runvisas neitraluri RerZi wverodan $\frac{H}{\sqrt{2}}$-iT qvemoTaa. SeiZleba damtkicdes, rom gamWimav ZabvaTa tolqmedi gaivlis neitraluri RerZis zemoT ganlagebuli kveTis samkuTxa nawilis simZimis centrze, e.i. G_1 wertilze, xolo mkumSav ZabvaTa tolqmedi gaivlis neitraluri RerZis qvemoT mdebare trapeciis simZimis centrze, anu G_2 wertilze,

$$M_p = \left(2-\sqrt{2}\right)\frac{YBH^2}{6}$$

am kveTisaTvis formis koeficientia

$$4 \cdot \left(2 - \sqrt{2}\right) \approx 2{,}34$$

koWebis kveTebs erTi simetriis RerZiT, rogorc axlaxan ganxilul SemTxvevaSi, SesaZloa hqondeT iseTi konturi, romlis aRwera mxolod ricxobrivad SeiZleba, magaliTad nax.2.2.4.1 g-ze naCvenebi kveTa. praqtikuli amocanebis umravlesobisaTvis mTels kveTaSi plastikuri deformaciebis gamomwvevi mRunavi momenti SeiZleba vipovoT martivi eqsperimentis daxmarebiT. liTonis an muyaos Txeli furclisagan amoWrian ganiv kveTas da planimetrirebiT mis farTobs gansazRvraven. plastikuri Runvisas neitraluri NN sibrtyis mdebareoba Semdegnairad SeiZleba ganvsazRvroT: furcels NN-ze gaWrian, NN-is zemoTa da qvemoTa farTobebi saWiroa toli iyos da danis wverze yoveli nawilis balansirebiT pouloben maTi simZimis Zalis centrebs anu G_1 da G_2-is. maSin $\frac{M_p}{Y}$ tolia kveTis farTobis G_1 da G_2 Soris manZilze namravlis naxevris. ganmtkicebadi masalisagan damzadebuli koWebisaTvis saWiroa gamoviyenoT grafikuli da grafikuli ricxviTi meTodebi.

iribi (asimetriuli) Runva xdeba maSin, roca modebuli mRunavi momentis sibrtye ganivi kveTis simetriis RerZis arc paraleluria da arc perpendikularuli. dacerebul plastikur Runvas ganixilavdnen iohanseni, harisoni, bareti da brauni.

nax. 2.2.4.1 d-ze mocemulia koWis A farTobis ganivi kveTis saerTo xedi nebismierad arCeuli neitraluri xaziT. Tu kveTa plastikuri deformaciebiT mTlianadaa moculi, maSin ZalTa wonasworobidan gamomdinare, mRunav ZabvaTa kveTisadmi marTobulobis wyalobiT, misi farTobis nawili A_T, romelzedac moqmedeben gamWimavi Zabvebi neitraluri RerZis cali mxridan, saWiroa iyos toli danarCeni nawilis A_C farTobisa, romelzedac moqmedeben mkumSavi Zabvebi meore mxridan. saWiroa daculi iyos piroba:

$$YA_T - YA_c = 0$$

maSasadame $A_T = A_c$.

yoveli mocemuli mimarTulebisaTvis, neitraluri RerZis mdebareoba SeiZleba mocemuli kveTis biseqtrisebis ojaxidan miviRoT.

gamWimav ZabvaTa $A_T Y$ tolqmedi gaivlis A_T farTobis G_T centrze, mkumSav ZabvaTa $A_C Y$ tolqmedi ki A_C farTobis G_C centrze. aqedan gamomdinare, kveTis sruli plastikuri momenti

$$M_p = rAY$$

sadac M_p -s mimarTuleba $G_T G_C$ monakveTis perpendikularulia.

minakveTis sigrZe $G_T G_C = 2r$, xolo misi Sua wertili cxadia, rom aris kveTis centri G.

G_T da G_C fokusebs centroidaluri fokusebi ewodeba (nax. 2.4.1 d,e). Tu es fokusebi cnobilia, maSin M_p-s povna nebismieri RerZis mimarT advilia Tu CavTvliT, rom r - monakveTi G-dan fokusamde – momentis mimarTulebis perpendikularulia. Semdgom, centroidaluri fokusebis mrudisadmi r radiusiT mocemuli wertilidan gavlebuli mxebi gansazRvravs neitraluri RerZis mimarTulebas. es gamomdinareobs iqidan, rom centriodanul fokusebs Soris mdebare mocemuli monakveTi aris mRunavi momentis mimarTulebis perpendikularuli. momijnave centroidaluri fokusis monakveTi asevea perpendikularuli momijnave mRunavi momentis mimarTulebis da amgvarad, zRvarze gadasvlisas, ori diametris boloebSi mdebare wertilebis SemaeTebeli wirisadmi gavlebuli mxebebi Sesabamisi mRunavi momentebis mimarTulebebis paralelurebi unda iyos da Sedegad, warmoSvan neitraluri xazi.

2.5 plastikuri Runva Semdgomi drekadi gantvirTviT (narCen ZabvaTa ganawileba, drekadi zambarireba, brtyeli deformacia)

araganmtkicebadi masalisagan damzadebuli marTkuTxa kveTis mqone koWi (ix. nax. 2.5.1) datvirTulia ise, rom ganivi kveTa aris drekadplastikuri. (ix. nax. 2.5.1. e). amisaTvis aucilebeli mRunavi momenti tolia $\frac{bh^2 Y}{4}$. koWis gantvirTva igive uaryofiTi $\frac{bh^2 Y}{4}$ -s sididis momentis damatebis tolfasia.

brtyeli daZabuli mdgomareobisas idealuri drekadplastikuri masalis koWSi narCen Zabvebs, roca M_E<M<M_p gansazRvraven iseve, rogorc axlaxans ganxilul SemTxvevaSi, roca M= M_p. nax. 2.5.1 b-ze naCvenebia (A,BDOD′B′A′A) Zabvebis ganawilebis epiura, romlebsac M warmoSobs ACOC′A′A ki aris drekadi zambarirebis Zabvebis ganawilebis epiura M momentis gantvirTvis dros.

Tu drekad-plastikuri Runvisas koWis neitraluri RerZis simrudis radiusia R, maSin $\frac{y}{R} = \frac{Y}{E}$. Tu CavsvavT y-s (2.3.3) gantolebaSi, miviRebT

$$M = b\left[3h^2Y - 4Y^3R^2 / E^2\right]/12 \qquad (2.5.1.)$$

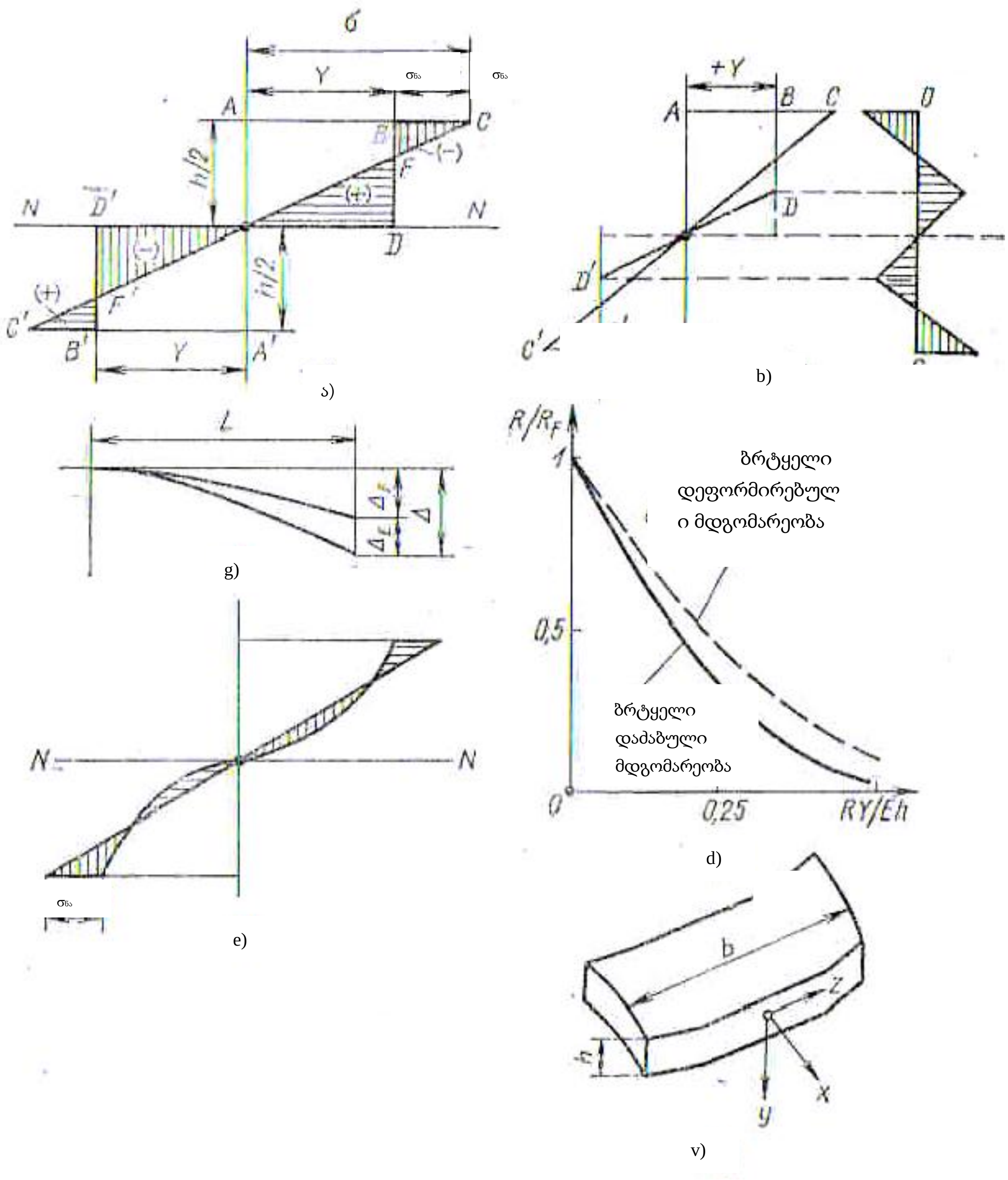
zambarirebis Sedegad radiusis simrudis cvlilebaa R_E da

$$M = EI / R = Ebh^2 / 12R_E \qquad (2.5.2)$$

nax. 2.5.1 g.-ze naCvenebia koWi, romelic M-is moqmedebis Sedegad miiRebs Δ CaRunvas. gantvirTvisas drekadi zambarireba aris Δ_E , maSin narCeni CaRunva $\Delta_F = \Delta - \Delta_E$ radganac $\Delta 2R \approx L^2$ maSin

$$\frac{1}{R_F} = \frac{1}{R} - \frac{1}{R_E} \text{ A an ANN } \frac{R}{R_F} = 1 - \frac{R}{R_E} \qquad (2.5.3)$$

sadac R_F narCeni simrudis radiusia.



nax. 2.5.1 idealuri drekadplastikuri masalis koWSi narCeni Zabvebis da drekadi zambarirebis Zabvebis ganawileba

(2.5.1) da (2.5.2) amoxsniT da Sedegebis (2.5.3)-Si CasmiT miviRebT:

$$\frac{R}{R_F} = 1 - 3\left(\frac{YR}{Eh}\right) + 4\left(\frac{YR}{Eh}\right)^3 \qquad (2.5.4)$$

$$\frac{R}{R_F} = \left(\frac{YR}{Eh} + 1\right) \cdot \left(2\frac{YR}{Eh} - 1\right)^2 \qquad (2.5.5)$$

roca $\frac{R}{R_F} = 0$ es aris sruli zambarireba, e.i. CaRunva drekadia. Tu $\frac{R}{R_F} = 1$, maSin zambarireba ar xdeba. nax. 2.5.1.d-ze naCvenebia $\frac{R}{R_F}$ -is $\frac{YR}{Eh}$ -ze damokidebulebis grafiki.

ukanasknel qveTavebSi Runva ganixileboda brtyeli daZabuli mdgomareobis pirobebSi. b da h zomebi daaxloebiT toli iyo da amitom CaRunvis sibrtyis marTobul sibrtyeSi didi Zabvebi ar warmoiSveboda. Tu b mravaljer metia h-ze, maSin ganivi kveTis simrudis radiusi 0-is tolia, Tu ar CavTvliT gverdiT waxnagebTan mdebare or mcire ares. koWis didi nawili (nax.2.5.1 e) brtyelia, amitom e_z deformacia b an z mimarTulebiT 0-is tolia. sufTa drekadi deformaciebisaTvis

$$e_z = 0 = \sigma_z - \nu\sigma_x / E \qquad (2.5.6)$$

σ_y yvelgan iTvleba 0-is tolad, ν - puasonis koeficientia,

$$e_x = (\sigma_x - \nu\sigma_z)/E = y/R$$

Sesabamisad $\sigma_x = Ey/R + \nu\sigma_z$ (2.5.7)

Tu CavsvavT σ_z –s (2.5.3)-dan (2.5.7)-Si da gardavqmniT, miviRebT: $\sigma_z = \frac{Ey}{(1-\nu^2)R}$ amitom

$$M_E = \int_{-h/2}^{+h/2} \sigma_x y dy = Eh^3/12R(1-\nu^2) = E'h^3/12R \qquad (2.5.8)$$

(2.5.8)-dan Cans, rom brtyeli deformirebuli mdgomareobisas Cven unda gamoviyenoT $E/(1-\nu^2) = E'$. brtyeli daZabuli mdgomareobis dros gamosayenebli E-s nacvlad. maSin gamosaxuleba Runvisas brtyeli deformirebuli mdgomareobisaTvis eqvivalenturia (2.5.5)-isa:

$$\frac{R}{R_F}=1-3\left(\frac{YR}{Eh}\right)\left(1-\nu^2\right)+4\left[\frac{YR}{Eh}\left(1-\nu^2\right)\right]^3 \qquad (2.5.9)$$

es grafikulad mocemulia (nax.2.5.1 d)-ze. SeiZleba SevniSnoT, rom denadobis Zabva Runvisas brtyeli brtyeli deformirebuli mdgomareobis dros mizesis pirobis mixedviT tolia $\left(2\sqrt{3}\right)Y$, treskas pirobis mixedviT ki Y. eqsperimentidan Cans, rom ukanaskneli Sedegi Seesabameba rbili foladis martiv Runvas.

realuri masalis koWSi narCeni Zabvebi SeiZleba Sefasdes amgvaradve, Zabva-deformaciis WeSmariti mrudis gamoyenebiT, Tumca gamoTvlebi, romlebic aucilebelia narCeni Zabvebis epiuris asagebad, damRlelia (nax.2.5.1 v) isini gamarTlebulia im SemTxvevaSi, Tu narCeni Zabvebi yvelgan naklebia, vidre masalis denadobis zRvari, rodesac nimuSs xelmeored vtvirTavT mopirdapire niSnis momentiT. am SemTxvevaSi denadobis Tavdapirveli zRvari iqneba naklebi, ramdenadac am ar SeiZleba bauSingeris efeqtis daviwyeba. principSi ar arsebobs aseve gansakuTrebuli sirTuleebi im SemTxvevaSic, Tu gvaqvs aramarTkuTxa kveTis mqone koWis Runva.

3.7. firfitis plastikuri Runvis ZiriTadi gantolebebis integreba

rogorc cnobilia, firfitis wonasworobis diferencialur gantolebas aqvs saxe:

$$M_r^2 - M_r M_\theta + M_\theta^2 = M_s^2 \qquad (3.7.1)$$

sadac $M_s=\sigma_s\frac{h^2}{4}$.

rogorc aqedan Cans, M_s xasiaTdeba denadobis zRvriT. integrebis gasaadvileblad SemovitanoT nadais funqcia ψ, romlis saSualebiTac mRunavi momentebi Semdegnairad gamoisaxeba:

$$M_r = \frac{2}{\sqrt{3}} M_\theta \cos\psi, \quad M_\theta = \frac{2}{\sqrt{3}} M_s \cos\left(\psi - \frac{\pi}{3}\right) \qquad (3.7.2)$$

nadai aRniSnul funqcias iyenebs ZabvaTa TeoriaSi. rogorc cnobilia, iTvleba rom plastikur zonaSi Zabvebis gamoTvlis amocana statikurad rkvevadia. sinamdvileSi es statikurad rkvevadoba pirobiTia, radgan Zabvebi gamoiangariSeba ara mxolod statikis erTi gantolebidan, aramed plastikurobis pirobidanac. nadais funqcias Cven gamoviyenebT firfitebis Runvis ZiriTadi gantolebebis integrebisaTvis.

amgvari xerxis gamoyenebiT wonasworobis diferencialuri gantoleba igivurad kmayofildeba. axla mRunavi momentebis intensivobis gamosaxulebebi (3.7.2) SevitanoT (3.7.1)-Si. martivi gardaqmnebis Semdeg miviRebT:

$$\frac{dr}{r} = \frac{\sin\psi d\psi}{\frac{\sqrt{3}}{2} \cdot \frac{Q \cdot M}{M_s} \sin\left(\psi - \frac{\pi}{6}\right)} \qquad (3.7.3)$$

am gantolebis integreba SesaZlebelia Caketili formiT, Tu Q=0, anu sufTa Runvis SemTxvevaSi, da im SemTxvevaSic, roca

$$Q = \frac{a}{r} \qquad (3.7.4)$$

sadac *a* mudmivi sididea. ukanasknel SemTxvevas (3.7.4) adgili eqneba maSin, roca wriuli fila datvirTulia Seyursuli ZaliT firfitis centrSi an konturis Sesabamisad koncentruli wris SigniT Tanabrad ganawilebuli datvirTviT. sxva saxis datvirTvebis

SemTxvevaSi wonasworobis diferencialuri gantolebis integreba unda Sesruldes ricxviTi meTodebiT. Cvens naSromSi am mizniT gamoyenebulia energetikuli meTodi. zeda Sefasebis kriteriumidan gamomdinare davubrundeT am amocanas. jer ganvixiloT SemTxveva Q=0 (3.7.3)-dan miviRebT:

$$\frac{dr}{r} = \frac{\sin\psi d\psi}{\sin\left(\psi - \frac{\pi}{6}\right)} \qquad (3.7.5)$$

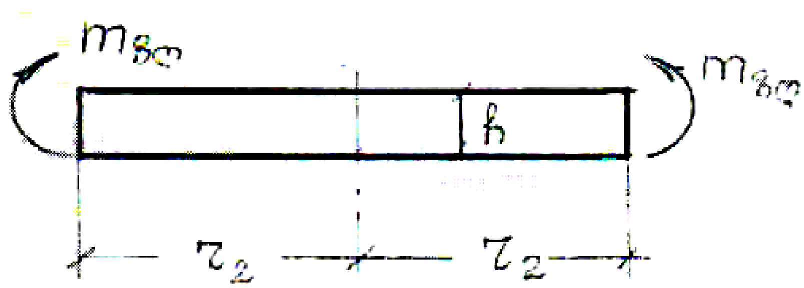
am gantolebis integrali gamoTvlilia Cvens mier da igi tolia

$$\frac{C}{r^2} = e^{\sqrt{3\psi}} \cdot \sin\left(\psi - \frac{\pi}{6}\right) \qquad (3.7.6)$$

aq C integrebis mudmivaa. gadavuideT am (3.7.5) gantolebis (3.7.6) amonaxsnis gamoyenebaze zRvruli datvirTvis gamosaTvlelad konkretuli amocanebisaTvis.

magaliTi 1

r_2 radiusis mqone wriuli firfita datvirTulia konturze Tanabrad ganawilebuli m_{zR} momentiT (nax.3.7.1)



nax. 3.7.1

cxadia, rom amgvari datvirTvis moqmedebis dros saqme gvaqvs sufTa RunvasTan, anu ganivi Zala $Q=0$.

sasazRvro pirobebs eqnebaT Semdegi saxe:

roca $r=0$, $M_r=M_\theta$ (3.7.7)

roca $r=r_2$, $M_r=m_{zR}$

pirveli saszRvro pirobis gamoyenebiT (3.7.2)-sa da (3.7.6)-is saSualebiT miviRebT

$$\psi=\frac{\pi}{6}; \qquad C=0 \tag{3.7.8}$$

da Sesabamisad r-is yvela mniSvnelobisaTvis $\psi=\frac{\pi}{6}$, amis gaTvaliswinebiT meore sasazRvro pirobidan davadgenT, rom

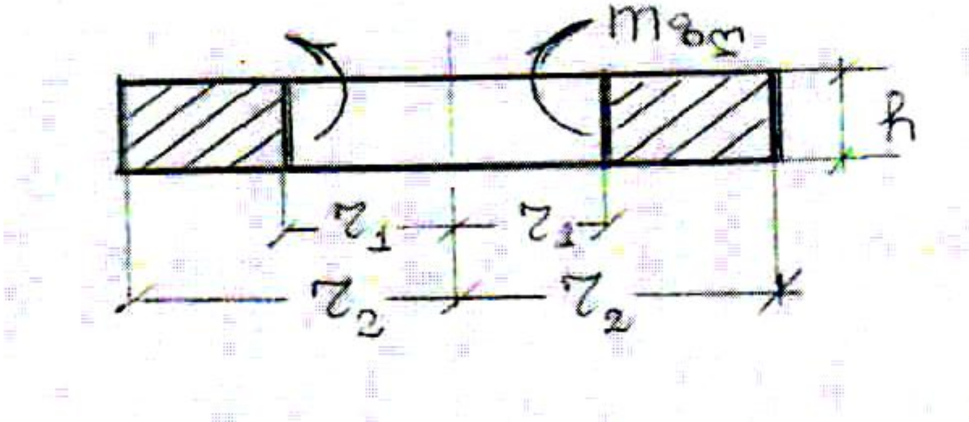
$$m_{zR}=M_s \tag{3.7.9}$$

magaliTi 2

rgoluri fila Sida radiusiT r_1 da gare radiusiT r_2 Sida konturze datvirTulia Tanabrad ganawilebuli m_{zR} momentiT (nax. 3.7.2) iseve, rogorc wina magaliTSi, aqac $Q=0$ xolo sasazRvro pirobebs eqnebaT Semdegi saxe roca $r_1=r$, $M_r=M_s$ $r_2=r$, $M_r=-M_s$ meore sasazRvro pirobidan (3.7.2)-isa da (3.7.6)-is gamoyenebiT miviRebT:

$$\psi_2=\frac{3}{2}\pi \quad c=-\frac{\sqrt{3}}{2}e^{\frac{3\sqrt{3}\pi}{2}}\cdot r_2^2 \tag{3.7.10}$$

aq gaTvaliswinebulia agreTve isic, rom gareTa konturze, sadac $r=r_2$, mRunavi momentis intensivoba M_s uaryofiTia.



nax. 3.7.2

pirveli sasazRvro pirobidan, kvlav (2) da (6) gamosaxulebebis gamoyenebiT davadgenT, rom

$$\frac{r_2^2}{r_1^2} = -\frac{2}{\sqrt{3}} e^{\sqrt{3}\left(\psi_1 - \frac{3}{2}\pi\right)} \cdot \sin\left(\psi_1 - \frac{\pi}{6}\right) \qquad (3.7.11)$$

xolo

$$m_{zR} = \frac{2}{\sqrt{3}} M_s \cos\psi_1 \qquad (3.7.12)$$

am gamosaxulebaSi ψ_1 aris ψ funqciis mniSvneloba firfitis Siga konturze. (3.7.3)-dan M_2/M_1 konkretuli SefardebisaTvis SesaZlebelia ψ_1-is gamoTvla, ris Semdegac (3.7.4)-is saSualebiT davadgenT momentis intensivobis zRvrul sidides.

3.7.12 gamosaxulebaze dakvirvebis Semdeg mivalT daskvnamde, rom momentis intensivobis maqsimums adgili iqneba, roca $\psi_1=2\pi$ da toli iqneba

$$m_{zR.\max} = \frac{2}{\sqrt{3}} M_s \qquad (3.7.13)$$

SevitanoT $\psi_1 = 2\pi$ mniSvneloba (3.7.3)-Si miviRebT

$$\frac{r_2^2}{r_1^2} = \frac{e^{\frac{\sqrt{3}}{2}\pi}}{\sqrt{3}} = (2{,}963)^2 \qquad (3.7.14)$$

aqedan gamomdinareobs Semdegi: rodesac rgolur filaSi gare radiusis Sefardeba Sida radiusTan $r_2/r_1 > 2{,}963$ datvirTvis intensivobis aranair sidides ar SeuZlia miiyvanos rgoluri firfita mzidunarianobis amowurvamde, radgan misi nawili yovelTvis drekad stadiaSi darCeba.

axla ganvixiloT (3.7.3)-is integrebis sakiTxi im SemTxvevisaTvis, roca Q=a/r. miviRebT:

$$\frac{dr}{r} = \frac{\sin\psi d\psi}{b - \sin\left(\psi - \frac{\pi}{6}\right)} \qquad (3.7.15)$$

sadac

$$b = \frac{\sqrt{3}}{2} \cdot \frac{a}{M_s} \qquad (3.7.16)$$

am diferencialuri gantolebis integrali warmodgeba Semdegi gamosaxulebis saxiT:

$$\ln\frac{C}{r^2} = \sqrt{3}\psi + \ln\left[b - \sin\left(\psi - \frac{\pi}{6}\right)\right] - \sqrt{3b}\int\frac{d\psi}{b - \sin\left(\psi - \frac{\pi}{6}\right)} \qquad (3.7.17)$$

aq C integrebis mudmivaa. (3.7.17)-is ukanaskneli integralis aReba damokidebulia b sididis erTTan Sedarebis Sedegze. roca b>1, miviRebT:

$$\int \frac{d\psi}{b-\sin\left(\psi-\frac{\pi}{6}\right)} = \frac{2}{\sqrt{b^2-1}} \operatorname{arctg} \frac{b\,tg\frac{1}{2}\left(\psi-\frac{\pi}{6}\right)-1}{\sqrt{b^2-1}} \qquad (3.7.18)$$

roca b<1, miviRebT:

$$\int \frac{d\psi}{b-\sin\left(\psi-\frac{\pi}{6}\right)} = \frac{2}{\sqrt{b^2-1}} \ln \frac{b\,tg\frac{1}{2}\left(\psi-\frac{\pi}{6}\right)-1-\sqrt{1-b^2}}{b\,tg\frac{1}{2}\left(\psi-\frac{\pi}{6}\right)-1+\sqrt{1-b^2}} \qquad (3.7.19)$$

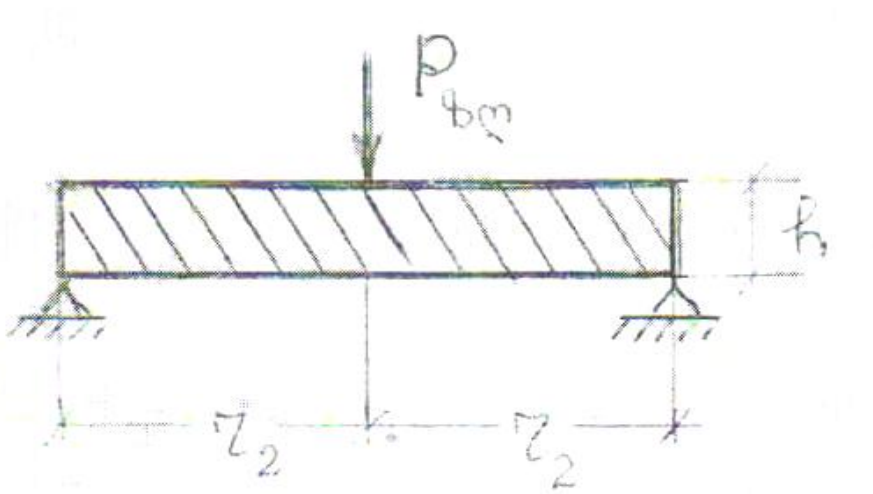
sasazRvro SemTxvevaSi, roca b=1 miviRebT:

$$\int \frac{d\psi}{b-\sin\left(\psi-\frac{\pi}{6}\right)} = tg\frac{1}{2}(\psi+3) \qquad (3.7.20)$$

am magaliTisaTvis miRebuli integralebi (3.7.18), (3.7.19) da (3.7.20) gamogvadgeba wriuli da rgoluri firfitebis sxvadasxva SemTxvevaSi zRvruli datvirTvebis dasadgenad.

magaliTi 3

wriuli fila dayrdobilia konturze da datvirTulia centrSi moqmedi Seyursuli P_{zR} ZaliT(nax. 3.7.3)



nax. 3.7.3

datvirTvis gansaxilvel SemTxvevaSi

$$Q = \frac{P_{zR}}{2\pi r} \qquad (3.7.21)$$

da maSasadame, imis gaTvaliswinebiT, rom

$$Q = \frac{a}{r} \text{ da } b = \frac{\sqrt{3}}{2} \cdot \frac{a}{M_s} \qquad (3.7.22)$$

miviRebT:

$$a = \frac{P_{zR}}{2\pi} \quad b = \frac{\sqrt{3}}{2} \cdot \frac{P_{zR}}{2\pi M_s} \qquad (3.7.23)$$

sasazRvro pirobebs mocemuli amocanebisaTvis eqnebaT saxe:

roca $r=0$ $M_r=M_s$

roca $r=r_2$ $M_r=0$

am pirobebidan vadgenT ψ-s mniSvnelobas filis centrSi da konturze roca r=0 $\psi_1 = \frac{\pi}{6}$ (3.7.24)

roca r=r_2 $\psi_2 = \frac{\pi}{6}$ (3.7.25)

gavaintegroT (3.7.15) gantoleba zeda meore sasazRvro pirobis gamoyenebiT, miviRebT:

$$\ln\frac{r}{r_2} = \int_{\pi/2}^{\psi} \frac{\sin\psi d\psi}{b - \sin\left(\psi - \frac{\pi}{6}\right)} \quad (3.7.26)$$

amave sasazRvro pirobidan gamomdinareobs, rom integralSi

$$\int_{\pi/2}^{\pi/6} \frac{\sin\psi d\psi}{b - \sin\left(\psi - \frac{\pi}{6}\right)} \quad (3.7.27)$$

radgan integrebis sazRvrebi sasrulia, maSin integralqveSa funqcia argumentis raRac mniSvnelobisaTvis usasrulo xdeba. zemoT moyvanili (3.7.15) da (3.7.25) gamosaxulebebidan SeiZleba davaskvnaT:

$$b - \sin\left(\psi - \frac{\pi}{6}\right) \geq 0 \quad (3.7.28)$$

radgan gansaxilvel intervalSi $\frac{\pi}{6} \leq \psi \leq \frac{\pi}{2}$ $\sin\left(\psi - \frac{\pi}{6}\right)$ gamosaxulebis maqsimaluri mniSvneloba aris $\frac{\sqrt{3}}{2}$ davadgenT, rom

$$b \geq \frac{\sqrt{3}}{2} \quad (3.7.29)$$

imisaTvis, rom integralqveSa funqcia usasrulo gaxdes, aucilebelia b-s sidide toli iyos $\frac{\sqrt{3}}{2}$ -is

$$b = \frac{\sqrt{3}}{2} \qquad (3.7.30)$$

am SemTxvevaSi integralqveSa funqcia iqceva usasrulobad, roca $\psi=\pi/2$. Tu davakavSirebT erTmaneTTan (3.7.23) da (3.7.30)-s miviRebT datvirTvis zRvrul mniSvnelobas

$$P_{zR} = 2\pi M_s \qquad (3.7.31)$$

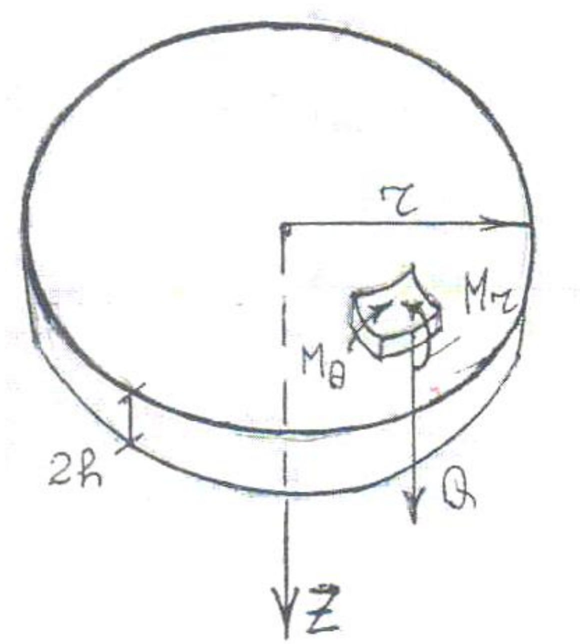
3.8. wriuli da rgoluri firfitebis zRvruli datvirTvebis gansazRvra

ganxiluli iqneba rogorc wriuli da rgoluri, agreTve sawyisi simrudis mqone firfitebis plastikuri Runvis deformaciebi. dasawyisisaTvis gamoviyenoT RerZsimetriuli datvirtvis magaliTi. nax.3.8.1-ze naCvenebia mrgvali fila, romlis sisqea 2h, datvirTva P=P(r), xolo r-radius veqtoria. r,θ,z cilindruli sistemis z RerZi mimarTulia qveviT. masala xist-plastikuria. drekadi filebis Runvis klasikuri Teoriis Tanaxmad ZiriTad debulebebs geometriuli xasiaTi aqvT, amitom isini samarTliania plastikuri Runvis SemTxvevaSic, anu kirxhofis qvemoTmoyvanili ori daSveba ZalaSi rCeba.

1. Sua zedapiri deformacias ar ganicdis;
2. Sua sibrtyis marTobebi deformaciis Semdeg Sualeduri sibrtyis perpendikularebi xdebian.

Zabvis mdgenelebi σ_z da τ_{rz} simciris gamo gaangariSebaSi SeiZleba ar iyos gaTvaliswinebuli. zemoT - Tqmulis gaTvaliswinebiT, saqme gvaqvs brtyel daZabul mdgomareobasTan. kveTebSi, sadac r=const, θ=const imoqmedeben mxolod mRunavi momentebi M_r da M_θ

$$M_r = \int_{-h}^{h} \sigma_r z dz; \qquad M_\theta = \int_{-h}^{h} \sigma_\theta z dz; \qquad Q = \int_{-h}^{h} \tau_{rz} dz; \tag{3.8.1}$$



nax. 3.8.1

wonasworobis diferencialur gantolebas aqvs saxe:

$$\frac{dM_r}{dr} + \frac{M_r - M_\theta}{r} = Q \tag{3.8.2}$$

mxebi Zabvebi wreze r=const awonasworeben gare datvirTvas, amotom:

$$Q = -\frac{1}{r}\int_{r_1}^{r} prdr \tag{3.8.3}$$

r_1 aris rgolis Sida radiusi. rgoluri filis SemTxvevaSi W=W(r) aris filis CaRunvis siCqare. siCqaris komponentebs aqvs Semdegi saxe:

$$\xi_r = z \cdot M_r \qquad \xi_\theta = z \cdot M_\theta \tag{3.8.4}$$

firfitis Sua sibrtyis denadobis siCqaris parametrebia:

$$M_r = -\frac{d^2W}{dr^2} \quad M_\theta = -\frac{1}{r}\frac{dw}{dr} \tag{3.8.5}$$

firfitis Runvis gantolebebi (mizesis denadobis pirobebidan) miiRebs saxes:

$$\xi_r = -\frac{\lambda'}{3}(2\sigma_r - \sigma_\theta); \quad \xi_\theta = \frac{\lambda'}{3}(2\sigma_\theta - \sigma_r) \tag{3.8.6}$$

nulisagan gansxvavebuli σ_r da σ_θ Zabvebis komponentebi akmayofileben denadobis Semdeg pirobas:

$$\sigma_r^2 - \sigma_r\sigma_\vartheta + \sigma_\theta^2 = \sigma_s^2 \tag{3.8.7}$$

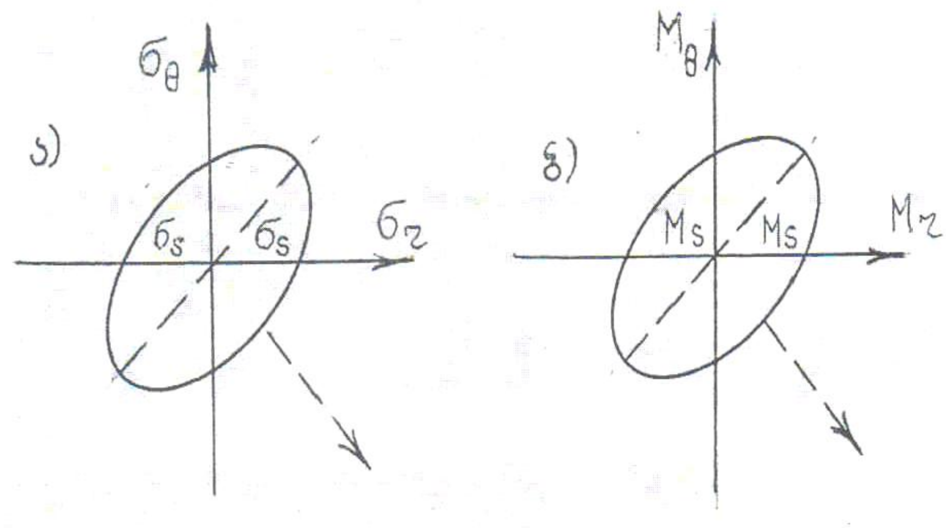
(3.8.6) gantolebebi iwereba Semdegi saxiT:

$$\xi_r = -\frac{\lambda'}{3}\frac{df}{\partial\sigma_r}; \quad \xi_\theta = \frac{\lambda'}{3}\frac{df}{\partial\sigma_\theta} \tag{3.8.8}$$

aq f-iT aRniSnulia denadobis pirobis gamosaxulebis marcxena mxare. sxvagvarad rom vTqvaT, deformaciis siCqaris veqtori denadobis mrudis normalia (nax. 3.8.2 wyvetili xazi). Sefardeba $\frac{\xi_r}{\xi_\theta}$ avRnoSnoT η-Ti. maSin (3.8.6)-dan miviRebT, rom normalis gaswvriv σ_θ, σ_r –is proporciulia, magram am SemTxvevaSi imave denadobis pirobidan gamomdinareobs Semdegi tolobebic:

$$\sigma_r = \pm f_1(\eta)\sigma_s \ \text{da} \ \sigma_\theta = \pm f_2(\eta)\sigma_s \tag{3.8.9}$$

aq f_1 da f_2 η-s garkveuli funqciebia.



nax. 3.8.2

maSasadame, Zabvebi σ_r da σ_θ normalis gaswvriv mudmivi arian yvela dadebiTi z-saTvis da icvlian niSans uaryofiTi z-saTvis. amasTan, isini ganicdian wyvetas Sua sibrtyeze (koWis Runvis analogia) da denadobis elifsze gamoisaxebian sapirispiro wertilebiT. maSasadame

$$M_r = \sigma_r h^2 \,; M_\theta = \sigma_\theta h^2 \qquad (3.8.10)$$

Sesabamisad gveqneba:

$$M_r^2 - M_r M_\theta + M_\theta^2 = M_s^2 \qquad (3.8.11)$$

sadac $M_s = \sigma_s h^2$ aris mRunavi momentis maqsimaluri mniSvneloba. es gantoleba warmoadgens garsebis plastikuri Runvis Sesabamisi gantolebis kerZo SemTxvevas. Tu (3.8.6)-Si Zabvebs Sesabamisi momentebiT SevcvliT, xolo deformaciebis siCqareebs simrudis η_r da η_θ mdgenelebiT, martivi gardaqmnebis Sedegad miviRebT:

$$\eta_r = \lambda^* \frac{\partial F}{\partial M_r}; \quad \eta_\theta = \lambda^* \frac{\partial F}{\partial M_\theta} \qquad (3.8.12)$$

aq F-iT aRniSnulia (3.8.11)-is marcxena nawili, xolo λ^* skalaruli namravlia. maSasadame, simrudis siCqaris vetori M_r, M_θ sibrtyeSi zRvruli simrudis normalia. aqedan gamomdinareobs, rom denadobis asocirebuli kanoni samarTliania iseTi ganzogadoebuli sidideebisaTvisac, rogorebicaa M_r, M_θ da gamrudebis siCqareebi η_r, η_θ.

axla (3.8.11)-is gamoyenebiT gamovricxoT M_θ da misi mniSvneloba SevitanoT wonasworobis (3.8.2) diferencialur gantolebaSi. miviRebT arawrfiv diferencialur gantolebas M_r-is mimarT.

$$\frac{dM_r}{dr} = \frac{1}{r}\left(\frac{1}{2}M_r \pm \sqrt{M_s^2 - \frac{3}{4}M_r^2}\right) = Q \qquad (3.8.13)$$

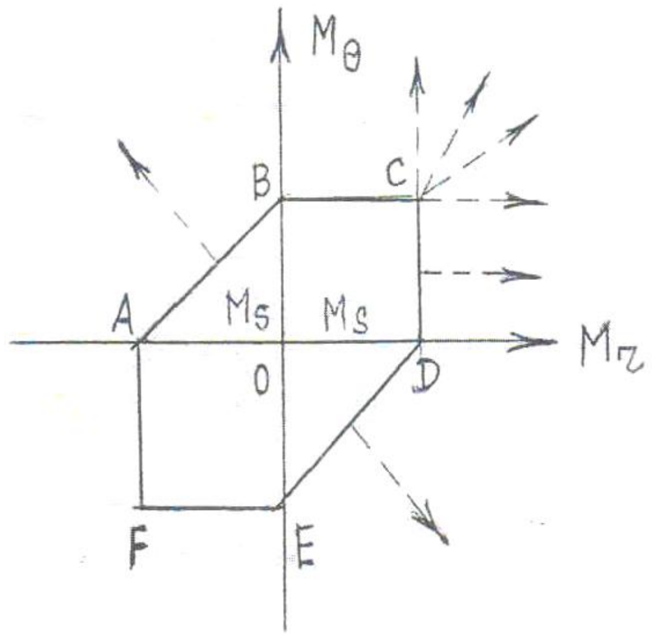
am gantolebis amonaxsni Sesabamisi sasazRvro pirobebisaTvis gansazRvravs zRvrul datvirTvas. (3.8.4) da (3.8.6) gantolebebis gardaqmniT miviRebT firfitis Runvis siCqaris diferencialur gantolebas:

$$\eta(2M_\theta - M_r)\frac{d^2W}{dr^2} = (2M_r - M_\theta)\frac{dW}{dr} \qquad (3.8.14)$$

am gantolebis integrebiT (cnobili M_r da M_θ momentebis da sasazRvro pirobis gamoyenebiT) (3.8.14) miiRebs saxes:

$$M_r - 2M_\theta = \pm\frac{2}{\sqrt{3}}M_s \qquad (3.8.15)$$

(3.8.13) gantolebis amoxsna seriozul siZneleebTan aris dakavSirebuli. amoxsnis gamartivebis saSualebas iZleva treskas plastikurobis piroba, romlis Tanaxmadac elifsi icvleba eqvskuTxediT. aseT SemTxvevaSi firfita wriul zonebad iyofa da TiToeul zonaSi denadobis piroba wrfivi kanoniT gamoixateba da integreba garTulebebis gareSe xdeba. amave dros sizuste didad ar irRveva. am cdomilebis Semcirebac SesaZlebelia, Tu Cawerili eqsvkuTxedebis nacvlad aviRebT Semowerili da Cawerili eqvskuTxedebis Sualedur eqvskuTxeds. amisaTvis sakmarisia σ_s –is nacvlad aviRoT $\sigma_s' \approx 1{,}08\sigma_s$ (nax. 3.8.3).



nax. 3.8.3

CaRunvis siCqare uwyveti unda iyos. deformaciaTa siCqareebi ki ξ_r da ξ_θ zogad SemTxvevaSi SeiZleba wyvetas ganicdidnen. wrewirs, sadac ξ_0 wyvetas ganicdis, ewodeba saxsruli wrewiri. am wrewirze $M_r = \pm M_s$. marTlac - ξ_θ ganicdis wyvetas, ξ_r ki SemousazRvrelia. denadobis asocirebuli kanonis Tanaxmad aseTi mdgomareoba SesaZlebelia eqvsvkuTxedis vertikaluri gverdebisaTvis, romelTa gaswvriv $M_r = \pm M_s$.

xist-plastikuri sxeulis sqemis Tanaxmad unda davuSvaT, rom firfitis nawili (raRac regionaluri zona) SeiZleba aradeformirebuli darCes da xistad gadaadgildes vertikaluri mimarTulebiT. aseT SemTxvevaSi zemoT moyvanili sasazRvro pirobebi ZalaSi rCeba. Camagrebis SemTxvevaSi ki gevqneba $\frac{dw}{dr} = 0$ da $M_r = \pm M_s$.

magaliTi 1

firfita Tavisuflad aris dayrdnobili. p intesivobis tvirTi ganawilebulia r radiusis SigniT (nax.3.8.4). drekadi Runvis SemTxvevaSi maqsimaluri normaluri Zabva aRiZvreba firfitis centrSi, sadac $r=0$ da pirvelad aqve aRiZvreba plastikuri deformaciac. aq $M_r=M_\theta=M_s$, xolo centrTan axlos gveqneba (nax.3.8.3)-is Sesabamisi erT-erTi plastikuri varianti - C, BC, CD. CD- varianti ewinaaRmdegeba wonasworobis gantolebas. (radgan CD- ze $M_r=M_s$ da $\frac{dM_r}{dr}=0$) meore mxriv $M_\theta<M_s$, $p>0$ da (3.8.2) diferencialuri gantolebidan miviRebT $r>c$, $p=0$.

$$M_r=\begin{cases}\dfrac{c_1}{r}-\dfrac{1}{6}Pr^2-M_s & \text{roca} \quad r\le c\\ \dfrac{c_2}{r}-\dfrac{1}{6}Pc^2-M_s & \text{roca} \quad r\ge c\end{cases} \qquad (3.8.16)$$

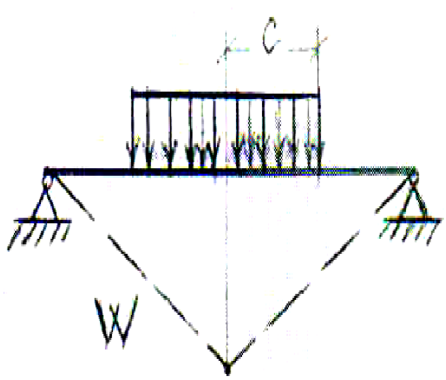
sadac C_1 da C_2 nebismieri mudmivebia. firfitis centrSi M_r –is SemosazRvrulobis gamo $C_1=0$, xolo M_r –is uwyvetobis pirobidan $r=c$ SemTxvevaSi SegviZlia vipovoT $C_2=\frac{1}{3}pc^2$. r–is zrdasTan erTad M_r mRunavi momenti mcirdeba, anu misi aRmniSvneli wertili mesame naxazze moZraobs C –dan B –sken. sayrdenze $M_r=0$, da realizdeba B varianti. filis danarCen nawilSi mimdinareobs BC varianti. zRvruli datvirTvis mniSvnelobis mosaZebnad ganvixiloT piroba $r=b$ da $M_r=0$. maSin

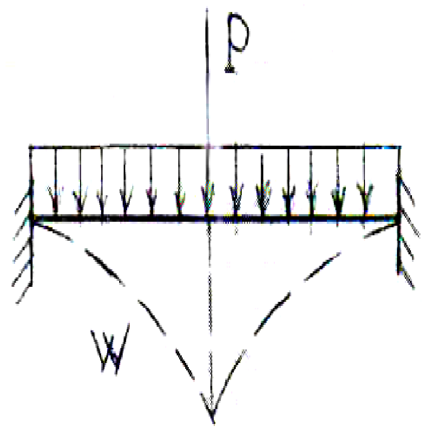
$$P^*=\frac{6b}{c^2(3b-2c)}M_s \qquad (3.8.17)$$

firfitis denadobis suraTi zRvrul mdgomareobaSi aixsneba Semdegnairad: denadobis asocirebuli kanonis Tanaxmad BC ubnisaTvis gveqneba $\eta=0$ anu $\frac{d^2w}{dr^2}=0$. aqedan gamomdinare, sasazRvro pirobisaTvis $W=0$, roca $r=b$, advilad movZebniT

$$W=W_0\left(1-\frac{r}{b}\right) \qquad (3.8.18)$$

sadac W_0 aris CaRunvis siCqaris mniSvneloba firfitis centrSi, romelic ar aris jer dadgenili. amrigad, denadobis procesSi firfita konusis formas iRebs.





nax. 3.8.4 nax. 3.8.5

roca datvirTva mTlian firfitazea ganawilebuli, maSin cxadia, rom c=b da zRvruli datvirTvis gamosaxuleba miiRebs Semdeg saxes:

$$P^* = \frac{6M_s}{b^2} \qquad (3.8.19)$$

amave SemTxvevisaTvis mizesis plastikurobis ganxilvis Sedegad miRebuli diferencialuri gantolebis ricxviTi integrebiT miRebuli zRvruli datvirTvis mniSvneloba iqneba

$$P^* = 6{,}5\frac{M_s}{b^2} \qquad (3.8.20)$$

magaliTi 2

wriuli firfita xistadaa Camagrebuli mTeli konturiT da masze moqmedebs Tanabrad ganawilebuli datvirTva (nax. 3.8.6) iseve rogorc wina SemTxvevaSi, centris irgvliv realizdeba BC varianti. r-is zrdis paralelurad M_r mcirdeba da xdeba nulis toli roca r=ρ. Semdeg xorciledeba varianti BA, romelic vrceldeba firfitaze konturamde r=b, sadac M_r=-M_s, anu dgeba varianti A, roca r≤p. (3.8.16)-is Tanaxmad gveqneba:

$$M_r = M_s - \frac{1}{6}pr^2 \qquad (3.8.21)$$

roca r=p, M_r=0 da $\rho^2 = \frac{6M_s}{p}$

im SemTxvevaSi, roca r>p, M_θ-M_r=M_s da wonasworobis diferencialuri gantolebidan miviRebT:

$$M_r = M_s \ln\frac{r}{p} - \frac{1}{4}P\left(r^2 - \rho^2\right) \qquad (3.8.22)$$

sadac gamoyenebulia piroba M_r=ρ. vTqvaT, konturze r=b, M_r=-M_s anu Camagrebis gaswvriv warmoiSva saxsari. am mdgomareobaSi mivdivarT gantolebamde

$$5 + 2\ln\frac{b}{\rho} = 3\frac{b^2}{\rho^2} \qquad (3.8.23)$$

aqedan gamovTvliT ρ-s da zRvrul datvirTvas. miviRebT ρ≈0,73b amgvarad

$$P^* = 11{,}3\frac{M_s}{b^2} \qquad (3.8.24)$$

amis Semdeg davubrundeT CaRunvis siCqaris gansazRvris sakiTxs. centralur zonaSi r≤ρ. iseve, rogorc pirvel magaliTSi, gveqneba:

$$\frac{d^2w}{dr} = 0;\ W = W_0 + C_1 r; \qquad (3.8.25)$$

sadac W_0 da C_1 nebismieri mudmivebia. AB variantis SemTxvevaSi, denadobis asocirebuli kanonis Tanaxmad:

$$\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr} = 0 \qquad (3.8.26)$$

konturze r=b ,w=0 da miviRebT:

$$w = C_2 \ln\frac{r}{b} \qquad (3.8.27)$$

aqac C_2 nebismieri mudmivia. cxadi xdeba, rom piroba $\frac{dw}{dr} = 0$ konturze ar sruldeba. amitom konturis gaswvriv marTlac warmoiqmneba plastikuri saxsari. C_1 da C_2 mudmivebi ganisazRvreba w-s $\frac{dw}{dr}$-is uwyvetobis pirobidan, roca $r=\rho$. CaRunvis siCqare firfitis centrSi w_0 gamouTvleli rCeba. CaRunvis forma naCvenebia 3.8.5 naxazze.

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