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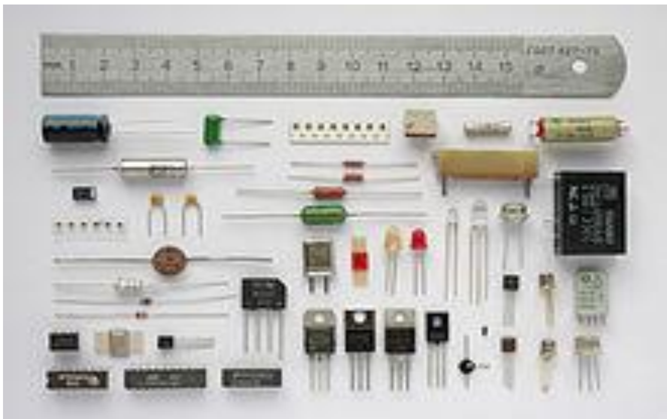
BASIC ELECTRICAL ENGINEERING
Theory and Practice



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A course in basic electrical engineering is among the key subjects in the syllabus of many engineering colleges and Universities. In fact, it is the basis of the subjects in which some of these colleges and Universities specialize.

It covers of electric circuits and its elements, main definitions, methods of calculation of direct current (d.c) and alternating current (a.c) circuits, d.c. and a.c. generators and motors, electrical measurement and some electronic elements and devises.

This textbook is meant mainly for students of power engineering and information as well as for other faculties of technical universities.

Dedicated to my lovely daughter Nino and granddaughter Qeti who are far away but always deep in my heart

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I n t r o d u c t i o n

1.1 The Field of Electrical Engineering

The field of electrical engineering involves the application of electricity to meet the needs of society. There are two primary uses of electricity: first, as a means to transfer electrical energy or power from one location to another; and second, to carry or transfer information.

For simplicity, one might assume that the traditional core of electrical engineering study could be divided into seven major specialty areas. They are:

1. Rower engineering
2. Electromagnetic
3. Communications
4. Computer engineering
5. Electronics
6. Systems
7. Controls.

Power engineering deals with the generation of electric power, its transmission over what are often very large distances, and the conversion of that power into forms that can do useful work. The generation of electric power most often is accomplished by the conversion of mechanical energy from a rotating shaft to electric energy in a generator. Many large power generating stations use some form of fuel to generate heat which produces steam. The steam is routed to a turbine which has blades on a shaft, which rotates by the action of the steam. This rotating shaft couples the energy to the generator. Hydroelectric power generating stations use the energy of water falling by gravity to turn the turbine blades. Often the water is backed up in a river by a dam in order to create a convenient site for a water to fall a large distance. Nuclear power generating stations use the heat generated by nuclear fission to produce steam to turn the turbine blades.

More recently, solar and wind energy has become a usable means of generating electric energy. Solar cells are carefully prepared slices of special materials fabricated in precise patterns and layers that directly convert the light from the sun into an electric current. Silicon solar cells as well as solar cells made from other materials are becoming more common as a source of electric power, particularly in remote locations.

Batteries are sources of electric power which derive their energy from a chemical reaction within the battery. Rechargeable batteries, such as the battery in your car, allow the chemical reaction to be reversed so that electric energy can be stored in the battery and then extracted at a later time.

Power in transmission lines, where it is distributed at a various substations to the users of the power. This power is used for heating, generation of electric light, driving various types of electric motors and many other applications.

The discipline of electromagnetism bridges the gap between applications of electricity for energy transfer and the remaining disciplines, which are primarily associated with information transfer. Electromagnetism deals with the interaction between magnetic fields, electric fields, and the flow of current. A coil of wire carrying an electric current generates a magnetic field; a piece of iron brought in proximity to this “electromagnet” will experience a force on it which can be made quite strong. This is the origin of the force used to turn electric motors and cause motion of other electromagnetic devices.

At direct current (d.c.) and low-frequency alternating current (a.c.) the electric and magnetic interactions remain relatively localized around the current-carrying wire. As frequency is increased, energy begins to radiate from the wire and propagate through the atmosphere as electromagnetic waves. These are waves that make possible radio communications, television, satellite communications, radar, and so on.

Electromagnetic waves propagating through space travel at the speed of light $c=3x10^8$ meters per second or 186,000 miles per second. Light is an electromagnetic waves within a particular range of frequencies; the exact frequency determines the color.

The field of **communications** focuses on the engineering and science required to transmit information from one place to another. Electronic communications systems often utilize unconfined electromagnetic waves for information transfer; however, it is also common for data to be transmitted by telephone wired, various cables, or optical fibers. One important question in the area of communications deals with the ways in which information is encoded on an electrical signal. The process of putting information on a carrier signal is called modulation or encoding and the process of extracting that information at the receiving end is called demodulation or decoding.

Computer engineering is one of the fastest-growing specialties within electrical engineering, and includes the design and development of computer hardware systems and the computer programs or “software” that control them. Computer systems range in size from the large “mainframes”, which are generally used for highly complex computing tasks, to special-purpose computers for engineering, business, accounting, banking, finance and many other purposes. Personal computers and workstations are being used at a rapidly increasing rate in industry, as well as for individual use. Engineering and science students in all disciplines will make use of the computer hardware and software developed by computer engineers in their specific field.

Electronics refers to the utilization of various materials in special configuration or structures to make devices which can valve “on” or “off” the flow of current. These devices can be interconnected to form circuits. Such electronic circuits may simultaneously control the flow of many different electric currents and be able to perform complex functions, such as the electronic computer just described.

The specialty of **system engineering** utilizes mathematical principles to model and describe complex systems and predict their performance based on engineering analyses. With such a mathematical description of the problem the engineer can optimize the system for a given set of conditions. System engineering principles can be applied to quantitatively address a wide variety of society’s problems.

Control systems are a very important class of systems, and the field of “controls” is widely studied within electrical engineering departments. Control systems are often electronic systems designed to provide fast and accurate “control” of electrical systems, chemical systems, hydraulic systems and others.

The field of electrical engineering is constantly in a state of change due to technology innovations which rapidly and continuously occur. There is, however, also a set of fundamental concepts and principles in electrical engineering forming a core body of knowledge and which make some understanding of all the innovations possible.

1.2. Basic Concepts

A. Charge. The force Between Two Charges

Electrical engineers deal primarily with charge, its motion, and the effects of that motion. **Electricity** is a word most often used in a nontechnical context to describe the presence of charge; the term electricity is used both to describe charge in motion (for example, through a wire, as an **electric current**) and stationary charge, **static electricity**.

Charge is a fundamental property of a matter and is said to be conserved-that is, it can neither be created nor destroyed. This means that if the charge moves away from one location, it must appear at another. There are two types of charge, positive charge and negative charge. Charge is the substance of which electric currents are made.

Charges near each other will attract each other or repel from each other according to the following rule: like charges repel each other; opposite charges attract each other.

The basic structure of an atom is held together by the attractive force between unlike charges.

Charge is designated by the symbol q (or Q- constant charge), and is measured in units of coulombs (C). The negative charge carried by a single individual electron is – $1.602x10^{-19}$ coulombs, and this is the smallest unit of charges that exists.

The force between two small clusters of charge (each one small enough to be considered a point) has been found to be described by the following equation:

$$F = k\ \frac{q_1(q_2)}{d^2} \tag{1.1}$$

Where q_1 is the charge at position 1 in coulombs, q_2 is the charge at position 2 in coulombs, d is the distance between the charges in meters and k is a constant of 8.99x109 Newton meter^2/C^2.

B. Conductors and Insulators

In order to put charge in motion so that it becomes an electric current, we must provide a path through which it can flow easily. In the vast majority of applications, charge will be carried by moving electrons along a path through which they can move easily. Materials through which charge flows readily are called **conductors.** Most metals, such as copper, are excellent conductors, and therefore are used for fabrication of electrical wires and the conductive path on electric circuit boards.

Insulators are materials which do not allow charge to move easily. Therefore, electric current cannot be made to flow through an insulator. Charge placed on an insulating material, such as the rubber comb, just stays there as static electricity, charge has great difficulty moving through it. Insulating materials are often wrapped around the center conducting core of a wire to prevent the charge from flowing off to some undesired place if the wire inadvertently touches some other object.Resistance will be defined quantitatively later; however, qualitatively a conductor has a low resistance to the flow of charge.

Semiconductors fall in the middle between conductors and insulators, and have a moderate resistance to the flow of charge

Table1.

Conductors	Semiconductors	Insulators
Silver	Silicon	Glass
Gold	Germanium	Plastic
Copper	Gallium Arsenide	Ceramic
Aluminum		Rubber

C. Current (charge in motion)

Electric **current** implies "charge in motion"; the term current is simply a measure of how much charge is moved per unit of time. Current is measured in **Amperes**, frequently called the **Amp** and abbreviated as **A**; one ampere is defined as the transfer of one coulomb in one second (that is, one ampere is equal to one coulomb/second).

Charge can be transported by various mechanisms. The most common is the movement of electrons through a conductor; however, positive ions flowing the opposite direction can transfer charge, as in the case in electrochemical reactions in battery or in electroplating. In addition, solid state electronic devices use semiconductors in which charge can be moved by electrons carrying negative charge, and "holes" carrying positive charge. The total current through any particular plane is the total net charge transferred divided by the time interval.

$$i=\frac{dq}{dt}=(d_{q^+}+d_{q^-})/dt, \qquad (1.2)$$

where d_{q^+} is the incremental positive charge transferred and d_{q^-} is the incremental negative charge transferred.

Current, a scalar quantity, requires a sign convention; if we assume that the direction we call "positive current" flow is the direction positive charge moves, then the above equation reveals that the total current is the sum of the rate of flow of positive charge in one direction and negative charge in the opposite direction.

The **sign convention**, which we will adopt here, is the following: Positive current will be defined as the net rate of flow of positive charge. Therefore, a wire conducting electrons to the left will be described as having positive current to the right.

In our world we encounter currents over many orders of magnitude. Lightning strikes consist of bursts of current that can be tens of thousands of amperes, while the current in a nerve pulse may be only picoamperes. In between is a wide range of currents. For example, typical household appliances require from 0.5 to 10 amperes; individual electronic circuits may require microamperes to milliamperes, and large industrial motors may require hundreds of amperes.

D. Energy. Potential. Voltage. Electromotive Force

Energy (w) is defined as the ability to do work. The unit of energy is joule (J).

As is known, **potential** at a point in an electrical field may be defined as numerically equal to the work done in bringing positive charge of one coulomb from infinity (where the force on a charge is zero) to that point against the electric field.

Voltage (potential difference) between two points is defined as the work required to move a unit charge from one point to the other:

$$v= \frac{dw}{dq} , \tag{1.3}$$

where w is the energy in joules, and q- the electric charge in coulombs.

The unit of voltage is the volt (V):

$$1V=1\frac{j}{c}$$

It is important to note that sometimes to characterize a source of electrical energy (battery or generator) the term “electromotive force” (emf) is used.

By electromotive force (emf) is meant the total potential difference established within the source between the two terminals when the source is not supplying any current (so that there is no internal voltage drop). The e.m.f. can be measured by connecting a suitable voltmeter across the terminals. But the potential difference is equal to the emf minus the internal voltage drop. In other words the terminal potential difference is not, as it depends on load current supplied by the source.

Also, the term e.m.f. has local significance i.e. it is spoken of with reference to the generator (battery) itself, but potential difference is distributive. For example, we cannot say that an e.m.f. 220 v exists across an electric bulb. It is the potential difference of 220 volts. In other words potential difference can be distributed away from its source of generation.

E. **Power**

Power is defined as the time rate at which energy w is produced or consumed, depending on whether the element is a source of power or a user of power, respectively. That is

$$P= \frac{dw}{dt} \tag{1.4}$$

which can be rewritten as

$$P=\frac{dw}{dt}=\frac{dw}{dq}\frac{dq}{dt}=vi \tag{1.5}$$

The above equation shows that power can be computed by product of the voltage across a circuit element and the current through it. Since both voltage and current can vary with time, the power p, is also a time varying quantity, and can be expressed as p (t).

Therefore, the change in energy from time t_1 to time t_2 can be found by integrating above equation, that is

$$w= \int_{t1}^{t2_1} pdt = \int_{t1}^{t2} vidt \tag{1.6}$$

Calculation of power requires the use of a consistent sign convention. The variable for the voltage v(t) is defined as the voltage across the element with the positive reference at the same terminal that the current variable i(t) is entering. The product of v and i, with their attendant signs, while determine the magnitude and sign of the power. If the sign of the power is positive, power is being absorbed by the element; if the sign is negative, power is being supplied by the element.

1.3. System of Units

The system of units used in this text is international system of units, which is normally referred to as SI standard system.

The abbreviations and symbols used to represent the various quantities studied in this text follow standard practice. Table 2 may be a useful reference if you encounter unfamiliar symbols.

Table 2

Standard Abbreviations and Symbols

ac	Alternating current
A	Ampere
C	Coulomb
db	Decibel
dc	Direct current
F	Farad
H	Henry
Hz	Hertz
j	Joule
m	meter
N	Newton
N-m	Newton-meter
Ω	Ohm
PF	Power factor
rad	Radian
RLC	Resistance-inductance-capacitance
rms	Root-mean-square
s	Second
S	Siemens
V	Volt
VA	Volt ampere
W	Watt

The standard prefixes which are employed in SI and, therefore, throughout our study of electrical engineering are shown in Fig.1.1.

10^{-12}	10^{-9}	10^{-6}	10^{-3}	1	10^{3}	10^{6}	10^{9}	10^{12}
I	I	I	I	I	I	I	I	I
Pico(p)	Nano(n)	Micro(μ)	Milli(m)		Kilo(K)	Mega(M)	Giga(G)	Tera(T)

Fig.1.1

1.4. Notation

Three of the most commonly used symbols in the study of electrical engineering are those for voltage (V,v), current (I,i), and electromotive force (E,e).

Small letters v,i,e indicate the instantaneous values as a function of time: v=v(t), i=i(t), e=e(t).

Capital letters V,I,E indicate constant values (for the dc circuits) or rms values (for alternating current circuits), correspondingly.

Vm, Im or Em are the maximum values (amplitudes) of a sinusoidally variable voltage, current or emf, respectively.

2. Electric Circuit. Laws of Electric circuit

2.1. Electric Circuit

A complete path which carries the electric current from the source of supply to the power-consuming apparatus and back again is called electric circuit. Electro-magnetic processes in an electric circuit can be described by means of such physical terms as electric current, voltage (potential differences) and electromotive force (e.m.f.)

In general case an electric circuit contains a power source (or sources), some kind of consumers, collectively called load, instruments, switches, and connecting lines or wires.

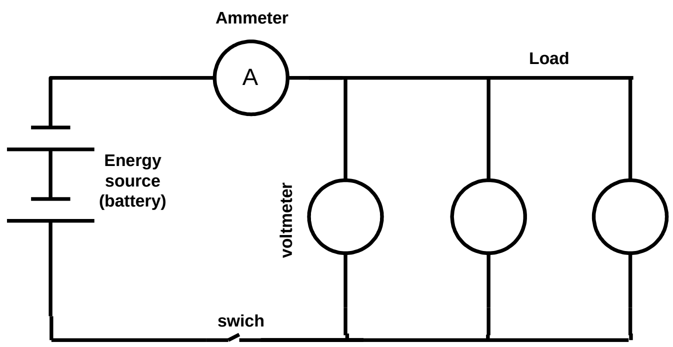


Fig.2.1

A power source converts energy in some non-electric form into electric energy. For example, generators convert mechanical energy into electrical energy; batteries convert energy of chemical processes into electricity, whereas thermocouples transform thermal into electrical energy.

The load that dissipates the energy converts electric energy into, say, mechanical (such as by d.c. or a.c. motors), thermal (such as be electric furnaces), or chemical (such as in electrolysis cells).

The switches, connecting wires, and instruments serve to convey electricity from the power supply to distribute it among the loads, and give indications about the status of the circuit as a whole and its elements.

A schematic of a simple d.c. circuit is shown in Fig.2.1.

The configuration of a circuit is specified by the geometrical (topological) concepts of a branch, a node (or junction), and a loop (or mesh).

A branch consists of one or several series-connected elements trough which the same current flows.

A node is a point where three or more branches meet.

A loop (or mesh) is a closed path passing over several branches so that any brunch or any node is encountered only once.

The equivalent circuit in Fig.2.2 has two nodes, and three branches of which the two contain one element each, and the third contains three elements.

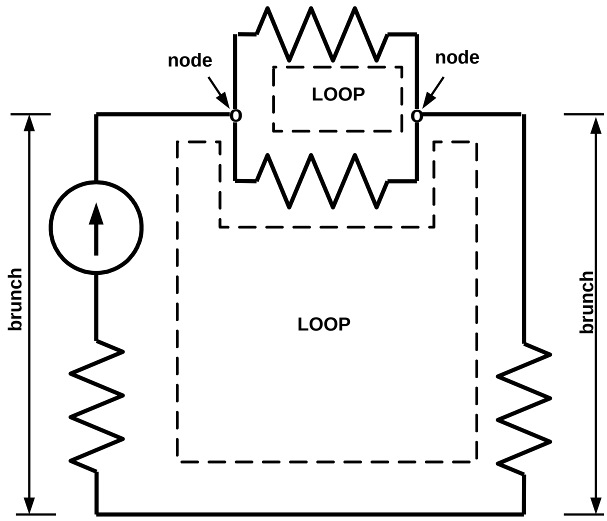


Fig.2.2.

2.2 . Ideal Circuit Elements

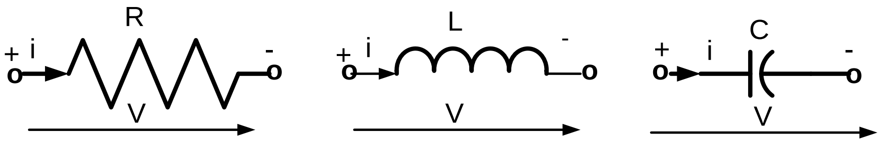
To make the analysis of electrical circuits simpler, we have defined some idealized circuit elements. These elements can be completely described by knowing the mathematical relation between the voltage across and the current through the element. Idealized active elements consist of sources. There are two types of sources, the voltage source and the current source. The voltage between the terminals of the ideal voltage source is determined be the value of the voltage source, regardless of the current passing through it, and regardless of any other circuit parameters. An ideal current source maintains a specified current through it, independent of the voltage developed across its terminals.

There are three ideal passive circuit elements: the resistor, the inductor, and the capacitor. All of these will be discussed in detail in later chapters; however, a brief introduction is provided here.

These three passive circuit elements differ electrically in the way in which the voltage across is related to the current through each of the elements. As you will see if v is the voltage across each element and I is the current through each element the relationship and the commonly used symbols are given in Fig.2.3. The constants, R, L and C, are known as the resistance (in ohms, Ω), inductance (in henries, H) and capacitance (in farads, F), respectively.

Symbol:

Resistor Inductor Capacitor



Relationship

v=Ri	v=L $\frac{di}{dt}$	v= $\frac{1}{C}\int idt$
i= $\frac{1}{R}$ v	i= $\frac{1}{L}\int vdt$	i=C $\frac{dv}{dt}$

Fig.2.3.

The resistor represents the part of a circuit component in which energy entering the element by the flow of current it is transformed to heat. Light can be emitted also if the resistive element becomes hot enough to glow.

The inductor represents a two-terminal electric element in which energy is stored in a magnetic field. Coils of wire such as those used to make an electromagnet, or the windings of wire in an electric motor, must be modeled using inductances.

The third passive circuit-element, the capacitor, stores energy in an electric field. The capacitor is often fabricated by two parallel conducting plates separated by an insulating layer.

These three circuit elements, combined with the active elements (sources), can be used to represent model, and study a wide range of electrical and electronic systems.

2.3. Ohm's Law. Joules Law of Electric Heating

Ohm's law is named for the German physicist Georg Simon Ohm, who is credited with establishing the voltage-current relationship for resistance.

Ohm's law states that **the voltage across a resistance is directly proportional to the current flowing through it**. The resistance, measured in ohms, is the constant of proportionality between the voltage and current.

A circuit element whose electrical characteristic is primarily resistive is called a resistor and is represented by the symbol shown in Fig.2.4. A **resistor** is a physical device that can be fabricated in many ways.

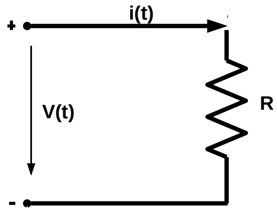


Fig.2.4.

The resistors, which find wide use in a variety of electrical applications, are normally carbon composition or wire wound. In addition, resistors can be fabricated using thick or thin films for use in hybrid circuits.

The mathematical relationship of Ohm's law is illustrated by the equation where $R \geq 0$.

$$v(t)=Ri(t) \tag{2.1}$$

In network calculations the true direction of currents in the circuit elements are not known in general case. Therefore, one has adopt an assumed positive direction of current flow in all circuit elements, which is chosen arbitrarily and is indicated by an arrow. In a load (of resistance R) the positive directions are usually assumed the same for both current and voltages (Fig.2.4.). In addition, note that we have assumed that resistor has a constant value and therefore that voltage-current characteristic is linear.

The symbol Ω is used to represent ohms, and therefore $1\,\Omega$ = 1 V/A.

It should be noted that the resistance offered by a conductor can be calculated by the formula

$$R=\rho\,\frac{l}{A}, \tag{2.2}$$

where l is the length of a conductor (in m), A-its cross-section (in mm^2), and ρ - the specific resistance or resistivity (in $\frac{\Omega mm^2}{m}$) Although we will assume here that the resistors are linear, it is important to realize that some very useful and practical elements to exist that exhibit a nonlinear resistance characteristic.

The power supplied to the terminals is absorbed by the resistor. Note that the charge moves from the higher (+) to the lower (-) potential as it passes through the resistor and the energy absorbed is dissipated by the resistor in the form of heat. The rate of energy dissipation is the instantaneous power, and therefore

$$p(t)=v(t)\,i(t) \tag{2.3}$$

which using Eq.(2.1), can be written as

$$p(t)=R\,i(t)=\frac{v^2(t)}{R} \tag{2.4}$$

This equation, called Joules law of electric heating, illustrates that the power dissipated in a resistor is a nonlinear function of either current or voltage and that is always a positive quantity.

The basic SI unit of power is the watt (w)

$$1\ w = 1\ j\ s^{-1} = 1\ V\ A$$

The basic SI unit of energy (or work) is joule (J)

$$1\ J = 1\ VAs,$$

however, it is too small for practical use. The practical unit of energy is the kilowatt hour (kWh). It is equal to the work done at a constant power of 1kW over 1h. Since 1Ws=1 J,

$$1Wh = 3600\ Ws = 3600J \text{ and}$$

$$1\ kWh = 3600000\ J.$$

Conductance, represented by the symbol G, is another quantity with wide application in circuit analysis. By definition, conductance is the reciprocal of resistance: that is

$$G=\frac{1}{R} \tag{2.5}$$

The unit of conductance is the Siemens and the relationship between units is

$$1\ \mathrm{S} = 1\ \frac{A}{V}$$

Using Eq.(2.5), we can write two additional expressions

$$i(t)=Gv(t) \tag{2.6}$$

$$p(t)=\frac{i^2(t)}{G}=Gv^2(t) \tag{2.7}$$

Equation (2.6) is another expression of Ohm's law, and Eq.(2.7) that of Joules law.

Two specific values of resistance, and therefore conductance, are very important: R=0 (G= ∞) and R=∞ (G=0). If the resistance R=0, we have what is called a short circuit. From Ohm's law

$$v(t)= R\ i(t)=0.$$

Therefore, v(t)=0, although the current could theoretically be any value. If the resistance R=∞, as would be the case with a broken wire, we have what is called an open circuit, and from Ohm's law

$$i(t)= \frac{v(t)}{R} = 0.$$

Therefore, the current is zero regardless of the value of the voltage across the open terminals.

2.4 Series and Parallel Connection of Resistors

A series circuit, such as shown in Fig. 2.5 is a closed loop of driver and resistors having the same current through every component in the loop.

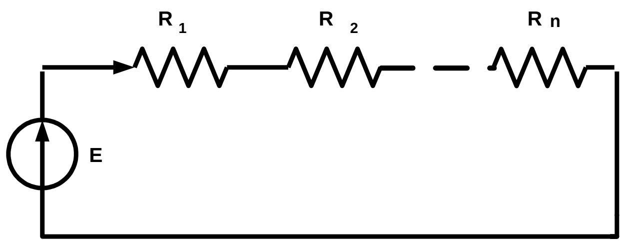


Fig.2.5

According to Kirchhoff's voltage law

$$E = IR_1 + IR_2 + ... + IR_n = IR_{eq},$$

Where the equivalent resistance of the series connected resistors is $R_{eq} = \sum_{k=1}^{n} R_k$. If several resistors are connected in parallel as shown in Fig.2.6, they also may be replaced by the equivalent resistor.

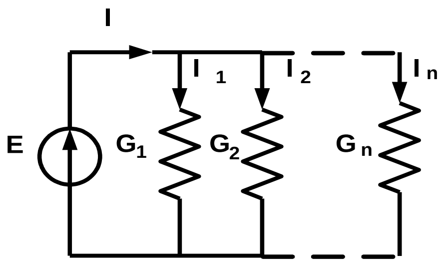


Fig.2.6

According to Kirchhoff's current law

$$I = I_1 + I_2 + \ldots + I_n = EG_1 + EG_2 + \ldots + EG_n = EG_{eq}$$

where the equivalent conductance of the parallel-connected resistors is

$$G_{eq} = \sum_{k=1}^{n} G_k$$

The equivalent resistance is

$$R_{eq} = 1/G_{eq}$$

If two resistors are connected in parallel

$$R_{eq} = \frac{1}{G_1 + G_2} = \frac{R_1 R_2}{R_1 + R_2}$$

2.5. Voltage and Current Divider

A voltage divider, also called a potential divider, is a device used to provide an output voltage that is lower than the input or driving voltage. A fixed voltage divider with two series-connected resistors is shown in Fig.2.7.

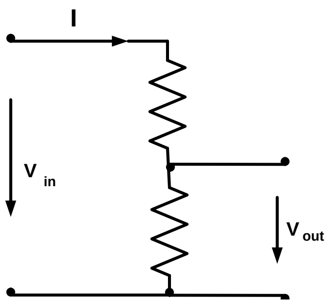


Fig.2.7

The output voltage is

$$V_{out} = IR_2 = V_{in} \frac{R_2}{R_1 + R_2}$$

A simple current divider is shown in Fig. 2.8, which consists of two resistors.

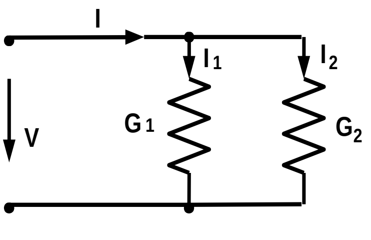


Fig2.8

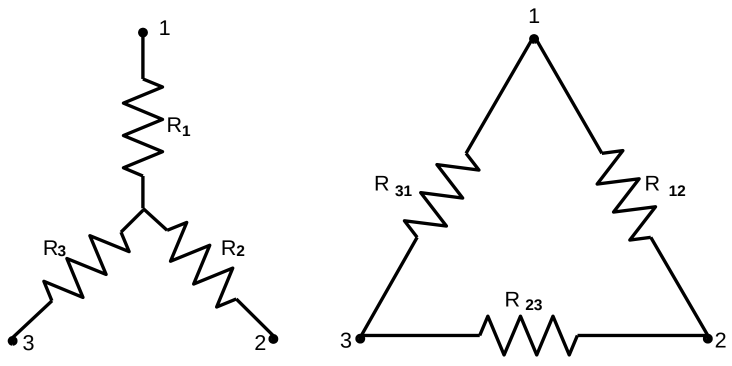
Currents in the resistors are

$$I_1 = VG_1 = \frac{I}{G_{eq}} = I \frac{G_1}{G_1 + G_2}$$

$$I_2 = I \frac{G_2}{G_1 + G_2}$$

2.6. Star (Wye)-Delta Trabsformation

The arrangement of the three branches in network of Fig. 2.9a is known as a star connection and the one in Fig. 2.9b as a delta connection



(a) (b)

Fig.2.9

The delta connection of the resistors may be replaced by the equivalent star connection or vice-versa, only on condition that the operation of the unconverted part of the network remains the same as before. It means that the currents, voltages and power in this part of the network do not change.

In the case of star circuit resistors R_1 and R_2 are connected in series between nodes 1 and 2. In the case of delta connection between the same nodes are in parallel resistors R_{12} and $(R_{23}+R_{31})$. Consequently, the resistance between nodes 1 and 2 will also be the same for both circuits

$$R_1 + R_2 = \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}$$

In the same way the resistance between nodes 2 and 3 is

$$R_2 + R_3 = \frac{R_{23}(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}}$$

and the resistance between nodes 3 and 1 is

$$R_2 + R_3 = \frac{R_{23}(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}}$$

Solution of these three equations gives the formulae by which the resistances of the star circuit can be found from the known resistors of the delta circuit

$$R_1 = \frac{R_{23}(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}} \qquad R_2 = \frac{R_{23}(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}} \qquad R_3 = \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}$$

Solution of the above three equations connection with R_{12}, R_{23}, R_{31} gives the formulae by which the resistors of the delta circuit can be found from the known resistors of the star circuit

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3} \qquad R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1} \qquad R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$

2.7 Kirchhoff's Laws

Kirchhoff's laws, named after the German scientist Gustav Robert Kirchhoff, are the basic laws governing the analysis of all electric circuits.

By Kirchhoff's first (or current) law, the algebraic sum of the currents at any node in an electric circuit is zero

$$\sum_{k=1}^{k=n} i_k = 0 \tag{2.8}$$

Or stated differently, the sum of the currents entering a node is equal to the sum of currents leaving the node. As an example, for the circuit shown in Fig.2.10

$i_1+i_2+i_4=i_3+i_5$ or

$i_1+i_2+i_4-i_3-i_5=0$

Note that we have assumed that the algebraic signs of the currents entering the node are positive and therefore that the signs of the currents leaving the node are negative.

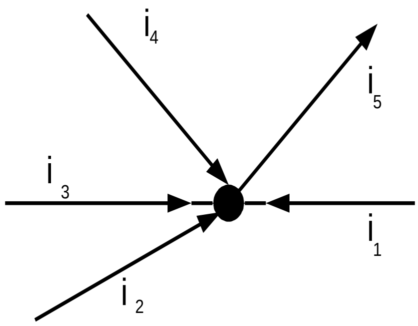


Fig.2.10

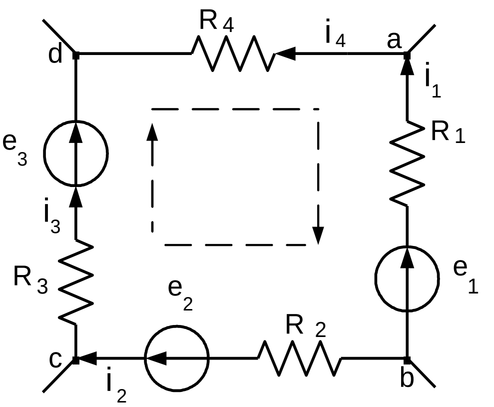


Fig.2.11

If we multiply the foregoing equation by -1, we obtain the expression

$-i_1-i_2-i_4+i_3+i_5=0$,

where, the leaving currents are positive and the entering currents are negative.

By Kirchhoff's second (voltage) law, in any closed loop of an electric circuit the algebraic sum of the voltages across the branches forming the loop is equal to the algebraic sum of the e.m.f's in that loop.

$$\sum_{k=1}^{k=n} e_k = \sum_{k=1}^{k=n} v_k \tag{2.9}$$

If the brunches consist of the resistive elements having the resistances of R_k, then according to the Ohm's law $v_k=R_k i_k$, and Kirchhoff's second law can be represented as follows:

$$\sum_{k=1}^{k=n} e_k = \sum_{k=1}^{k=n} R_k i_k \tag{2.10}$$

The currents and e.m.f's enter Eq.2.10 with a "+" sign if their directions are the same as that arbitrarily chosen for passing around the loop. As an example, for loop shown in Fig.2.11 and for the direction of passing around the loop indicated by the arrow, we have

$$-e_1+e_2+e_3=-R_1i_1+R_2i_2+R_3i_3-R_4i_4.$$

3.Direct Current Circuits

In the previous chapter, we looked at the movement of charges, showing that a net charge creates an electric field with differences in electric potential energy at different points in the field. When two points in a field with a potential difference are connected by a conducting material, electrons will flow spontaneously from one point to another. For instance, when the two terminals of a battery (a source of potential difference) are connected by a copper wire (a conducting material), electrons flow spontaneously from the negative terminal of the battery toward the positive terminal. This mass flow of electrons in a particular direction creates a **current**, which is the source of the circuits that we will examine in this chapter.

3.1 Mesh current method

The ***Mesh Current** Method*, also known as the ***Loop Current** Method*, is quite similar to the Branch Current method in that it uses simultaneous equations, Kirchhoff's Voltage Law, and Ohm's Law to determine unknown currents in a network. It differs from the Branch Current method in that it does *not* use Kirchhoff's Current Law, and it is usually able to solve a circuit with less unknown variables and less simultaneous equations, which is especially nice if you're forced to solve without a calculator. Let's see how this method works on the same example problem:

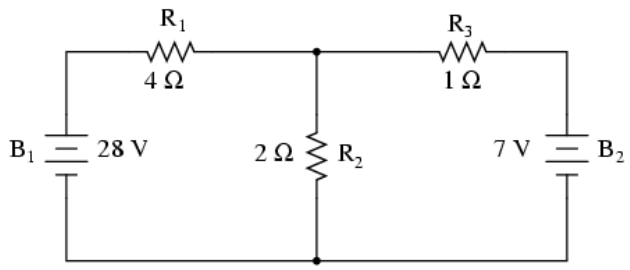


Fig.3.1

The first step in the Mesh Current method is to identify "loops" within the circuit encompassing all components. In our example circuit, the loop formed by B_1, R_1, and R_2 will be the first while the loop formed by B_2, R_2, and R_3 will be the second. The strangest part of the Mesh Current method is envisioning circulating currents in each of the loops. In fact, this method gets its name from the idea of these currents meshing together between loops like sets of spinning gears: The choice of each current's direction is entirely arbitrary, just as in the Branch Current method, but the resulting equations are easier to solve if the currents are going the same direction through intersecting components (note how currents I_1 and I_2 are both going "up" through resistor R_2, where they "mesh," or intersect). If the assumed direction of a mesh current is wrong, the answer for that current will have a negative value.

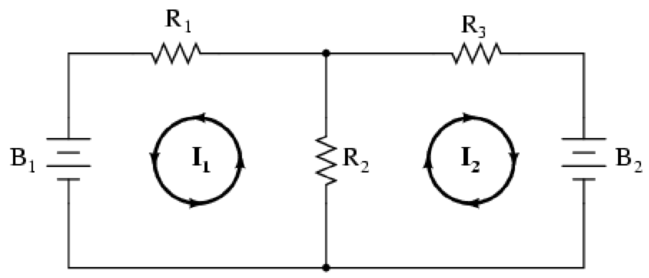


Fig.3.2

The next step is to label all voltage drop polarities across resistors according to the assumed directions of the mesh currents. Remember that the “upstream” end of a resistor will always be negative, and the “downstream” end of a resistor positive with respect to each other, since electrons are negatively charged. The battery polarities, of course, are dictated by their symbol orientations in the diagram, and may or may not “agree” with the resistor polarities (assumed current directions):

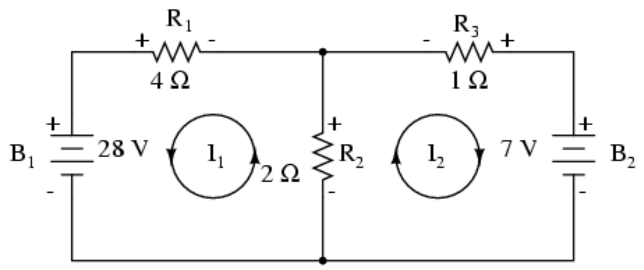


Fig.3.3

Using Kirchhoff's Voltage Law, we can now step around each of these loops, generating equations representative of the component voltage drops and polarities. As with the Branch Current method, we will denote a resistor's voltage drop as the product of the resistance (in ohms) and its respective mesh current (that quantity being unknown at this point). Where two currents mesh together, we will write that term in the equation with resistor current being the *sum* of the two meshing currents. Tracing the left loop of the circuit, starting from the upper-left corner and moving counter-clockwise (the choice of starting points and directions is ultimately irrelevant), counting polarity as if we had a voltmeter in hand, red lead on the point ahead and black lead on the point behind, we get this equation:

$$-28 + 2(I_1 + I_2) + 4I_1 = 0 \tag{3.1}$$

Notice that the middle term of the equation uses the sum of mesh currents I_1 and I_2 as the current through resistor R_2. This is because mesh currents I_1 and I_2 are going the same direction through R_2, and thus complement each other. Distributing the coefficient of 2 to the I_1 and I_2 terms, and then combining I_1 terms in the equation, we can simplify as such:

$$-28 + 2(I_1 + I_2) + 4I_1 = 0 \qquad \textit{Original form of equation}$$

. . . distributing to terms within parentheses . . .

$$-28 + 2I_1 + 2I_2 + 4I_1 = 0$$

. . . combining like terms . . .

$$\mathbf{-28 + 6I_1 + 2I_2 = 0} \qquad \textit{Simplified form of equation}$$

At this time we have one equation with two unknowns. To be able to solve for two unknown mesh currents, we must have two equations. If we trace the other loop of the circuit, we can obtain another KVL equation and have enough data to solve for the two currents. Creature of habit that I am, I'll start at the upper-left hand corner of the right loop and trace counter-clockwise:

$$-2(I_1 + I_2) + 7 - 1I_2 = 0 \qquad (3.2)$$

Simplifying the equation as before, we end up with: $-2I_1 - 3I_2 + 7 = 0$ Now, with two equations, we can use one of several methods to mathematically solve for the unknown currents I_1 and I_2:

$$-28 + 6I_1 + 2I_2 = 0$$

$$-2I_1 - 3I_2 + 7 = 0$$

. . . rearranging equations for easier solution . . .

$$6I_1 + 2I_2 = 28$$

$$-2I_1 - 3I_2 = -7$$

Solutions:

$I_1 = 5$ A

$I_2 = -1$ A

Knowing that these solutions are values for ***mesh*** currents, not ***branch*** currents, we must go back to our diagram to see how they fit together to give currents through all components:

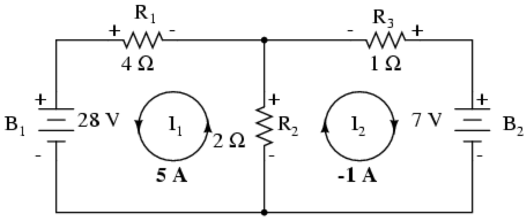


Fig.3.4

The solution of -1 amp for I_2 means that our initially assumed direction of current was incorrect. In actuality, I_2 is flowing in a counter-clockwise direction at a value of (positive) 1 amp:

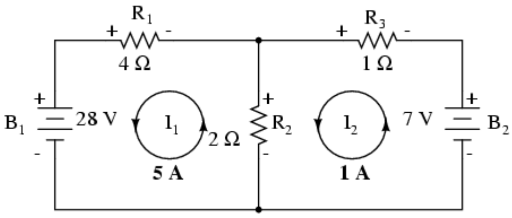


Fig.3.5

This change of current direction from what was first assumed will alter the polarity of the voltage drops across R_2 and R_3 due to current I_2. From here, we can say that the current through R_1 is 5 amps, with the voltage drop across R_1 being the product of current and resistance (E=IR), 20 volts (positive on the left and negative on the right). Also, we can safely say that the current through R_3 is 1 amp, with a voltage drop of 1 volt (E=IR), positive on the left and negative on the right. But what is happening at R_2? Mesh current I_1 is going "up" through R_2, while mesh current I_2 is going "down" through R_2. To determine the actual current through R_2, we must see how mesh currents I_1 and I_2 interact (in this case they're in opposition), and algebraically add them to arrive at a final value. Since I_1 is going "up" at 5 amps, and I_2 is going "down" at 1 amp, the *real* current through R_2 must be a value of 4 amps, going "up:"

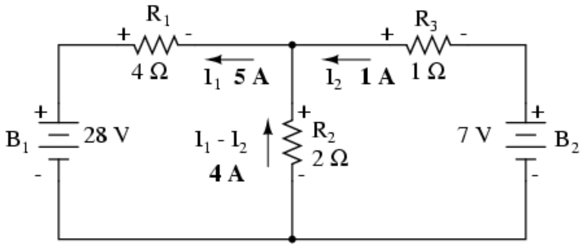


Fig.3.6

A current of 4 amps through R_2's resistance of 2 Ω gives us a voltage drop of 8 volts (E=IR), positive on the top and negative on the bottom. The primary advantage of Mesh Current analysis is that it generally allows for the solution of a large network with fewer unknown values and fewer simultaneous equations. Our example problem took three equations to solve the Branch Current method and only two equations using the Mesh Current method.

3.2 Node-Voltage Method

The node voltage method of analysis solves for unknown voltages at circuit nodes in terms of a system of KCL equations. This analysis looks strange because it involves replacing voltage sources with equivalent current sources. Also, resistor values in ohms are replaced by equivalent conductance in Siemens, G = 1/R. The Siemens (S) is the unit of conductance, having replaced the mho unit. In any event $S = \Omega^{-1}$. And S = mho (obsolete). We start with a circuit having conventional voltage sources (Fig.3,7). A common node E_0 is chosen as a reference point. The node voltages E_1 and E_2 are calculated with respect to this point.

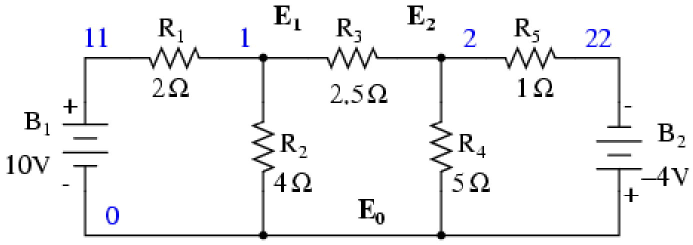


Fig.3.7

A voltage source in series with a resistance (Fig.3.8a) must be replaced by an equivalent current source in parallel with the resistance (Fig.3.8b). We will write KCL equations for each node. The right hand side of the equation is the value of the current source feeding the node.

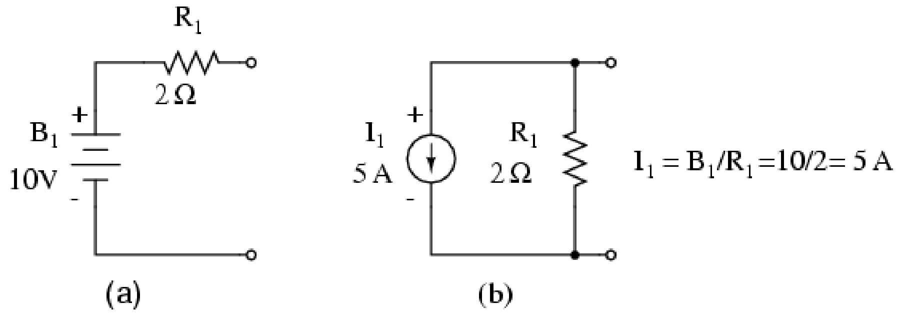


Fig.3.8

Replacing voltage sources and associated series resistors with equivalent current sources and parallel resistors yields the modified circuit (Fig.3.9). Substitute resistor conductance in Siemens for resistance in ohms.

$I_1 = E_1/R_1 = 10/2 = 5$ A

$I_2 = E_2/R_5 = 4/1 = 4$ A

$G_1 = 1/R_1 = 1/2\ \Omega = 0.5$ S

$G_2 = 1/R_2 = 1/4\ \Omega = 0.25$ S

$G_3 = 1/R_3 = 1/2.5\ \Omega = 0.4$ S

$G_4 = 1/R_4 = 1/5\ \Omega = 0.2$ S

$G_5 = 1/R_5 = 1/1\ \Omega = 1.0$ S

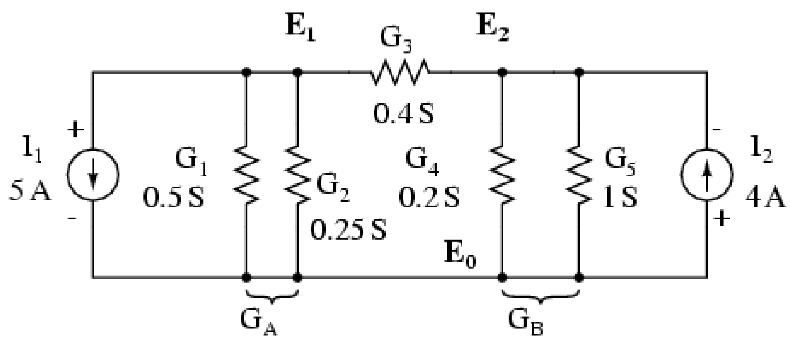


Fig.3.9

The Parallel conductance's (resistors) may be combined by addition of the conductance's. Though, we will not redraw the circuit. The circuit is ready for application of the node voltage method.

$$G_A = G_1 + G_2 = 0.5\text{ S} + 0.25\text{ S} = 0.75\text{ S}$$
$$G_B = G_4 + G_5 = 0.2\text{ S} + 1\text{ S} = 1.2\text{ S}$$

Deriving a general node voltage method, we write a pair of KCL equations in terms of unknown node voltages V_1 and V_2 this one time. We do this to illustrate a pattern for writing equations by inspection.

$$G_AE_1 + G_3(E_1 - E_2) = I_1 \qquad (3.3)$$
$$G_BE_2 - G_3(E_1 - E_2) = I_2 \qquad (3.4)$$
$$(G_A + G_3)E_1 \qquad -G_3E_2 = I_1 \qquad (3.3)$$
$$-G_3E_1 + (G_B + G_3)E_2 = I_2 \qquad (3.4)$$

The coefficients of the last pair of equations above have been rearranged to show a pattern. The sum of conductance's connected to the first node is the positive coefficient of the first voltage in equation (3.3). The sum of conductance's connected to the second node is the positive coefficient of the second voltage in equation (3.4). The other coefficients are negative, representing conductance's between nodes. For both equations, the right hand side is equal to the respective current source connected to the node. This pattern allows us to quickly write the equations by inspection. This leads to a set of rules for the node voltage method of analysis.

3.3. Thevenin's Theorem

Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single voltage source and series resistance connected to a load. The qualification of "linear" is identical to that found in the Superposition Theorem, where all the underlying equations must be linear (no exponents or roots). If we're dealing with passive components (such as resistors, and later, inductors and capacitors), this is true. However, there are some components (especially certain gas-discharge and semiconductor components) which are nonlinear: that is, their opposition to current *changes* with voltage and/or current. As such, we would call circuits containing these types of components, *nonlinear circuits*.

Thevenin's Theorem is especially useful in analyzing power systems and other circuits where one particular resistor in the circuit (called the "load" resistor) is subject to change, and re-calculation of the circuit is necessary with each

trial value of load resistance, to determine voltage across it and current through it. Let's take another look at our example circuit:

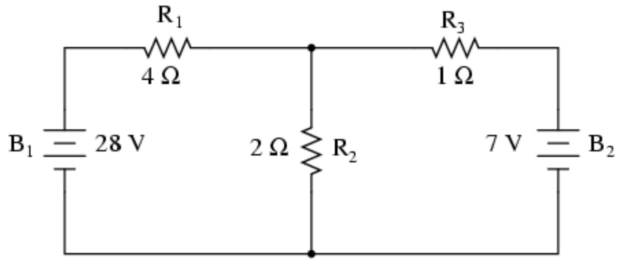


Fig.3.10

Let's suppose that we decide to designate R_2 as the “load” resistor in this circuit. We already have four methods of analysis at our disposal (Branch Current, Mesh Current, Millman's Theorem, and Superposition Theorem) to use in determining voltage across R_2 and current through R_2, but each of these methods are time-consuming. Imagine repeating any of these methods over and over again to find what would happen if the load resistance changed (changing load resistance is *very* common in power systems, as multiple loads get switched on and off as needed. the total resistance of their parallel connections changing depending on how many are connected at a time). This could potentially involve a *lot* of work! Thevenin's Theorem makes this easy by temporarily removing the load resistance from the original circuit and reducing what's left to an equivalent circuit composed of a single voltage source and series resistance. The load resistance can then be re-connected to this “Thevenin equivalent circuit” and calculations carried out as if the whole network were nothing but a simple series circuit:

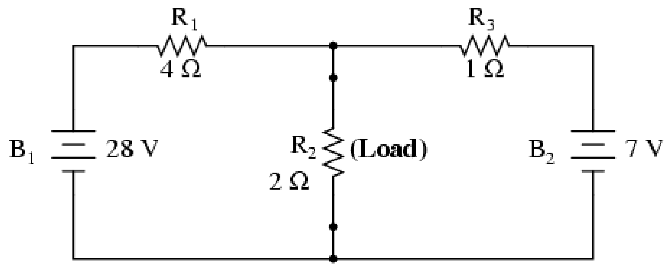


Fig.3.11

. . . after Thevenin conversion . . .

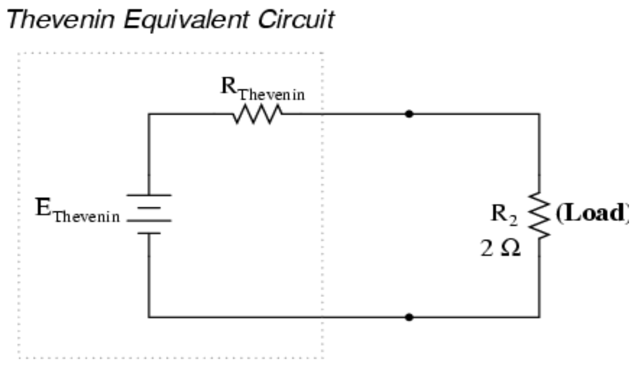


Fig.3.12

The “Thevenin Equivalent Circuit” is the electrical equivalent of B_1, R_1, R_3, and B_2 as seen from the two points where our load resistor (R_2) connects. The Thevenin equivalent circuit, if correctly derived, will behave exactly the same as the original circuit formed by B_1, R_1, R_3, and B_2. In other words, the load resistor (R_2) voltage and current should be exactly the same for the same value of load resistance in the two circuits. The load resistor R_2 cannot “tell the difference” between the original network of B_1, R_1, R_3, and B_2, and the Thevenin equivalent circuit of $E_{Thevenin}$, and $R_{Thevenin}$, provided that the values for $E_{Thevenin}$ and $R_{Thevenin}$ have been calculated correctly. The advantage in performing the “Thevenin conversion” to the simpler circuit, of course, is that it makes load voltage and load current so much easier to solve than in the original network. Calculating the equivalent Thevenin source voltage and series resistance is actually quite easy. First, the chosen load resistor is removed from the original circuit, replaced with a break (open circuit):

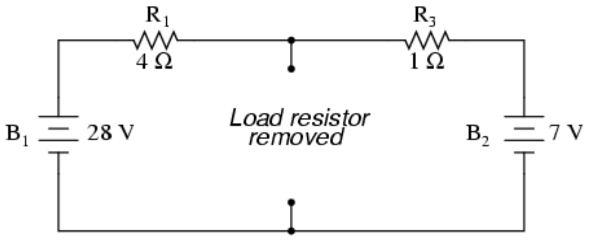


Fig.3.13

Next, the voltage between the two points where the load resistor used to be attached is determined. Use whatever analysis methods are at your disposal to do this. In this case, the original circuit with the load resistor removed is nothing more than a simple series circuit with opposing batteries, and so we can determine the voltage across the open load terminals by applying the rules of series circuits, Ohm's Law, and Kirchhoff's Voltage Law:

	R_1	R_3	Total	
E	16.8	4.2	21	Volts
I	4.2	4.2	4.2	Amps
R	4	1	5	Ohms

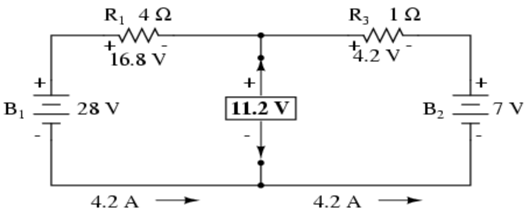


Fig.3.14

The voltage between the two load connection points can be figured from the one of the battery's voltage and one of the resistor's voltage drops, and comes out to 11.2 volts. This is our “Thevenin voltage” ($E_{Thevenin}$) in the equivalent circuit:

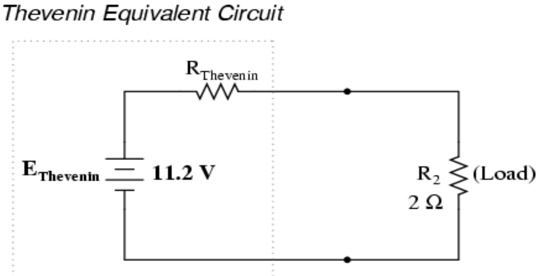


Fig.3.15

To find the Thevenin series resistance for our equivalent circuit, we need to take the original circuit (with the load resistor still removed), remove the power sources (in the same style as we did with the Superposition Theorem: voltage sources replaced with wires and current sources replaced with breaks), and figure the resistance from one load terminal to the other:

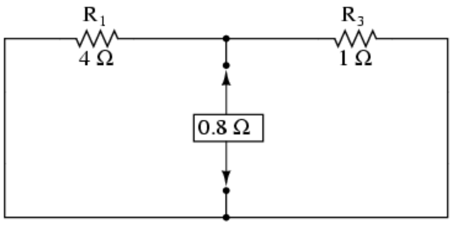


Fig.3.16

With the removal of the two batteries, the total resistance measured at this location is equal to R_1 and R_3 in parallel: 0.8 Ω. This is our "Thevenin resistance" ($R_{Thevenin}$) for the equivalent circuit:

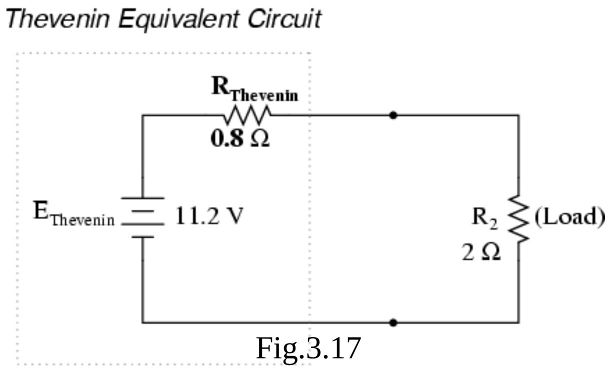


Fig.3.17

With the load resistor (2 Ω) attached between the connection points, we can determine voltage across it and current through it as though the whole network were nothing more than a simple series circuit:

	$R_{Thevenin}$	R_{Load}	Total	
E	3.2	8	11.2	Volts
I	4	4	4	Amps
R	0.8	2	2.8	Ohms

Notice that the voltage and current figures for R_2 (8 volts, 4 amps) are identical to those found using other methods of analysis. Also notice that the voltage and current figures for the Thevenin series resistance and the Thevenin source (*total*) do not apply to any component in the original, complex circuit. Thevenin's Theorem is only useful for determining what happens to a *single* resistor in a network: the load.

The advantage, of course, is that you can quickly determine what would happen to that single resistor if it were of a value other than 2 Ω without having to go through a lot of analysis again. Just plug in that other value for the load resistor into the Thevenin equivalent circuit and a little bit of series circuit calculation will give you the result.

4.Alternating Current Circuits

4.1. Equation of the Alternating Voltages and Currents

In general, an alternating current or voltage is one, the circuit direction of which reverses at regularly recurring intervals.

The shape of the curve obtained by plotting the instantaneous of voltage or current as ordinate against time as abscissa is called its waveform or wave shape.

An alternating current or voltage may not always take the form of symmetrical or smooth wave. But while it is possible for the manufacturers to produce sine-wave generators or alternators yet sine-wave is the ideal form thought by the designer and is the accepted standard. The waves deviating from the standard sine wave are termed as distorted waves. (Fig.4.1).

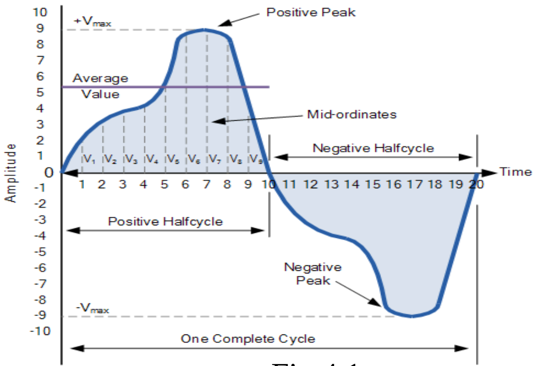


Fig.4.1

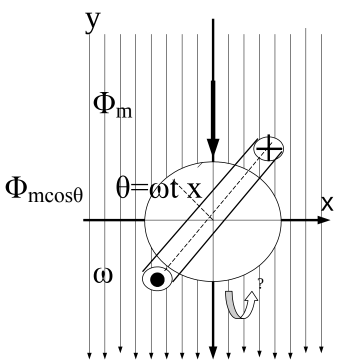


Fig.4.2

Consider a rectangular coil having N turns rotating in a uniform magnetic field with an angular velocity ω radian/second, as shown in Fig.4.2. Let time be measured from x-axis. Maximum flux Φ_M is linked with the coil when its plane coincides with x-axis. In time t seconds this coil rotates through an angle $\theta=\omega t$. In this deflected position, the component of the flux which is perpendicular to the plane of the coil is $\Phi=\Phi_M \cos \omega t$. Hence flux linkages" of the coil in this deflection position are $N\Phi= N\Phi_M \cos \omega t$.

According to Faraday's law of Electromagnetic Induction, the e.m.f. induced in the coil is given by the rate of change of flux-linkages of the coil. Hence, the value of the induced e.m.f. at this instant (i.e. when $\theta=\omega t$) or the instantaneous value of the induced e.m.f. is

$$e=-\frac{d}{dt}(N\Phi)=-N\frac{d}{dt}(\Phi_m\cos\omega t)=-\omega N\Phi_m(-\sin\omega t)=$$

$$=\omega N\Phi_m\sin\omega t=\omega N\Phi_m\sin\theta \quad (4.1)$$

When the coil has turned through 90^o i.e. when $\theta=90^o$, then $\sin\theta=1$, hence "e" has maximum value, say E_m. Therefore we get

$$E_m=\omega N\Phi_m=\omega N B_m A=2\pi fNB_m A, \text{ (volt)} \quad (4.2)$$

where Bm – maximum flux density in wb/m^2,

A – area of the coil in m^2,

f – frequency of rotation of the coil in rev/s.

Substituting this value of E_m in Eq. (4.1) we get

$$e=E_m\sin\theta=E_m\sin\omega t \quad (4.3)$$

Similarly, the equation of the induced alternating current is

$$i=I_m \sin \omega t \tag{4.4}$$

provided the coil circuit has been closed through a resistive load.

Since $\omega=2\pi f$, where f is the frequency of rotation of the coil, the above equations of the voltage and current can be written as

$$e= E_m \sin 2\pi ft= E_m \sin(\frac{2\pi}{T})\, t, \tag{4.5}$$

$$i= I_m \sin 2\pi ft= I_m \sin(\frac{2\pi}{T})\, t, \tag{4.6}$$

where T time-period of the alternating current or voltage is

$$T= \frac{1}{f} \tag{4.7}$$

It is clearly seen that induced e.m.f. is sine function of the time angle ωt and when e.m.f. is plotted against time, a curve similar to one shown in Fig.4.3 is obtained.

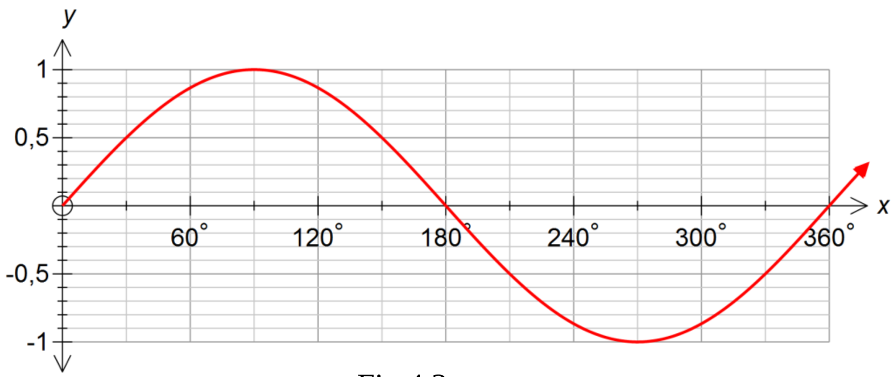


Fig.4.3

This curve is known as sine curve and the e.m.f. which varies in this manner is known as sinusoidal e.m.f.

The main characteristics of a sinusoidal quantity are as follows:

Cycle. – One complete set of positive and negative values of an alternating quantity is known as a cycle.

A cycle may also be sometimes specified in terms of angular measure. In that case, one complete cycle is said to spread over 360^o or 2π radians.

Time period. The time taken by an alternating quantity to complete one cycle is called its time period T. For example, a 50 Hz alternating current has a time period of $\frac{1}{50}$=0.002 sec.

Frequency. The number of cycles/second is called the frequency of the alternating quantity.

In fact, the frequency of the alternating voltage produced is a function of the speed and the number of the poles of the alternator. The relation connecting the above three quantities is given by

$$f = \frac{pN}{120}, \tag{4.8}$$

where N is revolution in r.p.m.

p is number of poles.

Amplitude. The maximum value, positive or negative, of an alternating quantity is known as its amplitude.

4.2. Phase and Phase Difference

By phase of alternating current is meant the fraction of the time period of that a.c. that had elapsed since it last passed through the zero position of reference. For example the phase of current at a point A is $\frac{T}{4}$ second, where T is the time period or expressed in terms of angle, it is $\frac{\pi}{2}$ radian (Fig.4.3). In electrical engineering, we are however more concerned with relative phases i.e. phase differences between different alternating quantities rather than with their absolute phases.

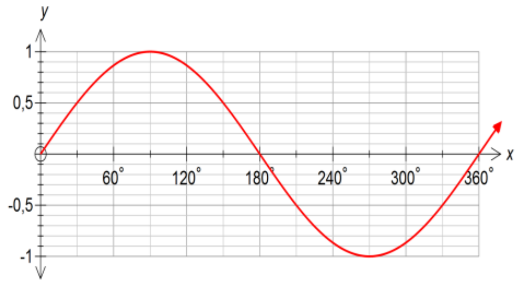


Fig. 4.4

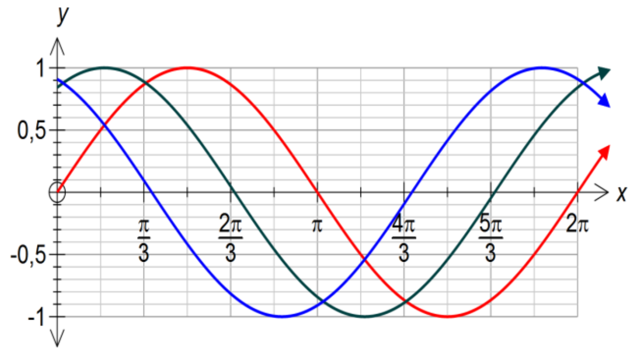


Fig.4.5

The three sinusoidal waves are shown in Fig.4.5. It is clearly seen that curves B and C are displaced from curve A by angles β and ($\alpha + \beta$) respectively.

Hence, it means that phase difference between A and B is β and between B and C is α, but between A and C is ($\alpha + \beta$). The statement, however, does not give indication as to which e.m.f. reaches its maximum value first. This deficiency is supplied by using the term “lag” or “lead”.

A leading alternating quantity is one which reaches its maximum (or zero) value earlier as compared with the other quantity.

Similarly a lagging alternating quantity is one which reaches its maximum or zero value later than the other quantity.

For example, in Fig.4.5 B lags behind A by β and C lags behind A by ($\alpha + \beta$).

The three equations for the instantaneous e.m.f.s in the Fig.4.5 are

$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin(\omega t - \beta)$$

$$e_C = E_m \sin[\ \omega t - (\alpha - \beta)\ ]$$

In Fig.4.6 quantity B leads A by an angle Φ. Hence their equations are:

$$v_A = V_m \sin \omega t$$

$$v_B = V_m \sin(\omega t + \Phi)$$

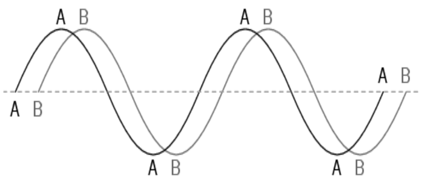


Fig.4.6

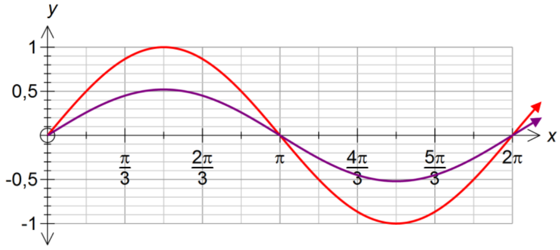


Fig.4.7

A plus (+) sign when used in connection phase difference denotes "lead", whereas a minus (-) sign denotes "lag".

If the two alternating (sinusoidal) quantities reach their maximum and zero values at the same time (i.e. they have equal phases) as shown in Fig.4.7 such quantities are said to be in phase with each other.

4.3. Root-Mean Square (R.M.S) Value

The r.m.s. value of an alternating current is given by that steady current (d.c) which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

It is also known as the effective or virtual value of a.c.

The standard form of a sinusoidal alternating current is

$$i = I_m \sin \omega t = I_m \sin \theta$$

The mean of squares of the instantaneous values of current over one cycle is

$\frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$ (integral of i squared with respect to θ)

The square root of this value is

$$\sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta}$$

Hence, the r.m.s. value of the alternating current is

$$I = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta} = \sqrt{\frac{I_m^{\ 2}}{2\pi}\int_0^{2\pi} \sin^2 d\theta} \quad \text{(putting } i = I_m \sin\theta\text{)}$$

Now $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ and

$$I = \sqrt{\frac{I_m^{\ 2}}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)d\theta} = \sqrt{\frac{I_m^{\ 2}}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{2\pi}} = \sqrt{\frac{I_m^{\ 2}}{4\pi}2\pi} = \sqrt{\frac{I_m^{\ 2}}{2}} = \frac{I_m}{\sqrt{2}}$$

or $I = 0.707\, I_m$ (4.9)

Hence, we find that for a symmetrical sinusoidal current r.m.s. value of current is equal to 0.707 multiply by a maximum value of current.

The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. values of alternating current and voltages respectively. In electrical engineering work, unless indicated otherwise, the values of the given current and voltage are always taken as the r.m.s. values.

It should be noted that average heating produced during one cycle is

$$RI^2 = (\frac{I_m}{\sqrt{2}})^2 R = \frac{1}{2} I_m{}^2 \qquad (4.10)$$

4.4. Average Value

The average value I_{av} of alternating current is expressed by that steady current which transfers across one circuit the same charge as is transferred by that alternating current.

In any case of symmetrical alternating current (i.e. one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by integrating the instantaneous value of current over half- cycle only. But in the case of unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.

The standard equation of an alternating current is

$i=I_m \sin A$

$$I_{av}=\frac{1}{\pi-0}\int_0^{\pi} id\theta = \frac{I_m}{\pi}\int_0^{\pi}\sin\theta dA = \frac{I_m}{\pi}\left|-\cos\theta\right|_0^{\pi} = \frac{I_m}{\pi}\left|+1-1\right| = \frac{2I_m}{\pi}$$

I_{av}= twice the max.current/π or

$$I_{av}=0.638\ I_m \qquad (4.11)$$

4.5. Vector Representation of Alternating Quantities

It has already been pointed out that an attempt is made to obtain alternating voltages and currents having sine wave forms. It is, however, cumbersome continuously to handle the instantaneous values in the form of equations of waves like $e=E_m\sin(\omega t+\psi_e)$, etc.

A conventional method is to employ vector method of representing these sine waves. These vectors may then manipulated instead of the sine functions to achieve the desired result.

In fact vectors are a short-hand for the representation of sinusoidal voltages and currents and their use greatly simplifies the problems in a.c. work. A vector quantity is a principal quantity which has both magnitude and direction. Such vector quantity is completely known only when particulars of its magnitude, direction and since in which it acts, are given. They are graphically represented by straight lines called vectors. The length of the line represents the magnitude of the alternating

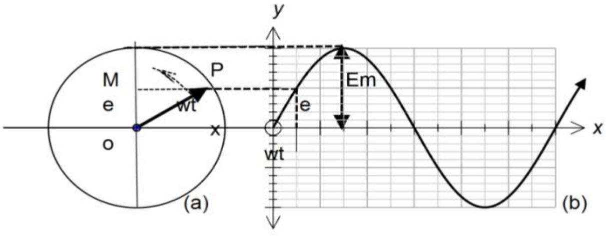


Fig.4.8

quantity, the inclination of the line with respect to some axis of reference gives the direction of that quantity and than arrow-head placed at one end indicates the direction in which the quantity acts.

The alternating voltages and currents are represented by such vectors rotating counter-clockwise with the same frequency as that of alternating quantity.

In Fig.4.8 OP is such a vector which represents the maximum value of the a.c. and its angle with x-axis gives its phase. Let the a.c. be represented by the equation $i = I_m \sin \omega t$. It will be seen that the projection of OP on y-axis at any instant gives the instantaneous value of that alternating current.

or $OM = OP \sin \omega t$

$I = OP \sin \omega t = I_m \sin \omega t$

In will be noted that a line OP can be made to represent an alternating current or voltage only if it satisfies the following conditions:

(1) Its length should be equal to that maximum value of the sinusoidal alternating quantity to a suitable scale.

(2) It should be in the horizontal position at the same instant as the alternating quantity is zero and increasing positively.

(3) Its angular velocity should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

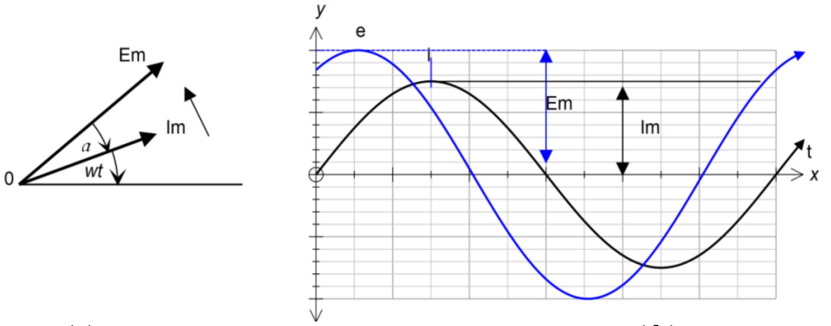
Instead of using maximum value as above it is more common practice to draw vector diagrams using r.m.s. values of alternating quantities. But it should be understood that in that case, the projection of the rotating vector on the y-axis does not give the instantaneous value of that alternating quantity.

4.6. Vector Diagrams of Sine Waves of Time Quantities

Two or more sine waves of the same frequency can be shown on the same vector diagram because the various vectors representing different waves all rotate counter-clockwise at the same frequency and so maintain a fixed position relative to each other. This is illustrated in Fig.9.9, where a voltage E_m and current I of the same frequency are shown. The current wave is supposed to pass upward through zero at the instant when t=0 while at the same time the voltage wave has already advanced by the angle α from its zero value. Hence, their equations can be written as

$$i = I_m \sin \omega t, \quad e = E_m \sin(\omega t + \alpha).$$

Sine waves of different frequencies cannot be represented on the same vector diagram in a still picture because due to difference in speed of different vectors, the phase angles between them will be continuously changing.



(a) (b)

Fig.4.9

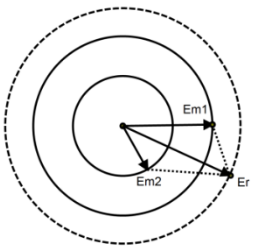
4.7 Addition of Two Alternating Quantities

In Fig.4.10 are shown two rotating vectors representing the maximum values of two sinusoidal voltage waves represented by

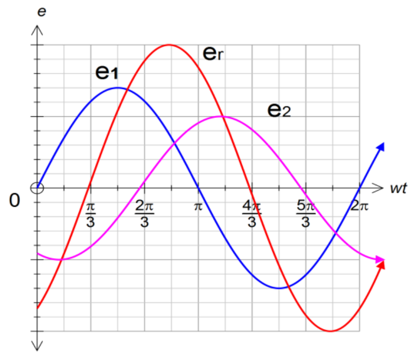
$$e_1=E_{m1}\sin\omega t \text{ and } e_2=e_{m2}\sin(\omega t-\psi_e).$$

It is seen that the sum of the two waves of the same frequency is another sine wave of the same frequency but of a different maximum value and phase. the value of the instantaneous resultant voltage v at any instant is obtained by algebraically adding the projections of the two vectors on the y-axis. If these projections are v_1 and v_2 than $v=v_1+v_2$ at that time.

The resultant curve has been drawn in this way by adding the ordinates. It is found that the resultant wave is a sine wave of the same frequency as the component waves but lagging behind E_{m1} by an angle ψ. The vector diagram of Fig.4.10 can be very easily drawn: Lay off E_{m2} lagging ψ_e behind E_{m1} and than complete the parallelogram. So E_m is obtained.



a)



b)

Fig.4.10

4.8. A.C. Through Pure Resistance Only

The circuit is shown in Fig.4.11. Let now consider that the circuit contains a resistance only and the applied voltage be given by the equation

$$v=V_m\sin\omega t. \tag{4.12}$$

Let R- is the ohmic resistance

i- is an instantaneous current

Obviously, the applied voltage has to overcame ohmic voltage drop only. Hence for equilibrium

$$v=Ri$$

Putting the value of v from above, we get $V_m\sin\omega t=Ri$

$$\text{or} \quad i=\frac{V_m}{R}\sin\omega t \tag{4.13}$$

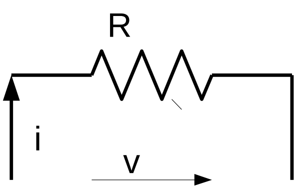


Fig.4.11

Current I is maximum when sin ωt is unity.

$$I_m = \frac{V_m}{R} \qquad (4.14)$$

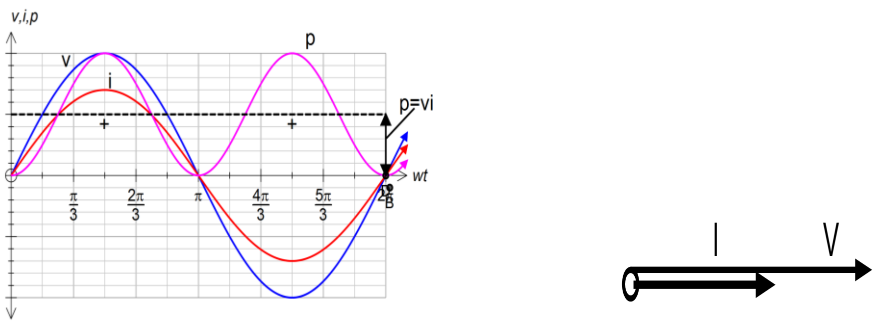
or, for r.m.s. values of the current and voltage

$$I = \frac{V}{R} \qquad (4.15)$$

Hence, equation (4.13) becomes

$$i = I_m \sin \omega t \qquad (4.16)$$

Comparing Eq. (4.16) and (4. 12), we find that the alternating voltage and current in the circuit with pure resistance are in phase with each other, as shown in Fig.4.12.a. It is shown by phasors $\overline{V} and \overline{I}$ Fig.4.12.b.



a) b)

Fig.4.12

<u>Power.</u> Instantaneous power

$$p = vi = V_m I_m \sin^2 \omega t = \frac{V_m I_m}{2}(1 - \cos 2\omega t),$$

or, substituting $V_m = \sqrt{2}V$ and $I_m = \sqrt{2}I$, we get

$$p = V\ I\ (1 - \cos 2\omega t) \qquad (4.17)$$

Power consists in a constant part VI and fluctuating part $-VI\cos 2\omega t$ of frequency doubly that of voltage and current waves. For a complete cycle, the average value of $VI\cos 2\omega t$ is zero. Hence, power for whole cycle is

$$P = \frac{1}{T}\int_0^T pdt = \frac{V_m I_m}{2} = VI = RI^2 = \frac{V^2}{R} \qquad (4.18)$$

It is seen from Eq. (4.17) that no part of the power cycle becomes negative at any time. This is because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive. In other words, the power supplied to the terminals is always absorbed by the resistor and the energy absorbed is dissipated by the resistor in the form of heat.

4.9. A.C. Through Inductance Only

Whenever an alternating voltage is applied to a pure inductive coil a back e.m.f. is produced due to the self-inductance of coil.

(By pure inductance is meant one has no ohmic resistance and hence no RI^2 loss. Pure inductance is actually not attainable through it is very nearly approached by a coil wound with such thick wire that its resistance is negligible. If it has some actual resistance then it is represented by a separate equivalent resistance joined in series with it).

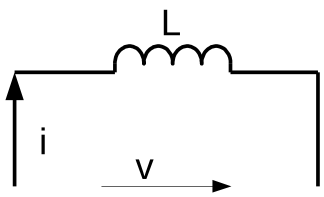


Fig.4.13

This back e.m.f., at every step, opposes the rise or fall of current through the coil. As there is no ohmic voltage drop, the applied voltage has to overcome this self-induced e.m.f. only. So, at every step

$$v = L\frac{di}{dt} \quad \text{and} \quad i = \frac{1}{L}\int_0^t v\,dt + i(0) \tag{4.19}$$

Now

$$v = V_m \sin \omega t,$$

and, supposing i (0) =0, the current flowing through the inductance will be

$$i = \frac{1}{L}\int_0^t v\,dt = \frac{V_m}{L}\int_0^t \sin \omega t dt = \frac{V_m}{\omega L}(-\cos \omega t) = \frac{V_m}{\omega L}\sin(\omega t - \frac{\pi}{2}) \tag{4.20}$$

The maximum value of i is

$$I_m = \frac{V_m}{\omega L} \tag{4.21}$$

or, dividing both sides by $\sqrt{2}$, for the r.m.s. values of the current an the voltage we get

$$I = \frac{V}{\omega L} \tag{4.22}$$

Here

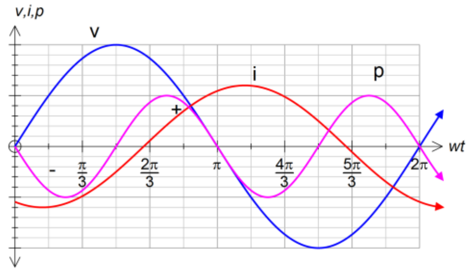
$$x_L = \omega L \text{ (ohm)} \tag{4.23}$$

plays the part of resistance. It is called the inductive reactance of the coil.

Hence, equation of the current becomes

$$i = I_m \sin(\omega t - \frac{\pi}{2}) \tag{4.24}$$

So, we find that the current flowing through a pure inductive coil lags the applied voltage by a quarter cycles or phase difference between the two is $\frac{\pi}{2}$ with voltage leading. (Fig.4.14., a,b).



(a)

(b)

Fig.4.14

Power. Instantaneous power

$$p=vi= V_m I_m \sin\omega t \sin(\omega t - \frac{\pi}{2}) = -V_m I_m \sin\omega t \cos\omega t = -\frac{V_m I_m}{2}\sin 2\omega t = VI\sin 2\omega t \quad (4.25)$$

Here, again it is seen that power wave is a sinewave of frequency double that of voltage and current waves. The maximum value of the instantaneous power is $\frac{V_m I_m}{2} = VI$

During the quarters of a period when the current and voltage have the same signs, the instantaneous power is positive and energy is stored in the magnetic field of the inductive element. However, during the quarters of a period when v and I have different signs the instantaneous power is negative, which means that instead of drawing energy from the source, the inductor supplies to the source.

Power for the whole cycle is zero

$$P = -VI\int_0^T \sin 2\omega t dt = 0 \quad (4.26)$$

which means that the average demand of power from the supply for a complete cycle is zero.

4.10. A.C Through Pure Capacitor

By a pure capacitor or capacitance is meant one that has neither resistance nor dielectric loss. If there is loss in capacitor then it may be represented by loss in comparatively high resistance joined in parallel with the pure capacitor. Let $v(t)$ be the voltage across the capacitor at any instant and $q(t)$ - the charge at that instant.

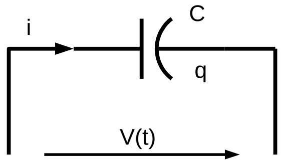


Fig.4.15

Then, according to the equation

$$i(t) = \frac{dq(t)}{dt} = C\frac{dv(t)}{dt} \quad (4.27)$$

putting the value of $v = V_m \sin\omega t$, we get

$$i = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin\omega t) = \omega C\cos\omega t$$

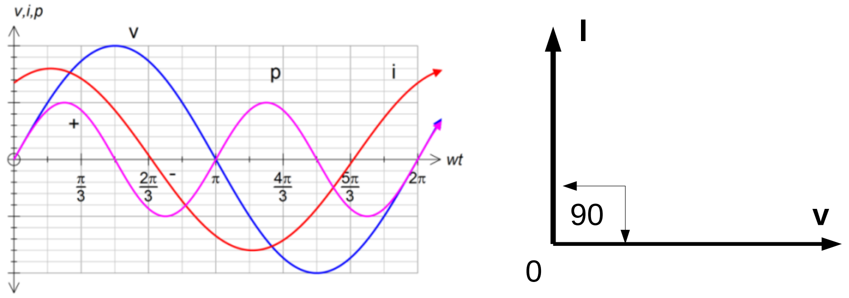
or
$$i = \frac{V_m}{\frac{1}{\omega C}}\sin(\omega t + \frac{\pi}{2}) = I_m \sin(\omega t + \frac{\pi}{2}) \qquad (4.28)$$

Obviously $I_m = \frac{V_m}{\frac{1}{\omega C}}$ (4.29)and $I = \frac{V}{\frac{1}{\omega C}}$ (4.30)

The denominator is known as capacitive reactance in ohms, if C is in farads and ω in rad/sec. It is denoted

$$X_c = \frac{1}{\omega C} \qquad (4.31)$$

It is seen that if applied voltage is given by $v = V_m \sin\omega t$, then current is given by $i = I_m \sin(\omega t + \frac{\pi}{2})$



a) b)

Fig.4.16

Hence, we find that the current in pure capacitor leads its voltage by a quarter cycle or phase difference between its voltage and current is $\frac{\pi}{2}$ with current leading (Fig.4.16.a). Vector representation is given in Fig.4.16., b.

Power. Instantaneous power is

$$p = vi = V_m I_m \sin\omega t \sin(\omega t + \frac{\pi}{2}) = V_m I_m \sin\omega t\cos\omega t = \frac{V_m I_m}{2}\sin 2\omega t = VI\sin 2\omega t \qquad (3.32)$$

Power for the whole cycle

$$P = VI\int_0^T \sin 2\omega t dt = 0 \qquad (3.33)$$

We find that in purely capacitive circuit, the average demand of power from supply is zero (as in purely inductive circuit). Again it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $\frac{V_m I_m}{2} = VI$. In capacitive element, the instantaneous power is positive during the interval when the applied voltage is increasing in magnitude (irrespective its direction). During this interval, the capacitor is being charged, and its electric field is storing energy. When the applied voltage is decreasing in magnitude, the instantaneous power is negative. Now the capacitor is discharging, and the energy stored up in its electric field is returned to the source.

4.11. Resistance, Inductance and Capacitance in Series

A pure resistance R (Ω), a pure inductance of L (Hn) and pure capacitance of C (F) connected in series are shown in Fig.4.17.

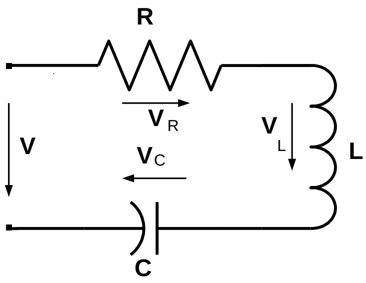


Fig.4.17

Let V be r.m.s. value of applied voltage, I- r.m.s. value of resultant current,

V_R=RI – voltage drop across R (in phase with I),

V_L=X_LI – voltage drop across coil L (leading I by $\frac{\pi}{2}$),

V_C=X_CI – voltage drop across C (lagging I by $\frac{\pi}{2}$).

These voltage drops are shown in phasor diagram of Fig.4.18. It is seen that $\overline{V_L}$ and $\overline{V_C}$ are 180^o out of phase with each other, i.e. they are in direct opposition to each other. Moreover, it has been assumed that V_L is greater than V_C in magnitude.

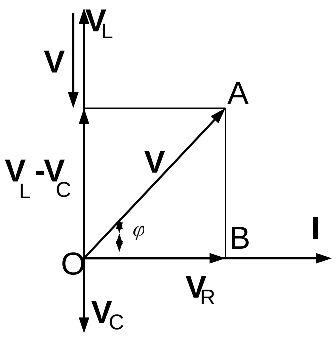


Fig.4.18

Subtracting $\overline{V_C}$ from $\overline{V_L}$ we get the net reactive drop

$$V_L - V_C = (X_L - X_C)\, I$$

The applied voltage $\overline{V}$ is represented by the phasor sum of $\overline{V_R}$ and $\overline{V_L - V_C}$.

Hence,

$$V = \sqrt{V_R{}^2 + (V_L - V_C)^2} = \sqrt{(RI)^2 + (X_L I - X_C I)^2} = \sqrt{\left[R^2 + (X_L - X_C)^2\right] I^2} =$$

$$= \sqrt{R^2 + (X_L - X_C)^2}\; I = ZI \qquad (4.34)$$

or

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)}} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{V}{Z} \tag{4.35}$$

The term

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \tag{4.36}$$

is known as the impedance of the a.c. circuit and the Eq. (4.35) represents Ohm's law for such circuit.

$$X = X_L - X_C = \omega L - \frac{1}{\omega C} \tag{4.37}$$

is the net reactance.

Phase angle is given by

$$\tan\ \varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{X}{R} \tag{4.38}$$

Hence, if applied voltage is given by

$$v = V_m \sin \omega t\,,$$

then current equation is

$$i = I_m \sin(\omega t - \varphi) \qquad \text{(see Fig.4.19)}$$

Remember that $X_C = \frac{1}{\omega C}$ is taken as negative. Therefore, when $X_L \succ X_C$, then $X \succ 0, \varphi \succ 0$ and a circuit is inductive in its effect, the current I lags behind the voltage in phase. When $X_L \prec X_C$, then $X \prec 0, \varphi \prec 0$, and the circuit is capacitive in its effect, the current in the circuit leads the voltage in phase. On the phasor diagram the positive value of the phase angle φ are counted counterclockwise from the current phasor $\overline{I}$; its negative value are counted clockwise.

When inductive reactance is equal to the capacitive reactance $X_L = X_C$ or $\omega L = \frac{1}{\omega C}$, the phase displacement $\varphi = 0$; the impedance of the circuit is a minimum $Z = R$, and the r.m.s. current is maximum: $I = \frac{V}{R}$.

The condition in a single-loop circuit containing an inductive element L, a capacitive element C and a resistive element R (Fig.4.17) such that $\varphi = 0$ ($\psi_u = \psi_i$), that is when the current and the voltage are in phase is called series (or voltage) resonance. The angular frequency at which the above condition ($\omega L = \frac{1}{\omega C}$) is observed is called the resonant frequency of the circuit $\omega_{res} = \frac{1}{\sqrt{LC}}$.

From the phasor diagram of Fig.4.18 we can also obtain the following relationships:

$$\cos\varphi = \frac{R}{Z}; R = Z\cos\varphi \tag{4.39}$$

$$\sin\varphi = \frac{X}{Z}\,;\, X = Z\sin\varphi \tag{4.40}$$

$$\tan\varphi = \frac{X}{R}\,;\, X = R\tan\varphi \tag{4.41}$$

where $Z = \sqrt{R^2 + X^2}$.

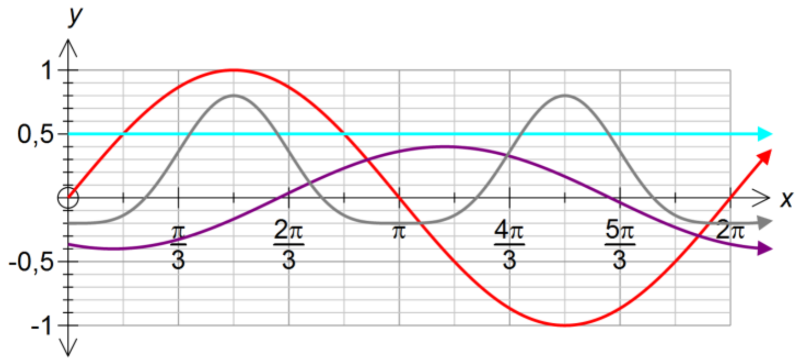


Fig.4.19

4.12. Instantaneous, Active, Reactive and Apparent Power of an A.C. Circuit

Assume that the voltage across and the current in a circuit (i.e. the passive one-port) are given by

$$v = V_m \sin\omega t \qquad \text{and} \qquad i = I_m \sin(\omega t - \varphi)$$

and find the instantaneous power in the circuit:

$$p = vi = V_m I_m \sin\omega t \sin(\omega t - \varphi) = \frac{V_m I_m}{2}\left[\cos\varphi + \cos(2\omega t - \varphi)\right] = VI\left[\cos\varphi + \cos(2\omega t - \varphi)\right] \tag{4.42}$$

Obviously, this power consists of two parts: a constant part $VI\cos\varphi$ and a pulsating component $VI\cos(2\omega t - \varphi)$, which has a frequency twice that of voltage and current.

Hence, the average power in a passive one-port over a period is given by

$$P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T VI\left[\cos\varphi + \cos(2\omega t - \varphi)\right]dt = VI\cos\varphi\,, \tag{4.43}$$

as $\int_0^T VI\cos(2\omega t - \varphi)dt = 0$

Equation (4.43) defines the active (or true) power in a circuit. It is seen to be proportional to the r.m.s. values of voltage and current, and $\cos\varphi$ is called the power factor. It should be noted that the active (or true) power is always positive and is independent of the sign of phase difference φ. It determines the energy conditions in a passive circuit in general, that is, the average rate of the irreversible conversion of energy in all resistive element of the passive circuit.

As was mentioned above

$$\cos\varphi = \frac{R}{Z}\,(\text{or}\,\tan\varphi = \frac{X}{R}) \tag{4.44}$$

On substituting for $\cos\varphi$ in Eq. (4.44), we obtain one more expression for the active (true) power

$$P = VI\cos\varphi = \frac{VIR}{Z} = RI^2$$

(4.45)

It should be noted that the product of the r.m.s. values of the voltage between the source terminals $V = E$, and the source current I defines apparent power of the source

$$S = VI = EI = ZI^2 \quad (4.46)$$

Active (true power is measured in watts, and apparent power in volt-amperes (VA). In power engineering, it is usual to measure apparent power in kilovolt-amperes, KVA.

Apparent power defines the service capabilities of many pieces of electric equipments (generators, transformers, electric machines and the like). As an example, for a generator the apparent power is equal to its maximum active (true) power that could be developed at $\cos\varphi = 1$. For most loads, however, $\cos\varphi \neq 1$. Therefore, the capacity of a source is not fully utilized.

For a proper analysis of power and energy relations in a circuit we introduce the concept of the reactive power of the source:

$$Q = EI\sin\varphi = VI\sin\varphi \quad (4.46)$$

By introducing reactive power, we can represent all the physical processes occur in all the inductive and capacitive elements of a passive circuit. Since $\sin\varphi = \frac{X}{Z}$, we get

$$Q = VI\sin\varphi = XI^2 \quad (4.47)$$

The reactive power may be positive or negative, depending on the sign of the angle φ. ($Q \succ 0, when \varphi \succ 0$) and ($Q \prec 0, if \varphi \prec 0$).

By comparing Eqs. (4.43) through (4.47), it is easy to see the following relation between the active, reactive and apparent power

$$S^2 = V^2I^2 = (VI\cos\varphi)^2 + (VI\sin\varphi)^2 = P^2 + Q^2$$

or $$S = \sqrt{P^2 + Q^2} \quad (4.48)$$

4.13. Parallel (G,L,C) Circuit

Fig. 4.20 shows an electric circuit which is a parallel combination of a resistive element, an inductive element and a capacitive element. Let the applied voltage be

$$v = V_m \sin\omega t = \sqrt{2}V\sin\omega t,$$

where V is the r.m.s. value of this voltage.

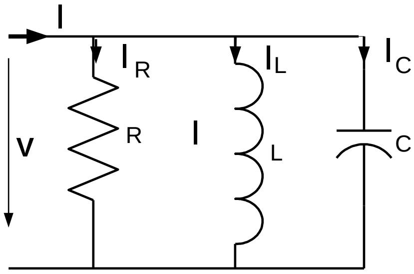


Fig.4.20

The r.m.s. values of the currents flowing through the branches can be represented as follows:

$$I_R = \frac{V}{R} = GV \text{ - current in } R \text{ (in phase with } V\text{)},$$

$$I_L = \frac{V}{\omega L} = B_L V \text{ - current in } L \text{ (lagging } V \text{ by } \frac{\pi}{2}\text{)},$$

$$I_C = \frac{V}{\frac{1}{\omega C}} = B_C V \text{ - current in } C \text{ (leading } V \text{ by } \frac{\pi}{2}\text{)},$$

where

$$G = \frac{1}{R} \text{ is the conductance of the resistive element,}$$

$$B_L = \frac{1}{\omega L} \text{ - inductive susceptance, and}$$

$$B_c = \omega C \text{ - capacitive susceptance}$$

These currents are shown in phasor diagram of Fig.4.21, assuming that $I_L \succ I_C$.

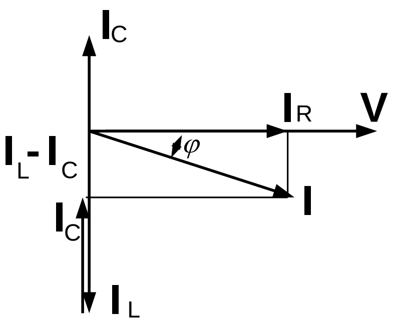


Fig.4.21

It is seen that $\overline{I_L}$ and $\overline{I_C}$ are 180^0 out of phase with each other, i.e. they are in direct opposition to each other.

Subtracting $\overline{I_C}$ from $\overline{I_L}$ we get the net reactive current

$$I_L - I_C = (B_L - B_C)V$$

The total circuit current is represented by the sum $\overline{I_R}$ and $(\overrightarrow{I_L - I_C})$.

Hence,

$$I = \sqrt{I_R{}^2 + (I_L - I_C)^2} = \sqrt{(GV)^2 + (B_L V - B_C V)^2} = \sqrt{G^2 - (B_L - B_C)^2} V = yV \qquad (4.49)$$

The term

$$y = \sqrt{G^2 - (B_L - B_C)^2} = \sqrt{G^2 - (\frac{1}{\omega L} - \omega C)^2} \qquad (4.50)$$

is known as the admitance of the a.c. circuit and the Eq.3.49 represents Ohm’s law for such circuit.

$$B = B_L - B_C = \frac{1}{\omega L} - \omega C \qquad (4.51)$$

is the net susceptance.

The phase difference is given by

$$\tan\varphi = \frac{I_L - I_C}{I_R} = \frac{\frac{1}{\omega L} - \omega C}{G} = \frac{B}{G} \tag{4.52}$$

Hence, if the applied voltage is given by

$$v = V_m \sin\omega t$$

then the current will be

$$i = I_m \sin(\omega t - \varphi) = yV_m \sin(\omega t - \varphi)$$

Remember that $B_C = \omega C$ is taken as negative.

Therefore, when $B_L \succ B_C$, i.e. $B \succ 0, \varphi \succ 0$, and, a current is inductive in its effect, the voltage leads the current in phase. When $B_L \prec B_C$, i.e. $B \prec 0, \varphi \prec 0$, the current is capacitive in its effect, the voltage lags behind the current in phase.

When the inductive susceptance B_L and capacitive susceptance B_C are equal to each other $B_L = B_C$, $(\frac{1}{\omega L} = \omega C)$, the phase difference $\varphi = 0$; the admittance of the circuit is a minimum $y = G$ and the r.m.s. current is minimum: $I = GV$.

The condition in a parallel (R, L, C) circuit containing an inductive element L, a capacitive element C and a resistive element R,(Fig.4.20) such that $\varphi = 0(B_L = B_C)$, that is when the current and the voltage are in phase, is called *parallel (or current) resonance.* Obviously, the resonant frequency is $\omega_0 = \frac{1}{\sqrt{LC}}$, as in the case of the voltage resonance.

From the phasor diagram of Fig.3.20, we can also obtain the following relationships:

$$\cos\varphi = \frac{G}{Y}, \quad G = Y\cos\varphi \tag{4.53}$$

$$\sin\varphi = \frac{B}{Y}, \quad B = Y\sin\varphi \tag{4.54}$$

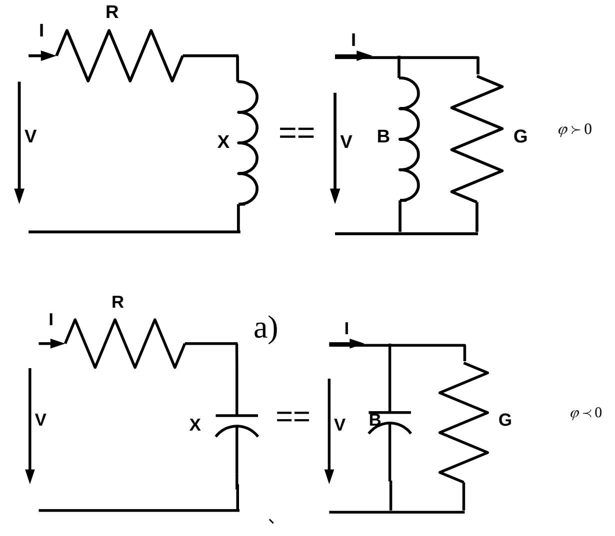
$$\tan\varphi = \frac{B}{G}, \quad B = G\tan\varphi$$

(4.55)

4.14. Transformation of a Series Circuit into a Parallel Circuit

The analysis of electric circuit can sometimes be markedly simplified by transforming a series connection into an equivalent parallel connection or vice versa. (Fig.4.22,a and b).

Two circuits are said to be equivalent if the input current I an the phase displacement φ between the voltage V and the current I are the same for both circuits, provided the voltage V across the terminals of the both circuits is the same.



b)
Fig.4.22

Comparing the expressions (4.39) and (4.40) (§ 4.11) to the expressions (4.53) and (4.54) (§4.13), respectively, and taking into account the obvious relationship between the impedance Z and admittance Y

$$\frac{V}{I} = Z = \frac{1}{Y}, \frac{I}{V} = Y = \frac{1}{Z} \tag{4.56}$$

we get

$$\cos\varphi = \frac{R}{Z} = \frac{G}{Y} \tag{4.57}$$

$$\sin\varphi = \frac{X}{Z} = \frac{B}{Y} \tag{4.58}$$

From Eqs.(4.57) and (4.58) it follows that conductance's and susceptances of the elements connected in parallel can be expressed in terms of the resistances of the elements connected in series in the following manner

$$G = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}, \tag{4.59}$$

$$B = \frac{X}{Z^2} = \frac{X}{R^2 + X^2} \tag{4.60}$$

Similarly, we can convert a parallel circuit into an equivalent series circuit with the help of the expressions given below:

$$R = \frac{G}{Y^2} = \frac{G}{G^2 + B^2} \tag{4.61}$$

$$X = \frac{B}{Y^2} = \frac{B}{G^2 + B^2} \tag{4.62}$$

4.15. A.C. Series-Parallel Circuit

An example of the a.c. series-parallel circuit to be calculated is shown in Fig.4.23,a.

The procedure leading to the impedance of the circuit is as follows.

The first step is to find the equivalent conductance's and susceptances of the parallel branches connected between points a and b by formulas:

$$G_2 = \frac{R_2}{R_2{}^2 + X_{C2}{}^2}, \qquad B_2 = \frac{X_{C2}}{R_2{}^2 + X_{C2}{}^2},$$

$$G_3 = \frac{R_3}{R_3{}^2 + X_{L3}{}^2}, \qquad B_3 = \frac{X_{L3}}{R_3{}^2 + X_{L3}{}^2}.$$

The circuit obtained after transformation is shown in Fig.4.23,b.

The second step: calculation of the equivalent conductance G_{ab}, susceptance B_{ab} and admittance Y_{ab} of the parallel part:

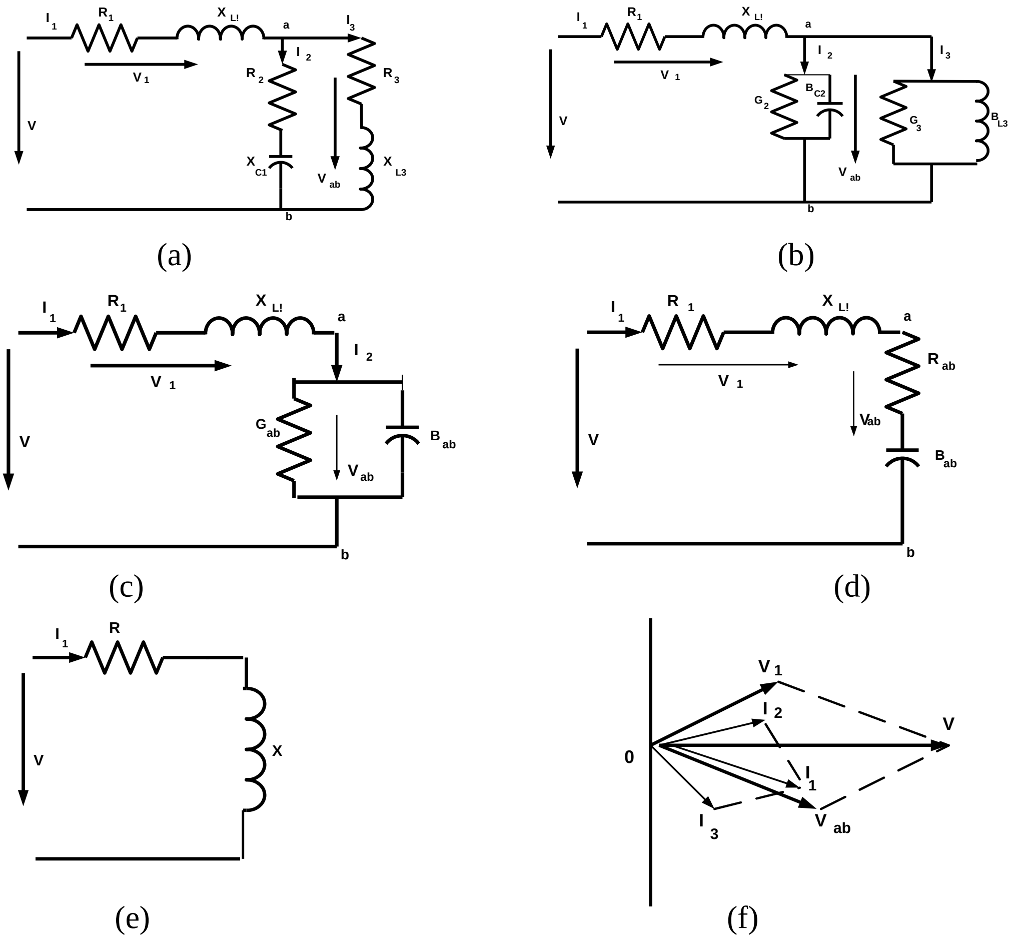


Fig.4.23

$$G_{ab} = G_2 + G_3, \quad B_{ab} = -B_{C2} + B_{L3}, \quad Y_{ab} = \sqrt{G_{ab}{}^2 + B_{ab}{}^2} \quad \text{(supposing } B_{C2} \succ B_{L3}\text{)}$$

The corresponding circuit is shown in Fig.4.23,c.

The third step: now the branch ab, containing conductance G_{ab} and susceptance B_{ab} in parallel must be converted into the equivalent branch containing element R_{ab} and X_{ab}, connected in series:

$$R_{ab} = \frac{G_{ab}}{Y_{ab}{}^2}, \qquad X_{ab} = \frac{B_{ab}}{Y_{ab}{}^2}$$

$$Z_{ab} = \sqrt{R_{ab}{}^2 + X_{ab}{}^2}$$

(see Fig.4.23,d)

The fourth step: calculating of the equivalent parameters of the given circuit (Fig.4.23,a):

$$R = R_1 + R_{ab} \ , \quad X = X_{L1} + X_{ab} \ , \quad Z = \sqrt{R^2 + X^2} \quad \text{(supposing } X_{L1} \succ X_{ab}\text{)}.$$

The equivalent circuit is represented in Fig.4.23,e.

The r.m.s. values of the currents, voltages and phase differences between them can be calculated by Ohm's law:

$$I_1 = \frac{V}{Z} \ , \qquad \tan\varphi = \frac{X}{R} \ ;$$

$$V_1 = Z_1 I_1 = \sqrt{R_1{}^2 + X_{L1}{}^2}\, I_1 , \qquad \tan\varphi_1 = \frac{X_{L1}}{R_1};$$

$$V_{ab} = Z_{ab} I_1 = \frac{I_1}{Y_{ab}} \ , \qquad \tan\varphi_{ab} = \frac{X_{ab}}{R_{ab}};$$

$$I_2 = \frac{V_{ab}}{Z_2} = \frac{V_{ab}}{\sqrt{R_2{}^2 + X_{C2}{}^2}} , \qquad \tan\varphi_2 = \frac{X_{C2}}{R_2} \ ;$$

$$I_3 = \frac{V_{ab}}{Z_3} = \frac{V_{ab}}{\sqrt{R_3{}^2 + X_{L3}{}^2}} , \qquad \tan\varphi_3 = \frac{X_{L3}}{R_3} \ .$$

A current and voltage phasor diagram for the circuit in question is shown in Fig.4.23,f.

4.16. Three Phase Circuits. Generation of Three-phase Voltages

The kind of alternating currents and voltages discussed in previous chapter are known as single-phase voltages and current; because they consist of a single alternating current and voltage wave.

Three-phase e.m.f.'s are generated by means of the three-phase alternator, which as distinct from a singl-phase generator consists of the three independent armature windings, which are 120^o electrical degrees apart (Fig.4.24). Hence, the e.m.f.'s induced in the three windings are 120^o apart in time phase.(as is represented in Fig.5.24,a). Each wave is assumed to be sinusoidal and having maximum value of E_m.

Equations of these waves are

$$e_a = E_m \sin\omega t \tag{4.63}$$

$$e_b = E_m \sin(\omega t - 120^o) \tag{4.64}$$

$$e_c = E_m \sin(\omega t - 240^o) \tag{4.65}$$

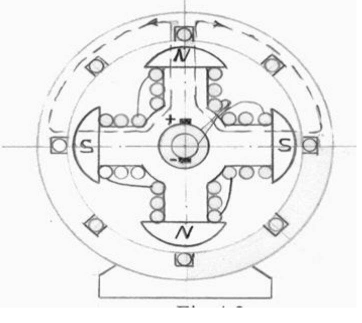


Fig.4.24

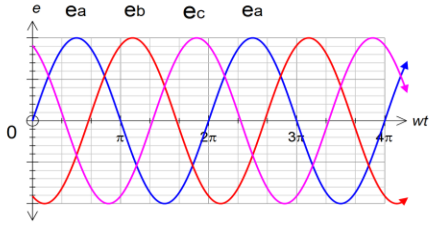


Fig.4.24,a

In the Fig.4.25 are shown the three vectors representing the r.m.s. voltages of the three phases: E_a, E_b, E_c. In present case $E_a = E_b = E_c = E$.

It can be shown by adding of the corresponding three vectors, that the sum of the three-phase e.m.f.'s is zero (Fig.4.25).

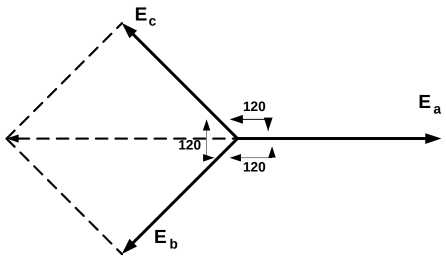


Fig.4.25

The three phases may be numbered 1,2,3, or a,b,c, or as is customary, they may be given three colours. The colours used commercially are red, yellow and blue. In that case sequence is R,Y,B. Obviously, in any three phase system there are two possible sequences in which three coil or phase voltages may pass through their maximum value i.e. 1, 2, 3 (or a,b,c; R,Y,B) or 1,3,2 (a,c,b; R,B,Y). By convention a,b,c is regarded as positive sequence and a,c,b as negative sequence.

4.17 Interconnection of Three Phases

If the three armature coils of the 3-phase alternator are not interconnected but are kept separate as shown in Fig.4.26, then each phase or circuit would need two conductors, the total number in that case being six. It means that each transmission cable would contain six conductors which will make the whole system complicated and expensive. Hence, the three phases are, generally, interconnected which results in substantial saving of copper. The general methods of interconnection are :

(a) Star or Wye(Y) connection and (b) Mesh or Delta (Δ) connection.

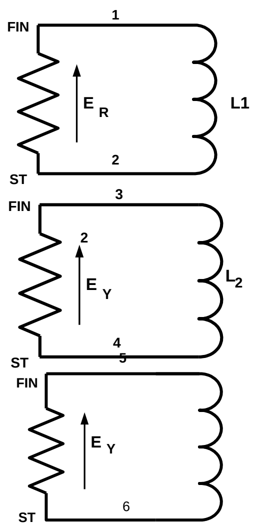


Fig.4.26

4.18. Star connection

In this method of interconnection, the similar ends, say, “start” ends of three coils (it could be “finishing” ends also) are joned together at point N as shown in Fig.4.28.

The point N is known as star point or “neutral” point. The three conductors meeting at point N are replaced by a single conductor known as neutral conductor as shown in Fig.3.28. Such an interconnected system is known as four-wire 3-phase system and is diagrammatically shown in fig.4.28.

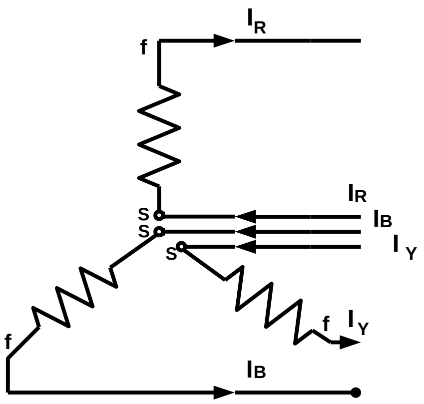


Fig.4.27

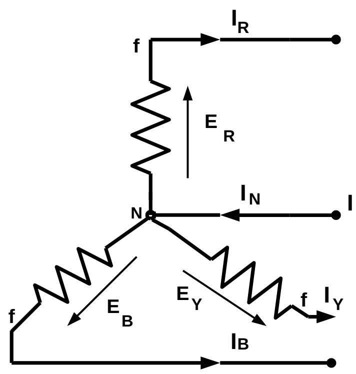


Fig.4.28

The point N is known as star point or “neutral” point. The three conductors meeting at point N are replaced by a single conductor known as neutral conductor as shown in Fig.3.28. Such an interconnected system is known as four-wire 3-phase system and is diagrammatically shown in fig.4.28.

If this 3-phase voltage is applied across a balanced symmetrical load, the neutral wire will be carrying three currents which are exactly equal in magnitude but are 120^o out of phase with each other. Hence, their vector sum is zero, i.e.

$$\overline{I_R} + \overline{I_Y} + \overline{I_B} = 0$$

The neutral wire, in that case may be omitted. The p.d. between any terminal (or line) neutral (or star) point gives the phase or star voltage. But the p.d. between any two lines gives the line voltage.

4.19 . Voltage and Currents in Star Connection

The voltage induced in each winding is called the “phase” voltage and current in each winding is likewise known as “phase” current. However, the voltage available between any pair of terminals (or outers) is called line voltage (V_L) and the current flowing in each line is called line current (I_L).

As seen from Fig.4.29, in this form of interconnection, there are two phase windings between each pair of terminals, but since their ends have been joined together, they are in opposition. Obviously, the instantaneous value of p.d. between any two terminals is difference of the two phase e.m.f.’s concerned. However, the r.m.s. value of this p.d. is given by the vector difference of two phase e.m.f’s. The vector diagram for phase voltages and currents in a star connection is shown in Fig.4.30, where a balanced system has been assumed.

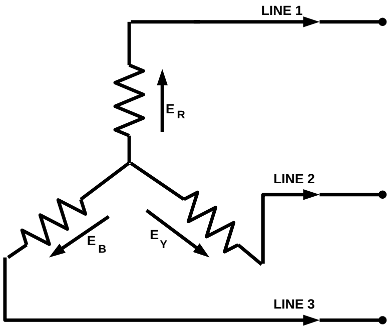


Fig.4.29

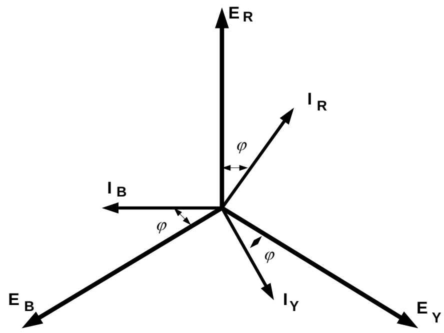


Fig.4.30

A 3-phase balanced load is that in which the loads connected across three phases are identical in magnitude and phase).

It means that $\overline{E_R} = \overline{E_Y} = \overline{E_{ph}}$

Line voltage $\overline{V}_{RY}$ between line 1 and line 2 is vector difference of $\overline{E}_R$ and $\overline{E}_Y$

$$\overline{V_{RY}} = \overline{E_R} - \overline{E_Y} \tag{3.66}$$

Similarly
$$\overline{V_{YB}} = \overline{E_Y} - \overline{E_B} \tag{3.67}$$

$$\overline{V_{BR}} = \overline{E_B} - \overline{E_R} \tag{3.68}$$

Hence $\overline{V}_{RY}$ is found by compounding $\overline{E}_R$ and $\overline{E}_Y$ reversed and its value is given by the diagonal of the parallelogram of Fig.4.31.

Obviously, the angle between $\overline{E}_R$ and $\overline{E}_Y$ reversed is 60^o. Hence if $E_R = E_Y = E_B = E_{ph}$, then

$$V_{RY} = 2E_{ph}\cos\frac{60}{2} = 2E_{ph}\frac{\sqrt{3}}{2} = \sqrt{3}E_{ph} \tag{4.69}$$

Similarly
$$V_{YB} = \left|\overline{E_Y} - \overline{E_B}\right| = \sqrt{3}E_{ph} \tag{4.70}$$

$$V_{BR} = \left|\overline{E_B} - \overline{E_R}\right| = \sqrt{3}E_{ph} \tag{4.71}$$

Now
$$V_{RY} = V_{YB} = V_{BR} = \sqrt{3}E_{ph} \tag{4.72}$$

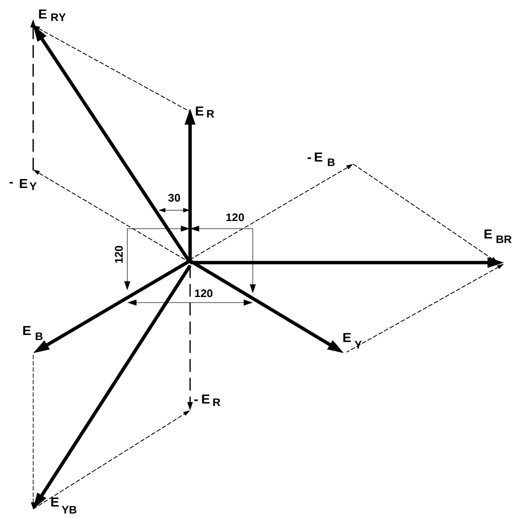


Fig.4.31

From Fig.4.31 it should be noted that

(a) Line voltages are 120^o apart,

(b) Line voltages are 30^o ahead of their respective phase voltages,

(c) The angle between the line current and the corresponding line voltages is $(30^o + \varphi)$ with current lagging.

From Fig.4.28 it is seen that each line is in series with its individual phase winding, hence, the line current in each line is the same as the current in the phase winding to which it is connected.

Current in line 1=I_R, current in line 2=I_Y, current in line 3=I_B. Since $I_R = I_Y = I_B = I_{ph}$ line current $I_L = I_{ph}$.

Power. The total power in the circuit is the sum of three phase power. hence

$$P = 3P_{ph} = 3E_{ph}I_{ph}\cos\varphi \tag{4.73}$$

If we take into account that

$$E_{ph} = \frac{V_L}{\sqrt{3}} \quad \text{and} \quad I_{ph} = I_L$$

Hence, in terms of line values the above expression becomes

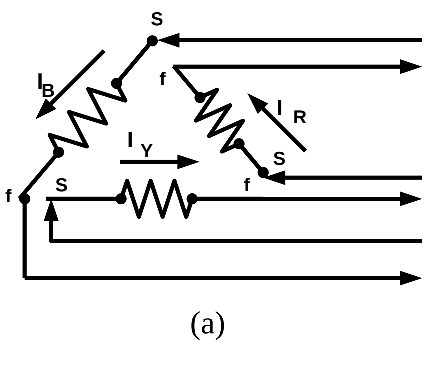
$$P = 3\frac{V_L}{\sqrt{3}} I_L \cos\varphi = \sqrt{3} V_L I_L \cos\varphi \tag{4.74}$$

It should be noted that φ is angle between phase voltage and line (phase) current.

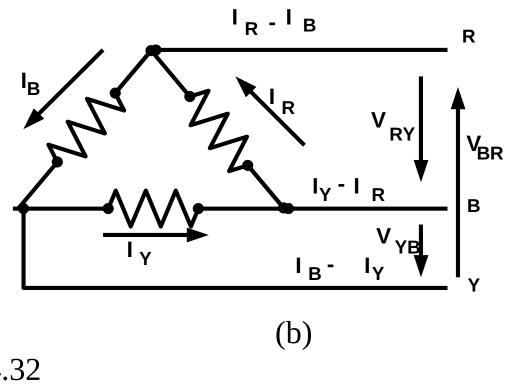
4.20. Delta (Δ) or Mesh Connection

In this form of interconnection, the dissimilar ends of the three-phase windings are joined together i.e. the "starting" end of one phase is joined to the "finishing" end of the other phase and so on as shown in Fig.4.32a,b. In other words, the three windings are joined in series to form a closed mesh as shown in Fig.4.32,a.

Three leads are taken out from the three junctions as shown in Fig.4.32,b and outward directions are taken as positive.



(a)



(b)

Fig.4.32

It might look as if this sort of interconnection results in short circuiting the three windings. However, if the system is balanced, then the sum of three voltages around the closed mesh is zero, hence, no current of fundamental frequency can flow around the mesh when the terminals are open. It should be clearly understood that at any instant, the e.m.f. in one phase is equal and opposite to the resultant of those in the other two phases.

This type of connection is also referred to as 3-phase, 3-wire system.

From Fig.4.32, b it is seen that there is only one phase winding completely included between any pair of terminals. Hence, in Δ- connection, the voltage between any pair of lines is equal to the phase voltage of the phase winding connected between the two lines considered. Since phase sequence is RYB (a,b,c or $1,2,3$), hence the voltage having its positive direction from R to Y leads by 120^o on that having its positive direction from Y to B. Calling the voltage between lines 1 and 2 as V_{RY} and that between lines 2 and 3 as V_{YB}, we find that V_{RY} leads V_{YB} by 120^o. Similarly, V_{YB} leads V_{BR} by 120^o as shown in Fig.4.33.

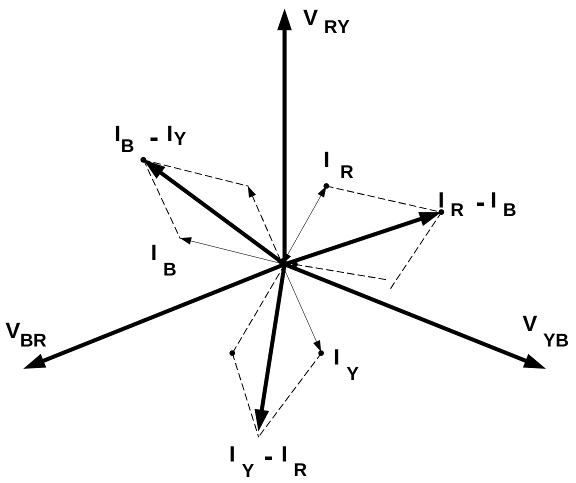


Fig.4.33

Let, $V_{RY} = V_{YB} = V_{BR} = V_L$ (line voltage), than it is seen that $V_L = V_{ph}$ From Fig.4.33 it will be seen that current in each line is the difference of the two phase currents flowing through that line.

For example, current in line 1 is:

$$\bar{I}_1 = \bar{I}_R - \bar{I}_Y \,, \tag{4.75}$$

That in line 2

$$\bar{I}_2 = \bar{I}_Y - \bar{I}_B \tag{4.76}$$

And in line 3

$$\bar{I}_3 = \bar{I}_B - \bar{I}_R \tag{4.78}$$

Current in line 1 is found by compounding $\bar{I}_R$ with $\bar{I}_B$ reversed and its value is given by diagonal of the parallelogram of Fig.4.33. The angle between $\bar{I}_R$ and $\bar{I}_B$ reversed (i.e. -$\bar{I}_B$) is 60^o. If $I_A = I_B = I_C = I_{ph}$ (phase currents) then currents in the line No.1 is

$$I_1 = 2I_{ph}\cos\frac{60^o}{2} = 2I_{ph}\frac{\sqrt{3}}{2} = \sqrt{3}I_{ph} \tag{4.79}$$

Similarly $\qquad I_2 = \sqrt{3}I_{ph} \qquad I_3 = \sqrt{3}I_{ph}$

Since all the line currents are equal in magnitude, i.e.

$$I_1 = I_2 = I_3 = I_L = \sqrt{3}I_{ph}$$

(4.80)

With reference to Fig.3.33 it should be noted that

(a) line currents are 120^o apart

(b) line currents are 30^o behind the respective phase current,

(c) The angle between the line current and the corresponding line voltage is $(30^o + \varphi)$ with the current lagging.

Power. Phase power

$$P_{ph} = V_{ph} I_{ph} \cos \varphi \tag{4.81}$$

Total power

$$P = 3V_{ph} I_{ph} \cos \varphi \tag{4.82}$$

So as $V_{ph} = V_L$, but $I_{ph} = \frac{I_L}{\sqrt{3}}$ in terms of line values the above expression for power becomes

$$P = 3V_L \frac{I_L}{\sqrt{3}} \cos \varphi = \sqrt{3} V_L I_L \cos \varphi \tag{4.83}$$

where φ is the power factor angle.

4.21 Comparison : Star and Delta Connection

(a) Star connection:

1. Alternators are usually star-wound for the reason that phase voltage has to be only $\frac{1}{\sqrt{3}}$ of the line voltage, whereas for a delta-winding, the phase voltage has to be equal to the line voltage. Now, the number of conductors per phase in the windings of an alternator or motor, for a given frequency and flux, is directly proportional to the phase voltage. Hence, for a given line voltage fewer turns /phase are required with Y-connection than with a delta-connection.

2. With star connection, the system of distribution mains can be arranged to suit both lighting and power circuit without using transformers. Heating and lighting circuits are put across neutral and any line wire, whereas the 3-phase motors, running at higher voltages can be joined across the lines directly.

3. Another advantage of y-connection is that the neutral point of the alternators can be (and usually is) earthed. In that case, the potential drop between each line and earth is equal to the phase voltage, i.e. $\frac{1}{\sqrt{3}}$ of the line voltage. Hence, if through fault, line conductor is earthed, the insulators will have to bear $\frac{1}{\sqrt{3}}$ of the line voltage (i.e. 57.7%) only. But in the case of a delta connection, if any line conductor is earthed, the insulators will have to bear full line voltage. Hence, there would be produced a correspondingly higher stress in the insulators with greater liability to break down.

(b) Delta Connection

1. The advantage of this connection is that transformers, in general, work more satisfactorily.

2. It is the only connection suitable for such machines as rotary converters.

3. This connection is much used for comparatively small low-voltage three-phase motors.

4.22. Comparison Between Single-and 3-phase Supply Systems

1. Power in a single phase system is pulsating (at twice the frequency of voltage). This is not objectionable for lighting or for small motors. But with large motors, pulsating power supply causes excessive vibration.
2. Single-phase motors (except commutator type) have no starting torque, hence they need ancillary apparatus for self-starting. This is unnecessary in the case of 3-phase motors working on 3-phase supply.
3. Power factor of a single-phase motor is lower than that of a 3-phase motor the same output and speed.
4. For a given size of frame the output of a 3-phase machine is greater than that of a single-phase motor.
5. To transmit a given amount of power at a given voltage at a given distance, 3-phase transmission requires $\frac{3}{4}$th weight of copper of a single-phase system.
6. Three-phase currents can produce rotating magnetic fields when passed through stationary coils (as in the case of induction and synchronous motors).

5. Mutual Inductance. Transformer

5.1 . Mutual Inductance in a Circuit

The study of inductance presents a very challenging but rewarding segment of electricity. It is challenging in the sense that, at first, it will seem that new concepts are being introduced. You will realize as this chapter progresses that these "new concepts" are merely extensions and enlargements of fundamental principles that you learned previously in the study of magnetism and electron physics. The study of inductance is rewarding in the sense that a thorough understanding of it will enable you to acquire a working knowledge of electrical circuits more rapidly.

Inductance is the characteristic of an electrical circuit that opposes the starting, stopping, or a change in value of current. The above statement is of such importance to the study of inductance that it bears repeating. Inductance is the characteristic of an electrical conductor that opposes change in current. The symbol for inductance is L and the basic unit of inductance is the Henry (H). One Henry is equal to the inductance required to induce one volt in an inductor by a change of current of one ampere per second. You do not have to look far to find a physical analogy of inductance. Anyone who has ever had to push a heavy load (wheelbarrow, car, etc.) is aware that it takes more work to start the load moving than it does to keep it moving. Once the load is moving, it is easier to keep the load moving than to stop it again. This is because the load possesses the property of inertia. Inertia is the characteristic of mass which opposes a change in velocity. Inductance has the same effect on current in an electrical circuit as inertia has on the movement of a mechanical object. It requires more energy to start or stop current than it does to keep it flowing.

If an electric circuit contains magnetically coupled coils the flux due to one links the other, and there is an e.m.f. of mutual inductance in each coil that must be taken into account.

When writing equations for a circuit with a mutual inductance we have to take into account the relative directions of the fluxes of self and mutual inductance. These directions can be ascertained if one knows the direction in which the coils are wound on their cores and the positive direction of the current through them.

5.2. Mutual Inductance in Series. Connection of Coils .

Fig.5.1 and Fig.5.3a shows two coils connected in series aiding and Fig.5.2 and Fig.5.3b - two coils connected in series opposition.

According to Kirchhoff`s voltage law for series aiding connection

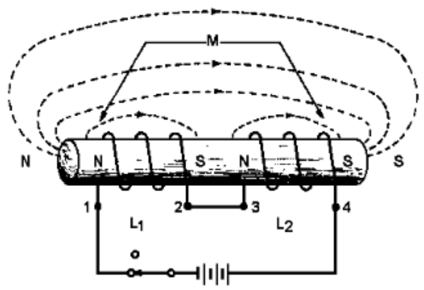


Fig.5.1

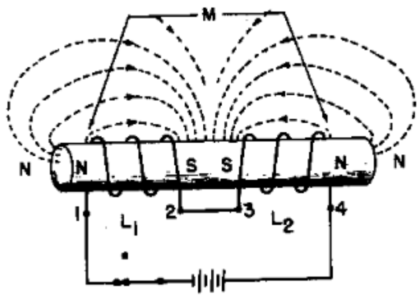


Fig.5.2

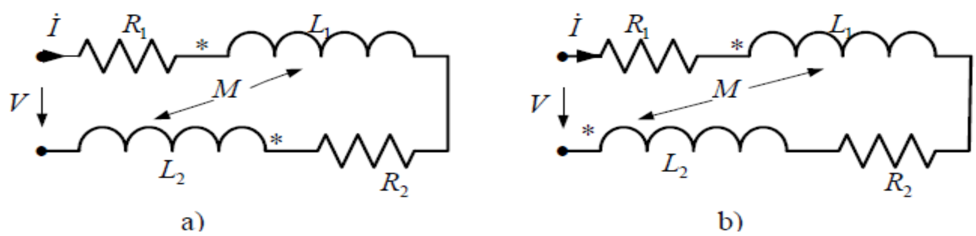


Fig.5.3

$$iR_1 + L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + iR_2 = v$$

In complex notation

$$\dot{I}\left[R_1 + R_2 + j\omega(L_1 + L_2 + 2M)\right] = \dot{E}$$

Fig.5.4 shows a vector diagram for series aiding connection, where $\dot{V}_1$ the complex voltage is across the first coil, and $\dot{V}_2$ is the complex voltage across the second coil.

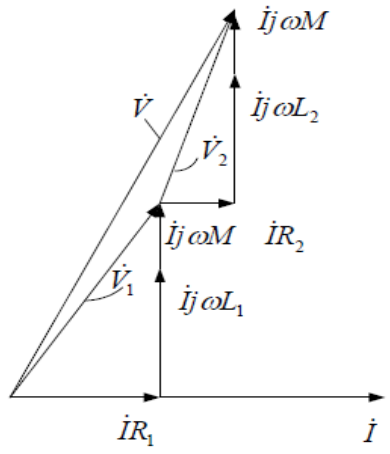


Fig.5.4

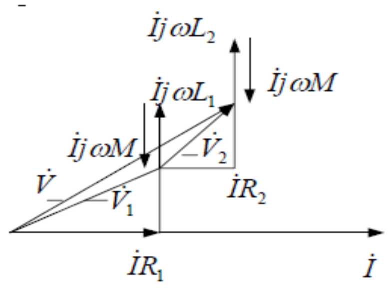


Fig.5.5

5.3. Working Principle of a Transformer

A transformer is a static (or stationary) piece of apparatus, by means of which, an electric power in one circuit is transformed to electric power (with the same frequency) in another circuit. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. The physical basis of a transformer is mutual induction between two circuits linked by a common magnetic flux. In its simplest form, it consists of two inductive coils which are electrically separate but magnetically linked through a path of low reluctance as shown in Fig.4.1. The two coils posse's high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the other coil in which it produces mutually-induced e.m.f. (according to the Faraday's Laws of Electromagnetic Induction i.e. $e = -M\dfrac{di}{dt}$).

If the second coil circuit is closed, a current flows in it and so electric energy is transferred from the first coil to the second coil.

The first coil, in which electric energy is fed from the a.c. supply mains, is called primary winding and the other, from which energy is drawn out, is called secondary winding.

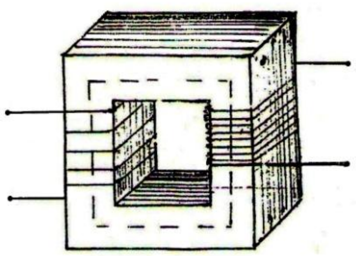


Fig.5.1

In brief, a transformer is a device that

(a) transformers electric power from one circuit to another,
(b) it does so without change of frequency,
(c) it accomplishes this by electromagnetic induction and
(d) where two electric circuits are in mutual inductive influence with each other.

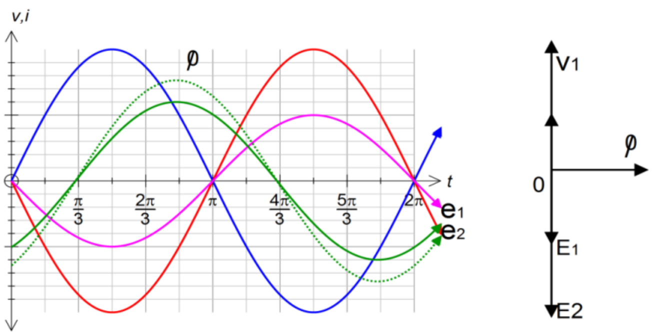
5.4. Elementary Theory of an Ideal Transformer

An ideal transformer is one which has no losses i.e. whose windings have no ohmic resistance, so there is no RI^2 loss and no core loss and in which there is no magnetic leakage.(for example superconductive transformers). In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core. It may, however, be noted that it is impossible to realize such a transformer in practice, yet for convenience, we will start with such a transformer and step by step approach an actual transformer.

Consider an ideal transformer (Fig.5.1) who's secondary is open and whose primary is connected to a sinusoidal alternating voltage v_1. This potential difference causes the flow of an alternating current in primary winding. Since primary winding is purely inductive and there is no output (secondary is open), the primary draws the magnetizing current I_μ only. The function of this current is merely to magnetic the core, it is small in magnitude and lags v_1 by 90^o. This a.c. I_μ produces an alternating flux Φ which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and hence is in phase with it. This changing flux is linked both with the primary and secondary windings. Therefore it produces self-induced e.m.f. in the primary. This self-induced e.m.f. E_1 is at every time, equal to and in opposition to v_1. It is also known as counter e.m.f. or back e.m.f. of the primary.

Similarly, there is produced in secondary an induced e.m.f. E_2 which is known as mutually-induced e.m.f. This e.m.f. is anti-phase with v_1 and its magnitude is proportional to the rate of change of flux and the number of secondary turns.

The instantaneous values of applied voltage, induced e.m.f's, flux and magnetizing current are shown by sinusoidal waves in Fig.5.2,a. Fig.5.2,b shows the vector representation of the effective values of the above quantities.



(a) (b)

Fig.5.2

5.5. E.M.F. Equations of a Transformer

Let N_1 is a number of turns in primary, N_2- the number of turns in secondary, Φ_m- maximum flux in the core (in Webbers) and is equal to $(B_m A)$, f - frequency of an a.c. input (in Hz).

As shown in Fig.5.3 the core flux increases from its zero value to maximum value Φ_m in one quarter of the cycle i.e. $\frac{1}{4}T = \frac{1}{4f}$ second. Average value of flux is $\frac{\Phi_m}{\frac{1}{4f}} = 4f\Phi_m$.

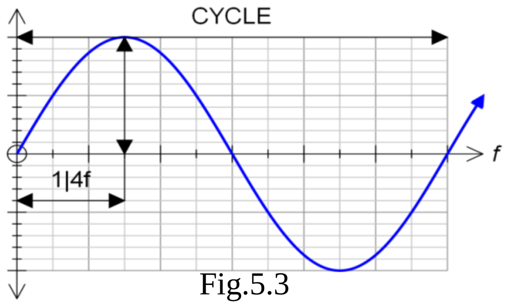


Fig.5.3

Now, rate of change of flux per turn means induced e.m.f. in volts.

Average e.m.f. induced over turn is $4f\Phi_m$ volt. If flux Φ varies sinusoidally, then r.m.s. value of induced e.m.f. is obtained by multiplying the average value with form actor

$$K_f = \frac{r.m.s.value}{averagevalue} = \frac{E}{E_{av}} = \frac{E_m}{\sqrt{2}} : \frac{2E_m}{\pi} = \frac{\pi}{2\sqrt{2}} \approx 1.11$$

R.m.s. value of e.m.f./ turn is:

$$\frac{E_1}{N_1} = K_f 4f\Phi_m = 4.44 f\Phi_m \quad \text{volt}$$

Now, r.m.s. value of induced e.m.f. in the whole of primary winding

$$E_1 = \frac{E_1}{N_1} N_1 = 4.44 fN_1 B_m A$$

Similarly, r.m.s. value of e.m.f. induced in secondary is

$$E_2 = 4.44 fN_2 \Phi_m = 4.44 fN_2 B_m A$$

In an ideal transformer on no load

$$V_1 = E_1 \quad \text{and} \quad V_2 = E_2$$

where V_2 is the terminal voltage.

From the above equations we get

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

This constant K is known as voltage transformation ratio.

If $N_2 \succ N_1, i.e., K \succ 1$, then transformer is called step-up transformer.

If $N_2 \prec N_1, i.e., K \prec 1$, then transformer is known as step-down transformer.

Again for an ideal transformer:

Input VA = Output VA $(S_1 = S\ )$

$$V_1 I_1 = V_2 I_2 \quad \text{or}$$
$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

5.6. Transformer on Load

When the secondary is loaded, secondary current I_2 is set up. The magnitude and phase of I_2 with respect to V_2 are determined by the characteristics of the load. Current I_2 is in phase with V_2 if load is noninductive, it lags if load is inductive and it leads if load is capacitive.

The secondary current sets up its own m.m.f. (magnetomotive force) and hence its own flux Φ_2 which is in opposition to the main no load primary flux which is due to $I_0 \approx I_\mu$. The secondary ampere-turns $N_2 I_2$ are known as demagnetizing ampere turns. The opposing secondary flux Φ_2 weakens the primary flux Φ_0 momentarily, hence primary back e.m.f. E_1 tends to be reduced. For a moment , V_1 gains the upper hand over E_1 and hence causes more current to flow in primary.

Let the additional primary current be $I_2^{'}$. It is known as load component of primary current. This current is antiphase with I_2. The additional primary m.m.f. $N_1 I_2^{'}$ sets up its own flux $\Phi_2^{'}$ which is in opposition to Φ_2 (but has same direction as Φ_0) and is equal to it in magnitude. Hence, Φ_2 and $\Phi_2^{'}$ cancel each other out, leaving the flux Φ_0 in the core only. So we find that the magnetic effects of secondary current I_2 are immediately neutralized by the additional primary current $I_2^{'}$ which is brought into existence exactly at the same instant as I_2.

Hence, whatever the load condition, the net flux passing through the core is approximately the same as at no load.

Therefore, we can conclude, that under full-load conditions the ratio of primary and secondary currents is constant.

5.7. Transformer Construction

The simple elements of a transformer consists of two coils having mutual inductance and a laminated steel core. The two coils are insolated from each other and from the steel core. Other necessary parts are: some suitable container for the assembled core and windings, a suitable medium for insulating the core and its windings from its conteiner, suitable bushings (either of porcelain, oil-filled or capacitor-type) for insulating and bringing out the terminals of windings from the tank.

In all types of transformers, the core is constructed of transformer sheet steel laminations assembled to provide a continuous magnetic path with minimum of air gap included. The steel used is of high silicon content, sometimes heat treated to produce a high permeability and a low hysteresis loss at the usual operating flux densities. The eddy current loss is minimised by laminating the core, the laminations being insulated from each other by a light coat of core-plate vanish or by an oxide layer on the surfface. The thickness of laminations varies from 0.35 mm for a frequency of 50 Hz to 0.5 mm for a frequency of 25 Hz.

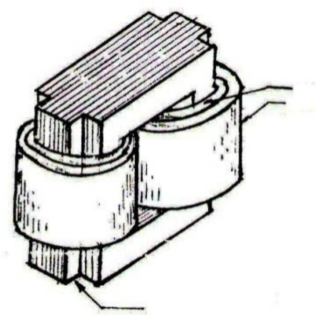


Fig.5.4
Core-type transformer

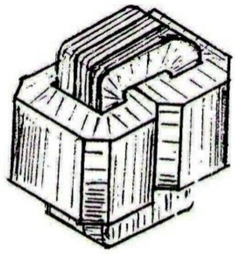


Fig.5.5
Shell-type transformer

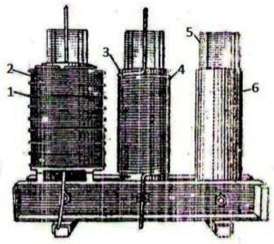
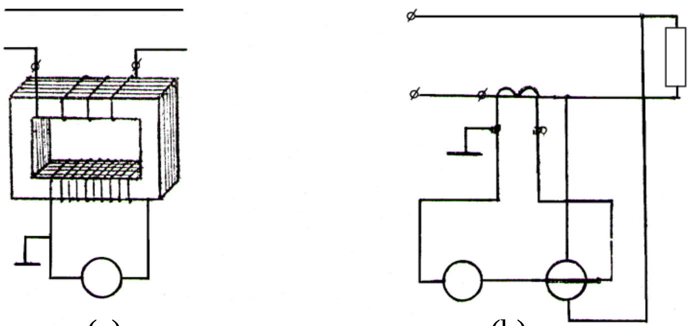


Fig.5.6
Three-phase power transformer

5.8. Measuring (Instrument) Transformers

There are two types of measuring transformers: 1. voltage or shunt-type transformer and 2. current or series transformer. Usually they are used to extend the range of a.c. instruments (i.e. voltmeters, ammeters, wattmeters, etc.) line multipliers and shunts in d.c. circuits.

1. Voltage (sunt-type) transformer. It is used in the high-voltage circuits for connection of parallel windings of voltmeters, wattmeters, energy-meters etc. The construction of the voltage transformer (Fig.5.7.,a,b) is similar to that of the low-power transformer. The primary winding consisting of many turns is connected across the high voltage supply in parallel. Secondary winding having small numbers of turns is joined to the parallel winding of the instrument.



(a) (b)

Fig.5.7

The primary voltage of the voltage transformer is equal to the nominal voltage of the supply, and secondary voltage usually equals to 100 v.

The ratio of nominal voltages of the primary and secondary windings is called the transformation ratio of the voltage transformer

$$K_{V.T} = \frac{V_{1N}}{V_{2N}} = \frac{N_1}{N_2}$$

2. Current (series) transformer. It is used in the high voltage devices for connecting of current (series) coils of ammeters, wattmeters, energy-meters etc. (Fig.5.8).

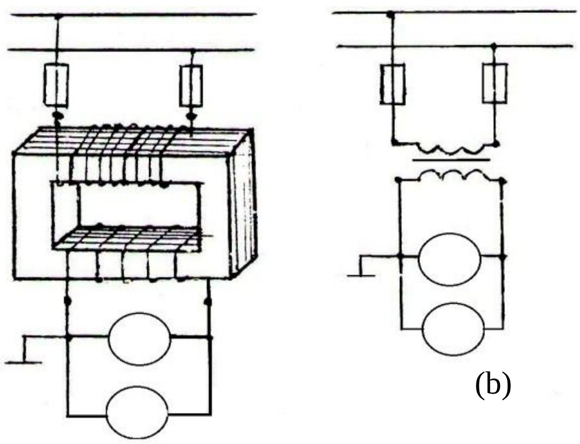
The primary of the current transformer is joined to measuring circuits in series. Terminals of the secondary winding are connected with current coils in series so that through them flows the same current.

The number of turns of the secondary winding of the current transformer usually is much more than that of primary.

The ratio of the current transformer is given as

$$K_{C.T} = \frac{I_{1N}}{I_{2N}},$$

where I_{1N} is a nominal value of the primary current, and I_{2N} is a nominal value of the secondary current.



(a) (b)

Fig.5.8

It must be noted that in current transformers the independent value is not the primary voltage (as in case of voltage transformers) but the current to be measured.

Usually the nominal value of the secondary current is equal to 5 A.

In order to receive the real quantity of the measured current the ammeter reading must be multiplied by the ratio $K_{C.T}$.

6. Induction Motor

6.1. General Principle

Of all the a.c. motors, the 3-phase induction motor is the one which is extensively used for various kinds of industrial drives. It has following main advantages as well as disadvantages.

Advantages

(1) It has very simple and extremly rugged, almost unbreakable construction.
(2) It cost is low and is very reliable.
(3) It has sufficiently high efficiency and a reasonable good power factor.
(4) It requires minimum of maintenance.

(5) It starts up from rest and needs no extra starting motor and has not to be synchronized. Its starting arrangement is simple, especially for squirrel-cage rotor type motor.

Disadvantages

(1) Its speed can’t be varied without sacrificing some of its efficiency.
(2) Gust like direct current shunt motor its speed decreases somewhat with increase in load.
(3) Its starting torque is somewhat inferior to that of a d.c. shunt motor.

6.2. Construction

An induction motor consists of two main parts: (a) a stator and (b) a rotor.

a) **The stator of an induction motor** (Fig.6.1) is in principle made up of a number of stampings which are slotted to receive the windings. The stator Carrie, a 3-phase winding and is fed from a 3-phase supply. It is wound for a definite number of poles, the exact number of poles being determined by the requirements of speed. Greater the number of poles, lesser the speed and vice versa.

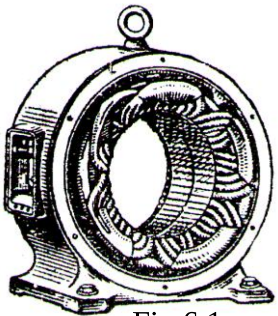


Fig.6.1

Squirrel-cage rotor (Fig.6.2). Almost 90% of induction motors are squirrel-cage type, because this type of rotor has the simplest and most rugged construction imaginable and is almost indestructible. The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors, which, it should be noted clearly, are not wires but consists of heavy bars of copper or aluminum. One bar is placed in each slot, rather the bars are inserted from the end when semi closed slots are used. The rotor bars are brazed or electrically welded to two heavy and stout short-circuiting end-rings, thus giving, what picturesquely called a squirrel-cage construction.

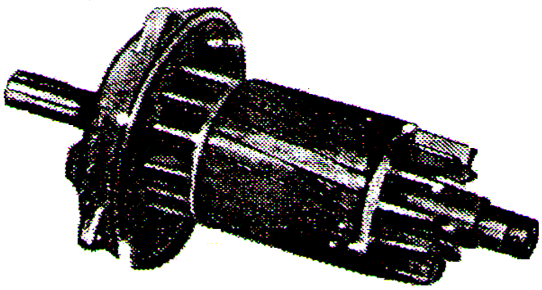


Fig.6.2

It should be noted that the rotor bars are permanently short circuited on themselves; hence it is not possible to add any external resistance in series with the rotor for starting purposes.

Phase Wound Rotor (Fig.6.3)

This type of rotor is provided with 3-phase, double-layer, distributed winding consisting of coils as used in alternators. The rotor is wound for as many poles as the number of stator poles and is always wound 3-phase.

The three phases are starred internally. The other three winding terminals are brought out and connected to three insulated slip-rings (Fig.5.3) mounted on the shaft with brushes resting on them. These three brushes are further externally connected to a 3-phase star-connected rheostat. This makes possible the introduction of additional resistance in the rotor circuit during the starting period for increasing the starting torque of the motor and for changing its speed/torque/current characteristics. When running under normal conditions, the slip-rings are automatically short-circuited by means of a metal collar

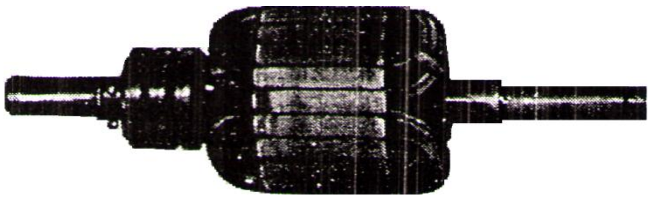


Fig.6.3

which is pushed along the shaft and connects all the rings together. Next, the brushes are automatically lifted from the slip-rings to reduce the frictional losses and wear and tear. Hence, it is seen that under normal running conditions, the wound rotor is short-circuited on itself just like squirrel-cage rotor.

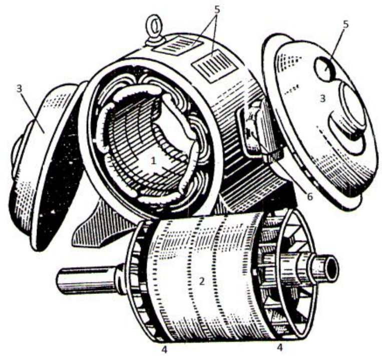


Fig.6.4

General view of a dismantled squirrel-cage induction motor is shown in Fig.6.4.

6.3. Production of a Rotating Field

It can be shown that when a 3-phase winding is energized by a 3-phase supply then the resultant flux produced rotates in space around the stator as though actual magnetic poles were being rotated.

The magnitude of the resultant flux is $1.5\Phi_m$, where Φ_m is the maximum value of the flux due to any phase. It rotates round the stator synchronously i.e. with synchronous speed $N_s = 120 f / p$, where p is the number of motor poles and f the frequency of the a.c. supply.

6.4. Principle of Operation

The reason why the rotor of an induction motor is set up into rotation is as follows: when the 3-phase stator windings are fed by 3-phase supply, then as said above a magnetic flux of constant magnitude but rotating at synchronous speed is set up. The flux passes through the air-gap, past the rotor surface and so cuts the rotor conductors which, as yet, are stationary. Due to the relative speed between the rotating flux and the stationary conductors, an e.m.f. is induced in the later, according to Faraday's Law of Electro-magnetic Induction. The frequency of the induced e.m.f. is the same as the supply frequency. Its magnetude is proportional to the relative velocity between the flux and the conductors and its direction given by Flemings Right-hand Rule. Since, The rotor bars or conductors form a closed circuit, rotor current is produced, whose direction, as iven by Lenz's Law, is such as to oppose the very cause producing it. In this case, the cause which produces the rotor current is the relative velocity between the rotating flux of the stator and stationary rotor conductors. Hence, to reduce the relative speed, the rotor starts running in the same direction as that of the flux and tries to catch up with the rotating flux.

6.5. Slip. Frequency of Rotor Current

In practice, the rotor never succeeds in "catching-up" with the stator flux. If it really did so, then there would be no relative speed between the two, hence no current, and no torque to maintain rotation. That is why the rotor runs at a speed which is always less then the speed of the stator field. The difference in speed depends upon the load on the motor.

The difference between the synchronous speed N_s and the actual speed N of the rotor is known as slip.

Actually, the term "slip" is descriptive of the way in which the rotor "slips back" from synchronism

$$\%\ slip\ \ s = \frac{N_s - N}{N_s} 100$$

Sometimes, $N_s - N$ is called the slip speed.

When the motor is stationary, the frequency of rotor current is the same as the supply frequency. But when the rotor starts revolving, then the frequency depends upon the relative speed or on slip-speed. Let at any slip-speed the frequency of the rotor current is f'. Then

$$N_s - N = \frac{120 f'}{p},$$

Also

$$N_s = \frac{120 f}{p}$$

Dividing one by other, we get

$$\frac{f'}{f} = \frac{N_s - N}{N_s} = s, \quad f' = sf .$$

6.6. Relation between Torque and Slip

It can be proved, that at normal speeds, close to synchronism the torque aproximatively will be

$$T \approx \frac{s}{R_r},$$

where R_r - rotor resistance/phase.

or if $R_r = const$, then

$T \equiv s$

Hence, for low values of slip, the torque/slip curve is approximately a straight line. As slip increases (for increasing load of the motor) the torque also increases and becomes maximum when $s = \frac{R_r}{X_r}$ (X_r - rotor reactance/phase). This torque is known as “pull out” or “break-down” torque.

As the slip further increases (i.e. motor speed falls), then

$$T \equiv \frac{s}{(sX_r)^2} \equiv \frac{1}{s}$$

Hence, the torque/slip curve is a rectangular hyperbola. So we see that beyond the point of maximum torque, any further increase in motor load results in decrease of torque developed by the motor.

The result is that the motor slows down and eventually stops. The circuit – breakers will be tripped open if the circuit has been so protected. In fact, the stable operation of motor lies between the values of $s = 0$ and that corresponding to maximum torque. The operating range is shown shaded in Fig.6.5.

It is seen that although maximum torque does not depend on R_r yet the exact location of $T_{\max}$ is dependent on it. Greater the R_r greater is the value of slip at which maximum torque occurs.

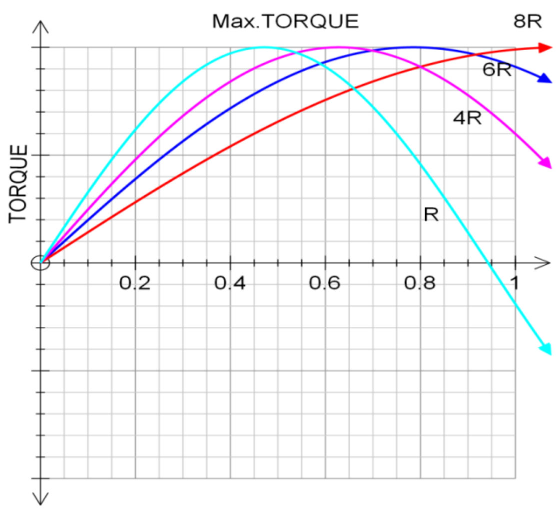


Fig.6.5

7. Alternators and Synchronous Motors

7.1 Basic Principle and Construction

A.C. generators or alternators (as they are usually called) operate on the same fundamental principle of electromagnetic induction as d.c. generators. They also consist of an armature winding and a magnetic field. But there is one important difference between the two. Whereas in d.c. generators, the armature rotates and the field system is stationary, the arrangement in alternators is just the reverse. In their case, standard construction consists of armature windings mounted on a stationary element called "stator" and field windings on a rotating element called "rotor". The details of construction are shown in Fig.7.1.

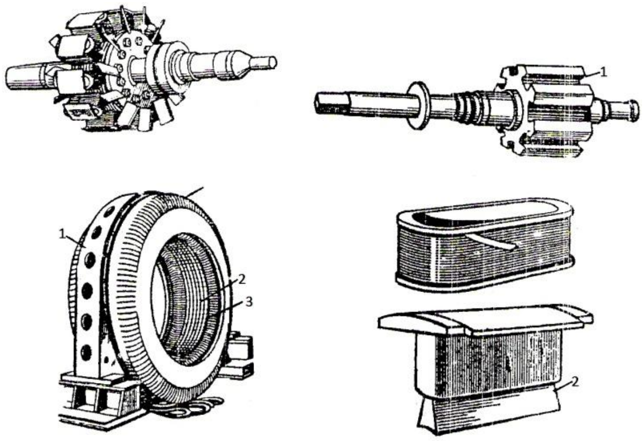


Fig7.1

(a) Stator

It consists of a cast-iron frame which supports the laminated armature core having slots on its periphery for housing the 3-phase winding.

(b) Rotor

There are two types of rotor:

(1) Salient (or projecting) pole type

It is like a flywheel which has a large number of alternate North and South poles bolted to it. The magnetic wheel is made of cast iron or steel of good magnetic quality. The magnetic poles are excited by a small d.c. generator mounted on the shaft of alternator itself.

Such rotors are used in low and medium-speed alternators which are characterised by their large diameters and short axial lengths.

(2) Smooth Cylindrical Type

It consists of a smooth solid forged-steel cylinder having a number of lots milled out at intervals along the outer periphery for accommodating field coils. Two or four regions corresponding to the control polar areas are surrounded by the field windings placed in slots. Obviously, in this case, the poles are non-salient i.e. they do not project out from the surface of the rotor.

Such rotors are used in steam-turbine driven alternators i.e. turbo-alternators or turbo-generators which run at very high speed and are characterized by their small diameters and very long axial lengths.

7.2. Principle of Operation

When the rotor is rotated by the prime-motor, the stator winding or conductors are cut by the magnetic flux of the rotor poles. Hence, an e.m.f. is induced in the rotor conductors. The frequency of this induced e.m.f. is given by $f = \frac{PN}{120}$ (where P is the total number of the magnetic poles, N is the rotative speed of the rotor in r.p.m., f is the frequency of generated e.m.f. in Hz.

N is known as the synchronous speed, because it is the speed at which an alternator must run in order to generate an e.m.f of the required frequency.

In fact, for a given frequency and given number of poles, the alternator will have to run at the following speeds.

Number of Poles	2	4	6	12	24	30
Speed (r.p.m.)	3000	1500	1000	500	250	200

Referring to the above equation we get $$P = \frac{120f}{N}$$

It is clear from the above equation, that because of slow rotative speeds of engine-driven alternators, their number of poles is much greater as compared to that of the turbo-generators which run at very high speed.

It can be found that R.m.s. value of e.m.f./phase will be

$$E = 4.44\, f\Phi T$$

where f is the frequency of induced e.m.f. in Hz., Φ - flux/pole in webers, T is the number of coils or turns per phase.

If the alternator is star connected ,then the line voltage is $\sqrt{3}$ times the phase voltage (as found from the above formula).

7.3. Synchronous Motor. Construction and Application.

A synchronous motor is electrically identical with an alternator. A synchronous machine may be used, at least theoretically, as an alternator when driven mechanically or as a motor when driven electrically, just as in case of d.c. machines.

Essential mechanical elements of such a motor are the same as those of an alternator. The rotor poles are excited by a small d.c. shunt generator mounted on the rotor shaft.

Synchronous motors are rarely used in practice. The synchronous motors are applicable in the following cases:

1. In view of their high efficiency, these motors can be advantageously employed for loads where constant speeds are desirable.
2. Since these motors have high efficiency and can be built in low speeds, they are well suited for direct connection to reciprocating compressors.

3. Over-excited synchronous motors are most commonly used for power factor improvement of lagging industrial loads. When employed in such a role, they are referred to as synchronous capacitors.

4. They are generally used for improving voltage regulation of the long transmission lines.

7.4. Comparison Between Synchronous and Induction Motors

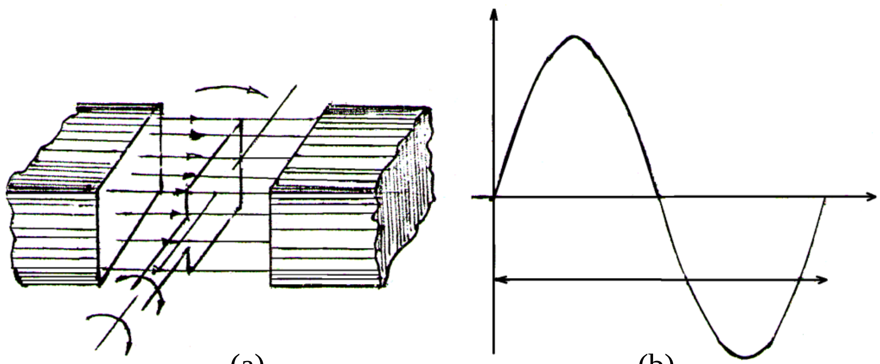
1. Whereas synchronous motor runs only at synchronous speed, an induction motor never runs with synchronous speed.
2. Synchronous motor can be operated under a wide range of power factors both lagging and leading. But induction motors always run with lagging power factor.
3. A synchronous motor is not inherently self-starting whereas induction motor is.
4. The torque of a synchronous motor is much less affected by changes in applied voltage than that of an induction motor.
5. A d.c. excitation is required by a synchronous motor but not by an induction motor.
6. Synchronous motors are usually more costly and complicated than induction motors.

8. Direct Current (D.C) Generators

8.1. Generator. Principle. Simple loop Generator

An electrical generator is a machine which converts mechanical energy or power into electrical energy or power.

This energy conversion is based on the principle of production of the induced e.m.f. As is known, whenever a conductor cuts magnetic flux induced e.m.f. is produced in it according to Faraday's Law of Electromagnetic Induction. This alternating e.m.f will cause an alternating current to flow if conductor circuit is closed. But it (a.c.) is, obviously, different from a direct current (d.c.), which continuously flows in one and the same direction. It should be noted that a.c. not only reverses its direction, it does not even keep its magnitude constant while flowing in any one direction. The two half-cycles may be called positive and negative half-cycles respectively (Fig.8.1.b).



(a) (b)

Fig.8.1.

For making the flow of current unidirectional in the external circuit, the slip-rings are replaced by slip-ring (Fig8.2.) which are made out of a conducting cylinder. The cylinder is cut into two halves or segments insulated from each other by a thin sheet of mica or some other insulating material.

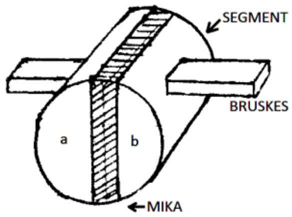
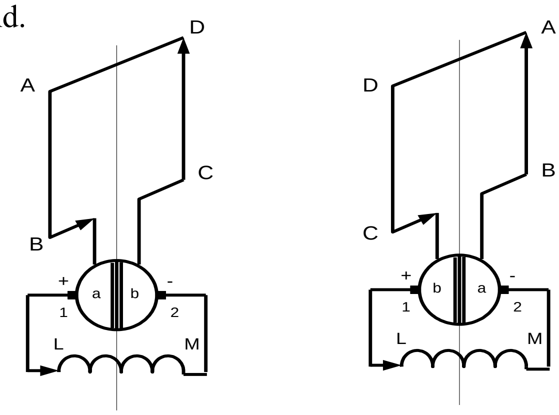


Fig.8.2.

As before, the coil ends are joined to these segments on which rest carbon brushes. It is seen (Fig.8.8.a) that in first half revolution, current flows along ABLMCD i.e. brush number 1, which is in contact with segment "a", acts as positive end of the supply and brush number 2 and "b" as the negative end.



(a) (b)

Fig.8.3.

In the next half revolution (Fig.8.3.b) the direction of the induced current in the coil is reversed. But at the same time, the positions of segments "a" and "b" are also reversed with the result that that brush number 1 comes in touch with that segment which is positive, i.e. segment "b". Hence, the current in the load resistance again flows from L to M. The wave form of the current through the external circuit is as shown in Fig.8.4. This current is unidirectional but not continues like pure direct current.

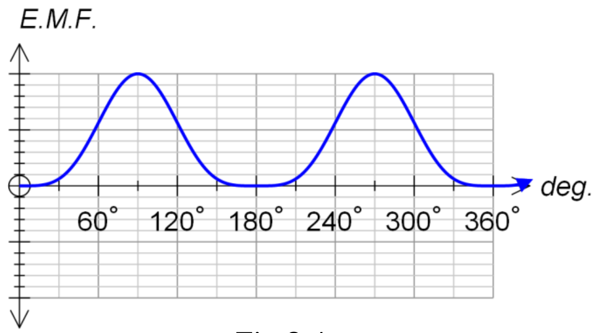


Fig.8.4

It should be noted that the position of brushes is so arranged that the change over of segments “a” and “b” from one brush to the other takes place when the plane of the rotating coil is at right angles to the plane of the lines of flux because in that position the induced e.m.f. in the coil is zero.

another important point to remember is that even now the current induced in the coil sides is alternating as before. It is only due to rectifying action of the split rings (also called commutator) that it becomes unidirectional in the external circuit. Hence, it should be clearly understood that even in the armature of a direct current generator the induced current is alternating.

8.2. Practical Generator

As was shown above the basic essential part of an electrical generator are: 1. magnets causing magnetic field and 2. a conductor or conductors which can move so as to cut the flux. But the construction of an actual generator is more complex and consists of the following main parts:

Yoke (Fig.8.5). The outer frame or yoke serves double purpose: 1. it produces mechanical support for the poles and acts as a protecting cover for the whole machine. 2. it carries the magnetic flux produced by poles.

In small generators where cheapness rather the weight is the main consideration, yokes are made of cast iron. But for large machines usually cast steel or rolled steel is employed.

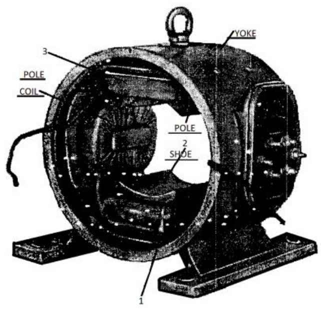


Fig.8.5.

Pole cores and Pole shoes. The fields magnets (Fig.8.5) consists of pole cores and pole shoes. The pole shoes serve two purposes: (1) the spreaad out the flux in the air gaps and also being of larger cross-section reduce the reluctance of the magnetic path and (2) they support the exciting coils.

Pole Coils. The field coils or pole coils which consists of copper wire or strip, are former wound for the correct dimension. Then the former is removed and the wound coil put into place over the core.

When current is passed through these coils, they electromagnetise the poles which produce the necessary flux that is cut by the revolving armature conductors.

Armarure core. It houses the armature conductors or coils and causes them to rotate and hence cut the magnetic flux of the field magnets (Fig.8.6 and Fig.8.7) . In addition to this, its imoirtannt

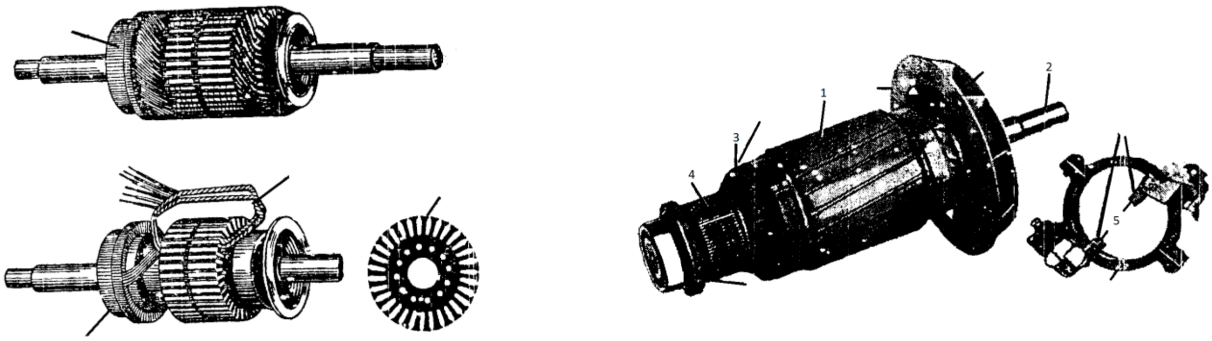


Fig.8.7

function is to provide a path of very low resistance to the flux passing through the armature from a N -pole to a S -pole.

It is cylindrical or drumshaped and is built up of usually circular sheet discs or laminations approximately 0.064 mm thick. (Fig.8.6,c)

The purpose of using laminations is to reduce the loss due to eddy currents. Thinner the laminations, greater is the resistance offered to the induced e.m.f., smaleer the current and hence less the RI^2 loss in the core.

Armature Windings (Fig.8.6.b). The armature windings are usually former wound. These are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Karions conductors of the coils are insulated from each other. The conductors are placed in the armature slots which are lined with tough insulating material. This slot insulators placed in the slot and secured in place by special hard wooden of fibre wedge.

Commutator. The function of the commutator is to facilitate the collection of current from armature conductors. As Shown in previous Art.8.1 it rectifies i.e. converts the alternating current induced in the armature into unidirectional current. It is of cylindrical structure (Fig.8.1) and is built up of wedge-shaped segments of high-conductivity hard-drown or drop-forged copper. These segments are insulated from each other by thin layers of mika.

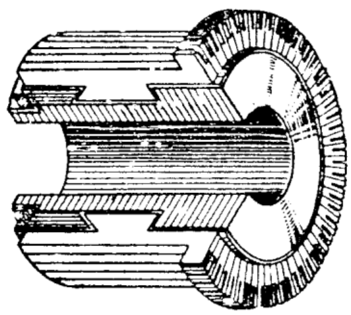


Fig.8.8.

Brushes and Bearings. The brushes (Fig.7.7), whose function is to collect current from commutator, are usually made of carbon and are in the shape of rectangular block. These brushes are housed in brush-holders. Becouse of their reliability, ball bearings are frequently employed through for heavy duties, roller bearings are preferable.

8.3. Types of Generators

Generators are usually classified according to the way in which their fields are excited. Generators may be divided into

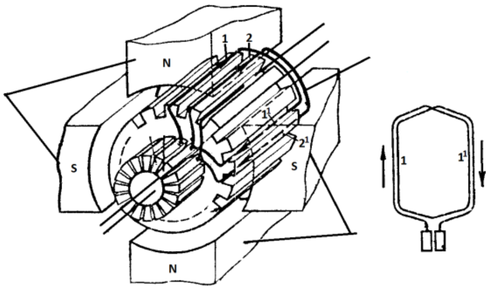


Fig.8.9

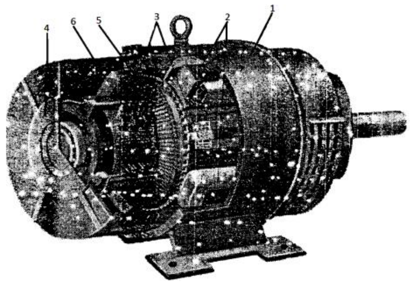


Fig.8.10

(a) **separately-excited and (b) self-excited generators**.

(a) **Separately-excited generators** are those whose field magnets are energised from independent external source of direct current. It is shown diagrammatically in Fig.8.11.

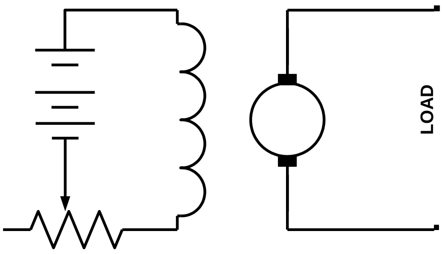


Fig.8.11

(a) **Self-excited generators** are those whose field-magnets are energised by the current produced by the generators themselves. Due to residual magnetism there is always present some flux in the poles. When the armature is rotated, some e.m.f. and hence some induced current is produced which is partly or fully passed through field coils thereby strengthening the residual pole flux further.

There are three types of self-excited generators named according to the manner in which their field coils (or windings) are connected to the armature.

1. **Shunt Wound**. The field windings are connected across or parallel with the armature conductors, and have full voltage of generator applied across them. (Fig.8.12)

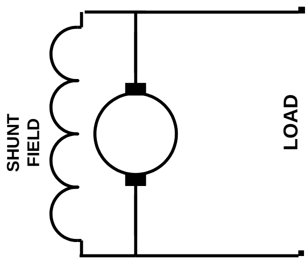


Fig.8.12

The field copil cosists of many turns of fine gauge. Such generators are in much common use.

2. **Series Wound**. In this case, the field windings are joined in series with the armature conductors(Fig.8.13). As they carry full load current,they consists of relatively few turns of thick wire or strip. Such generators are rarely used exept for special purposes.

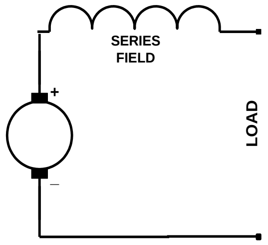


Fig.8.13

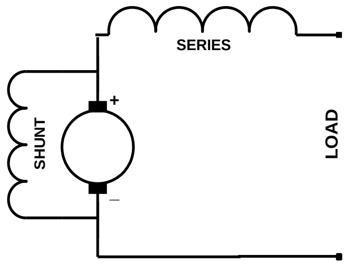


Fig8.14

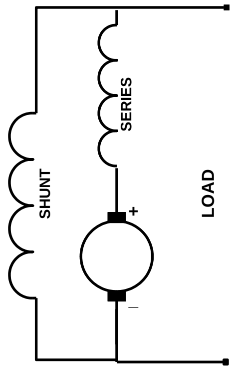


Fig.8.15

3. **Compound Wound**. It is a combination of a few series and a few shunt windings and can be either short-shunt or long-shunt as shown in Fig.8.14 and Fig.8.15 respectively.

8.4 Characteristics of D.C. Generators.

Following are the most important characteristics or curves of a D.C. Generator:

1. Open Circuit Characteristic (O.C.C)

It is also known as Magnetic Characteristic or No-load Saturation characteristic. It shows the relation between the no-load generated e.m.f. in the armature E_o and the field or exciting current I_f at a given fixed speed. In fact, it is just the magnetization curve for the material of the electromagnets. Its shape is practically the same for all generators whether separately or excited or self-excited (Fig.7.16).

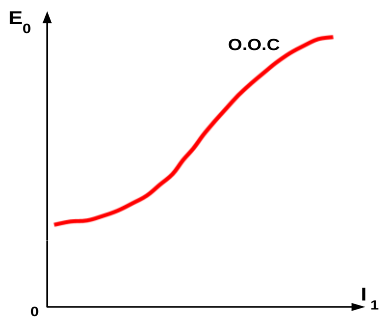


Fig.8.16.

1. **Internal or Total Characteristic $E(I_a)$.**

It gives the relation between e.m.f. E actually induced in the armature and the armature current I_a . This characteristic is of interest mainly to the designer.

2. **External Characteristic V(I).**

It also is referred to as a performance characteristic or sometimes voltage regulation curve. It gives relation between the terminal voltage V and the load current I. This curve lies below the internal characteristic because it takes into account the voltage drop over the armature circuit resistance. The values of V are obtained by subtracting $R_a I_a$ from corresponding values of E. This characteristic is of great importance in judging the suitability of a generator for a particular purpose. It may be obtained by simultaneous measurements with a suitable voltmeter and ammeter on a loaded generator.

8.5. External Characteristics of D.C. Generators

1. **Series generators .** Since in this generator the field windings are in series with the armature, hence they carry the full armature current I_a =I which is also the load current. Because of that a series generator is a variable voltage generator, i.e. its voltage increases with load current. Series generators are rarely used in practice except for same special purposes (Fig.8.17, curve 1).
2. **Shunt Generator.** On no-load the shunt generator gives its full open circuit voltage V=E_o. Then due to armature reaction, especially at higher loads, and ohmic voltage drop the characteristic drops down (Fig.8.17,curve 2). If load resistance is decreased, then armature current increases up to a certain value. After that any decrease in load resistance is not accompanied by any increase in load current as might be expected. Rather, The load current is decreased and the curve turns back as shown in Fig.7.17,curve 3.

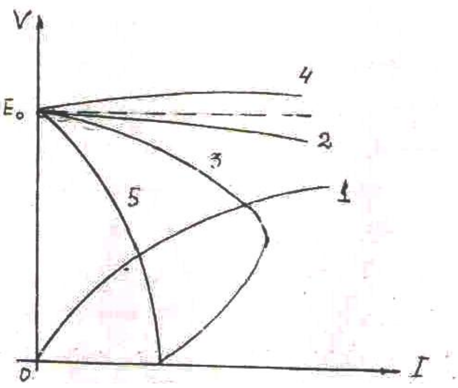


Fig.8.17

If the load resistance is too small,than the generator is short circuited and hence there is no generated e.m.f. due to heavy demagnetization of the main poles.

It will be seen from Fig.8.17 ,curve2, that a shunt generator gives its greater voltage at no load, the voltage V falling off as output current is increased. However, the fall in voltage from no load to full-load is small and the terminal potential differance (p.d) can always be maintained constant by adjusting the shunt field regulator.

3. **<u>Compound Generator</u>**. The form of external characteristic depends on the method of the field coils connection. If the series and shunt windings are so joined that their flaxes are additive i.e. in the same direction, then the magnetic flux increases with the load current. Hence, the e.m.f. increases as well.

9. Direct Current (d.c) Motors

9.1. Motor Principle

An electric motor is a machine, which converts electrical energy into mechanical energy. Its action is based on the principle that when a current-carrying conductor is placed in a magnetic field it experiences a mechanical force whose direction is given by left hand rule (it states that if the left hand is so placed in a magnetic field that lines of flux leaving N-pole enter the palm perpendicularly and the four fingers are in the direction of flow of the current in the conductor, then thumb gives the direction of the force, see Fig9.1)

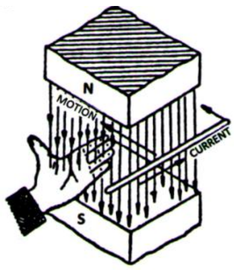


Fig.9.1

If l is the length of the conductor lying within the field and I ampere the current carried by it, then the magnitude of the force exerted on the conductor is

$$F = BlI$$

(where F-the force in newtons , B-magnetic flux density, in tesla, l- the length of the conductor,m). In general ,when the conductor lies at an angle θ with a field of flux density B then.

$$F = BIl\sin\theta \quad \text{Newton}$$

Constructional, there is no basic difference between a d,c. generator and a d. .c motor . In fact the same d. c. machine can be used interchangeably as a generator, or as a motor. When operating as a generator, it is driven by a mechanical machine and it develops voltage which in turn produces a current flow in an electric circuit. When operating as a motor ,it is supplied by electric current and it develops torque which in turn produces mechanical rotation . D. C. motors are also, like generators ,shunt-wound or series wound or compound wound.

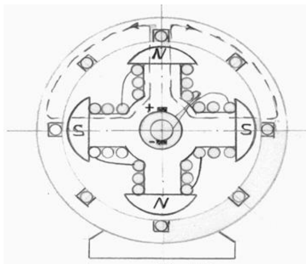


Fig.9.2

In Fig 9.2 is shown a part of a multipolar d.c motor. When its field magnets are excited and its armature conductors are supplied with current from the supply mains, they expiries a force tending to rotate the armature. Armature conductors under N-pole are assumed to carry current downwards (crosses) and those under S – poles to carry current upwards (dots).

By applying Fleming's left-hand rule, the direction of the force in each conductor can be found. It is shown

$$F = BIl\sin\theta \quad \text{Newton}$$

Constructional, there is no basic difference between a d.c. generator and d.c. motor. In fact the same d.c. machine can be used interchangeably as a generator or as a motor. When operating as a generator, it is driven by a mechanical machine and it develops voltage which in turn produces a current flow in an electric circuit. When operating as a motor, it is supplied by electric current and it develops torque which in turn produces mechanical rotation. D.C. motors are also like generators, shunt-wound or series wound or compound-wound.

In Fig.9.2 is shown a part of a multipolar d.c. Motor. When its field magnets are excited and its armature conductors are supplied with current from the supply mains, they expiries a force tending to rotate the armature. Armature conductors under N-pole are assumed to carry current downwards (crosses) and shoes under S-poles to carry current upwards (dots). By applying left hand rule, the direction of the force in each conductor can be found. It is shown by small arrows placed above each conductor. It will be seen that each conductor experiences a force F which tends to rotate the armature in only –clockwise direction. Those forces collectively produce a driving torque which sets the armature rotating. It should be noted that the function of the commutation in the motor is the same as in generator.

By reversing current in each conductor as it passes from one pole to another, it helps to develop a continuous and unidirectional torque.

9.2 The Back E.M.F

When the motor armature rotates, the conductors also rotate and fence cut the flux. In accordance with the law of electromagnetic induction, e.m.f. is induced in them whose direction , as found by Flemings right-hand rule is in opposition to the applied voltage (Fig.9.3).

Because of its opposing direction, it is Referred to as back e.m.f E . The Equivalent circuit of a motor In shown in Fig.8,3 b. The rotating armature generating the back e.m.f E_b Is like a battery of e.m.f E, a battery e.m.f. E_{bat} put across a supply mains of V volts. Obviously, V has to drive Ia against the opposition E_b. The power required to overcame this opposition is $E_b I_a$ watts.

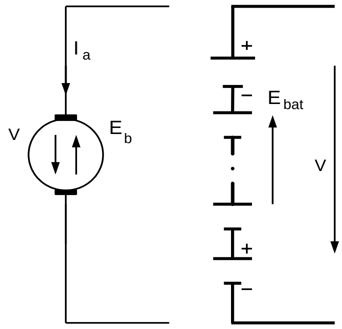


Fig.9.3

Back e.m.f depends, among other factors ,upon the armature speed. If speed is high, E_b is large, hence, armature current I_a is small. If the speed is less, then E_b is less, hence more current flows which develops more torque. So we find that E_b acts like a generator, i.e. it makes a motor. Sift- regulating so that it draws as much current as is just necessary.

9.3. Motor Characteristics

The characteristic curves of motor are those curves which show relation between the following quantities: (1) Torque and armature current, i.e $T=f_1(I_a)$ characteristic, (2) Speed and armature current, i. e $N=f_2(I_a)$ characteristic, (3) speed and torque i.e . $N=f_3(T)$ characteristic, which can be found from (1) and (2) above.

While discussing motor characteristics the following relations should always be kept in mind.

(a) $T= \Phi I_a$, (b) $I_a= (V-E_b)/R_a$ (c) $N= E_b/\Phi$

Mechanical characteristics of the shunt (1) series (2) and compound (3) motors, having considerable importance are shown in Fig.9.4.

Analysis of the above characteristics show that (a) speed of a shunt motor is sufficiently constant, (b) for the same current input, its starting torque is not as high as that of a series motor. Hence, it is used(1) the speed has to be maintained approximately constant from no-load to full-load. (2) When it is required to drive the load at various speeds any one spread being kept constant for a relatively long period. The shunt regulators enable the required speed control to be obtained easily and economically.

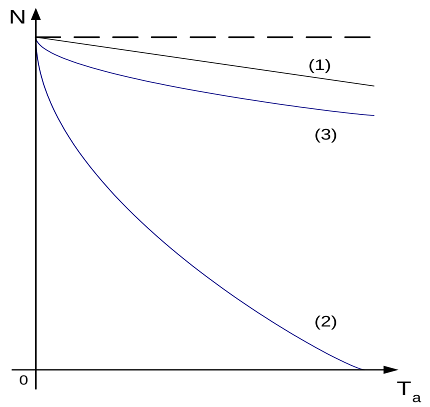


Fig.9.4

Therefore the shunt motors are applicated for driving a constant-speed line –shifting, lathes, centrifugal pumps, machine tools ,blowers, and fans reciprocating fans etc.

The series motors have (1) a relatively huge starting torque, (b) low speed at high load and dangerously high speed at low loads.

Hence, such motors are used:(a) when a large starting torque is required, (b) when the motor can be directly coupled to a load such as fan whose torque increases with speed, (c) it constancy of speed is not essential, then, in fact, the decrease of speed with increase of load, has advantage that the power absorbed by the motor does not increase as rapid by a torque. For instance, when torque is doubled the power approximately increases by 50 to 60% only,(d) a series motor should be not used where there is possibility of the load decreasing to a very small value.

Thus it should not be used for driving centrifugal pumps or for belt drive of any kind.

Because of that the series motors can be used for traction work, i,e electrical locomotives, rapid transit systems, trolley cars etc, cranes shoats, conveyors.

Since in the compound motors series excitations helps shunt excitation, there mechanical characteristics lie in between those of shunt and series motors Fig.9.4.

Such machines are used where series characteristics are required and where, in addition, the load is likely to be removed totally such as in some types of coal cutting machines or for driving heavy machine tools which have to tune sudden deep cut quite often. Due to shunt winding, speed will not became excessively high but due to series windings it will be able to tune heavy load. On conjunction with fly-wheel supplies (functioning as a load equalizer) it is employed where there are sudden temporary loads in rolling mills. The fly –wheel supplies its stored kinetic energy when motor slow down due to sudden heavy load. And when removal of a load, motor speeds up it gathers up its kinetic energy.

9.4. Speed Control of D.C. Motors

It can be shown that the speed of a motor is given by the relation

$$N = K\frac{V - R_a I_a}{\phi} r.p.m$$

Where R_a –armature circuit resistance (k= const). It is obvious that the speed can be controlled by varying (a) flux per pole ϕ (flux control) and (b) resistance of armature circuit R_a (rheostat control).

These methods as applied to shunt and series motors will be discussed below.

9.5 Speed Control of shunt Motors

a. Variation of flux or flux control method.

As seen from above ,by decreasing the flux, the speed can be increased and vice visa. The flux of a d.c. motor can be changed by changing I_{sh} with the help of a shunt field rheostat (Fig 9.5).

b . Armature or rheostatie control method.

This method is used when speeds below the no- load speed are required. As the supply voltage is normally constant, the voltage across the armature is varied by inserting a variable rheostat or resistance (called controller resistance) in series with the armature circuit as shown in Fig 9.6 . Aa. controller resistances increased, p.d. across the armature is decreased , thereby decreasing the armature speed.

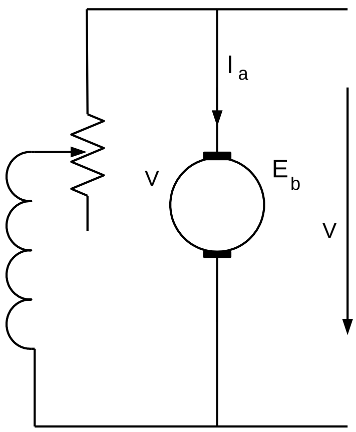


Fig.9.5

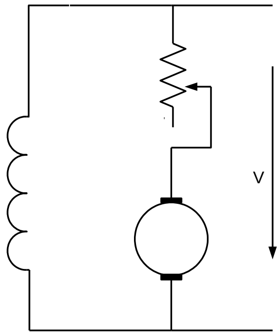


Fig.9.6.

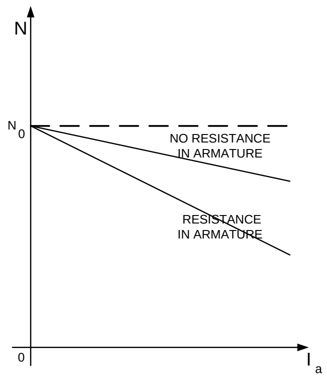


Fig.9.7

9.6. Speed Control of series Motor

a. <u>Flux control method</u> . Variations in the flux of series motor can be brought about in any one of the following ways.

(1**) <u>Field divertors</u>** . The series windings are shunted by a variable resistance known as field diverter (Fig 9.8) any desired amount of current can be passed trough the diverter by adjusting its resistance. Hence , the flux can be decreased and consequently the speed of the Motor increased .

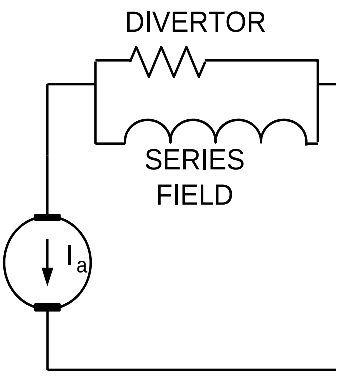


Fig.9.8

(2)**<u>Armature diverter</u>** . a diverter across The armature can be used for giving speed lower than the normal speed(Fig 9.9)

For a given constant load torque, if I_a is reduced due to armature diverter, then ϕ must increase. This results in an increase in current taken from the supply (which increases the flux) and a fall in speed. The variations in speed can be controlled by varying the diverter resistance.

3. <u>Tapped field control</u>. This method is often used in electric traction and shown in Fig.9.10. The number of series field turns in the circuit can be changed at will as shown. With full field, ,the motor runs at its minimum speed which can be raised in steps by cutting out some of series turns.

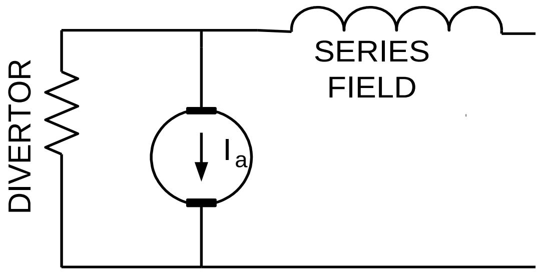


Fig 9.9

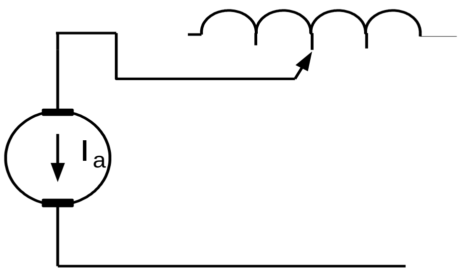


Fig 9.10

4.<u>Parallering field coils</u>. In this method, used for fan motor, several speed can be obtained by regrouping the field coil as shown in Fig 9.11.

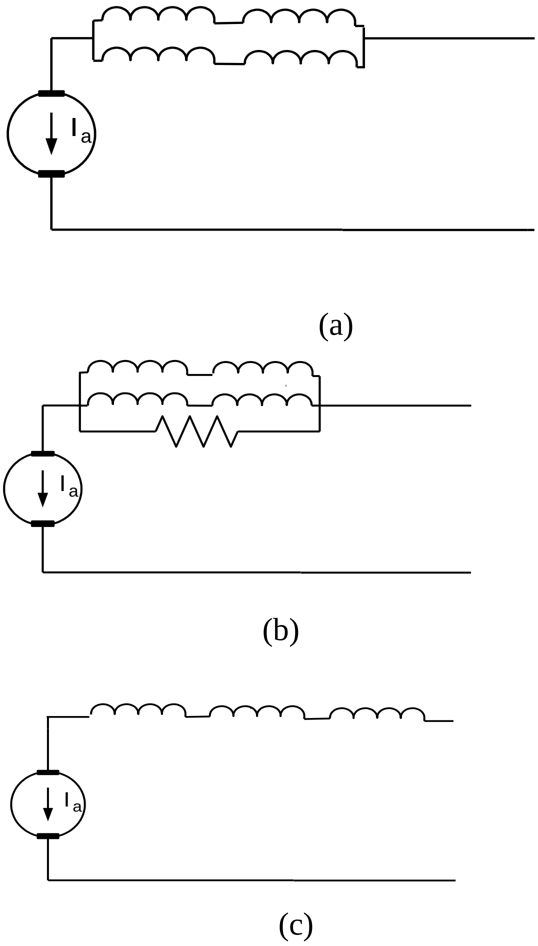


Fig 9.11

It is seen that for a 4 –pole motor three fixed speed can be obtained.

b<u>. Variable resistance in series witch motor armature.</u> By increasing the resistance in series with the armature (Fig 9.12) the Voltage applied across the armature terminal can be decreased.

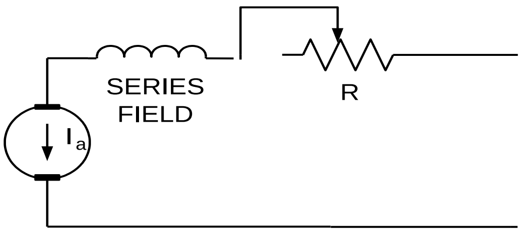


Fig.9.12

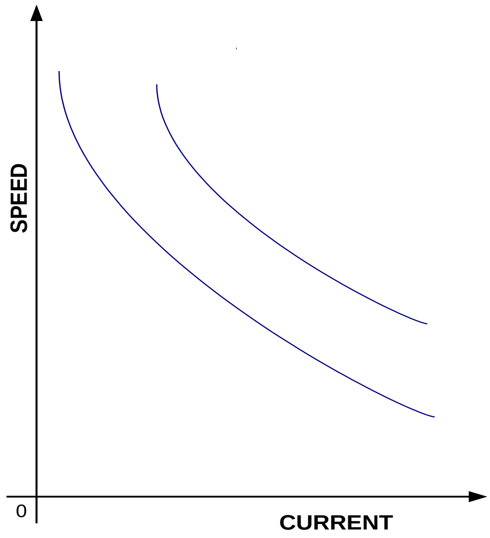


Fig.9.13

Motors convert electromagnetic energy into energy of motion or kinetic energy. Michael Faraday was the first person to create a device that used an electromagnet with permanent magnets to apply or create a directed force. The motor principle may be stated follows: when a current carrying conductor is located in an external magnetic field perpendicular to the conductor, the conductor experiences a force that is both perpendicular to both itself and the external magnetic field. In class a demonstration set-up can constructed to show this principle. A current carrying wire is suspended in a magnetic field and when the circuit is closed the wire moved out of the field lines of the magnet. If initially the wire moved into the horse magnet, when the current was reversed the wire moved out of the horseshoe magnet. See as viewed below.

10. Electrical Measurements

10.1. General

In most Colleges and Universities a course of lectures on Electrical Engineering is accompanied by a program of laboratory work designed to enable the student to apply a theoretical knowledge of electrical engineering to practical situations.

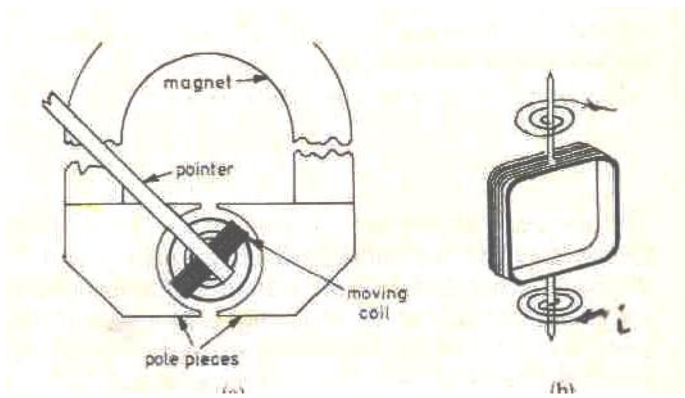
The student will encounter in the laboratory a range of voltmeters, both analogue and digital, as well as oscilloscopes, bridges and miscellaneous other instruments. It is important when making a measurement that one should know

- How the instrument disturbs the conditions in the circuit on, which the measurement is being made.
- What parameter is actually being measured or indicated, for example is it the mean value, the root mean square value or the peak value of a current or voltage?
- What effects the waveform an frequency of the current or voltage being measured have on the instrument indication.
- So that the student can be aware of the limitations of some of the basic measuring instruments likely to be encountered, a brief review of the principles of operation and characteristics of some instruments follow. It is not intended to be an introductory course in Electrical Measurements, but it is hoped that it will encourage the student to ask the questions "What am I measuring?" and "Am I using the right instrument?"

10.2. The Moving Coil Instrument

The Moving Coil Instrument measures direct current. It is used, in conjunction with rectifiers for the measurement of alternating current and voltage, as the basis for most multi-range deflection instruments.

The essential features of a moving coil instrument are shown in Fig.10.1.



(a) (b)

Fig.10.1.

A strong U-shaped permanent magnet has soft-iron pole pieces of the shape shown in Fig.10.1a, so that a soft-iron cylindrical core fits into the space between them. This form of constructions results in an almost uniform magnetic field in the gap between the pole pieces and the core.

The moving coil, mounted on a light aluminum frame, moves in the gap, the current being led in and out of the coil by two hair springs as shown in Fig.9.1b which also provide the controlling torque. Damping is provided by eddy currents induced in the aluminum former on which the coil is wound.

The deflecting torque on the coil is given by

$$\text{Torque} = (B\ I\ A\ N) \tag{10.1}$$

Where B= the flux density in the gap, I= the current flowing in the moving coil, A= the effective area of the moving coil, N= the number of turns on the moving coil.

The controlling torque provided by the springs is proportional to the angular deflection θ, so that when the moving coil system is at rest

$$\Theta\ \alpha\ (BAN)I \tag{10.2}$$

From which it may be seen that the deflection is proportional to the current, giving a linear scale calibration.

10.2.1 The Moving Coil Instrument as a Voltmeter

When the instrument is used as a voltmeter it is connected in series with a resistor R_{se} as shown in figure 10.2a.

The value of the resistor depends on the current sensitivity of the instrument and the voltage range required. For example if the meter gives full scale deflection when passing a current of I_m amperes a total resistance (including the meter resistance) of V_1/I_m ohms will be required to produce full scale deflection for V_1 volts, that is $R_{se} = (V_1/I_m - R_m)$ ohms, where R_m is the resistance of the moving coil. The voltage sensitivity of the meter is normally expressed as $1/I_m$ ohms per volt; thus a 50 µA instrument has a voltage sensitivity of 20000 Ω/V and the value of $(R_{se} + R_m)$ for a full scale deflection of 100 V would be 2MΩ.

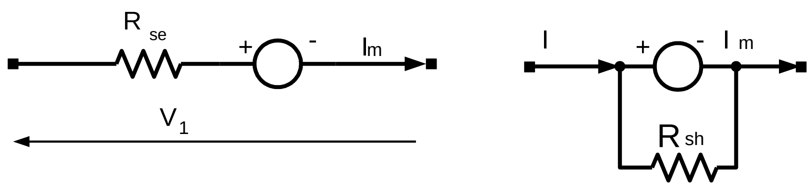


Fig.10.2

The shunting effect, or loading, on the circuit being measured must be considered when selecting an instrument for any particular measurement.

10.2.2 The Moving Coil Instrument as an Ammeter

When a moving-coil instrument is used as an ammeter it is usually necessary to connect a resistor in parallel with the meter as shown in figure 9.2b. If the meter gives full scale deflection for a current I_m amperes and the meter resistance is R_m ohms, the value of the shunt resistor R_{sh} , in parallel with the meter, to produce full scale deflection for a current of I amperes is

$$R_{sh} = \frac{R_m I_m}{I - I_m} \quad \text{Ohms} \qquad (10.3)$$

The resistance of the shunted instrument is

$$\frac{R_{sh} R_m}{R_{sh} + R_m} \text{ Ohms} \qquad (10.4)$$

And, before making a measurement, the effect of this resistance on the circuit conditions should be checked.

10.2.3 The Measurement of Alternating Current and Voltage Using a Moving Coil Instrument

A moving-coil meter may be used in conjunction with a rectifier to measure alternating voltages and currents at power and audio frequencies. An arrangement including a full-wave rectifier is shown in figure 10.3.

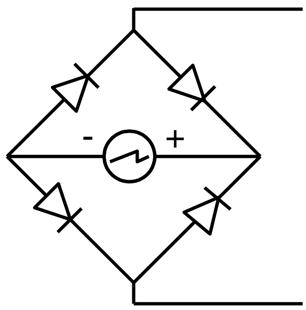


Fig.10.3

The instrument will measure the mean or average value of the rectified current flowing through it. It is the usual practice for rectifier instruments designed for use with sinusoidal waveforms to have the scale calibrated in terms of the r.m.s. value.

It is important to realize that the application of nonsinusoidal waveforms will lead to erroneous results.

10.3. The Moving-Iron Instrument

The Moving-Iron Instrument measures both direct and alternating current. In spite of this advantage it is not as widely used for light-current work as the moving-coil instrument since the latter is capable of greater sensitivity and higher accuracy.

There are several types of moving-iron instrument, but the principles of operation are demonstrated by referring to the repulsion type of instrument shown in figure 10.4. The current to be measured passes through the coil, along the axis of this coil is the shaft, pivoted at its ends, and carrying a small iron plate.

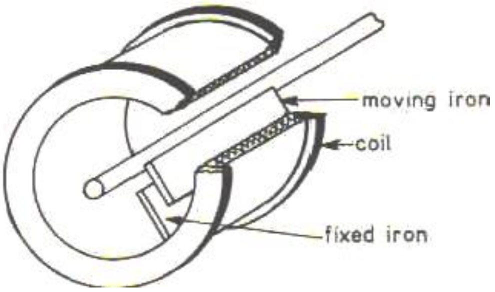


Fig.10.4

A second plate is fixed inside the coil adjacent to the moving iron plate. When a current flows in the coil the iron plates are magnetized in the same direction, there is a force of repulsion between the plates and the shaft carrying the moving iron plate and the pointer rotates.

The controlling torque is provided by a spring and damping is by means of an air chamber damper since the moving-iron instrument, unlike the moving-coil instrument, has no in-built damping.

The deflecting torque is given by

$$Torque = \frac{1}{2} I^2 \frac{dL}{d\theta} \tag{10.5}$$

Where ($dL/d\theta$) is the rate of change of inductance with angular deflection of the moving iron plate. Hence the deflection $\theta \; \alpha \; I^2 (dL/d\theta)$.

The instantaneous torque is proportional to the square of the instantaneous current, so that the average torque is a function of the mean square current and the instrument can be calibrated in terms of the r.m.s. current. In theory the meter will indicate the r.m.s. current regardless of its waveform; in practice however there may be some waveform errors, but these will not be considered here.

To obtain high sensitivity a large number of turns is required on the coil; this implies an appreciable coil resistance resulting in a large voltage drop across the instrument.

10.4 The Electrodynamic Instrument

Wattmeter's for use at power frequencies are usually of the electro-dynamic air-cored type in which a moving coil is located in the magnetic field of two series-connected fixed coils. The arrangement is shown in figure 10.5.

The expression for the torque is

$$Torque = I_1 I_2 \frac{dM}{d\theta} \tag{10.6}$$

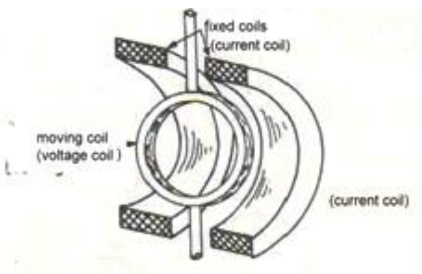


Fig.10.5

Where I_1 and I_2 are the currents in the fixed and moving coils respectively and (dM/dθ) is the rate of change of mutual inductance between the fixed and moving coils with angular deflection of the moving coil.

If the currents are

$$i_1 = I_{m1} \sin \omega t \tag{10.7}$$

$$i_2 = I_{m2} \sin(\omega t - \phi) \tag{10.8}$$

Then the instantaneous torque is

$$I_{m1} I_{m2} \sin \omega t \sin(\omega t - \phi) \frac{dM}{d\theta} \tag{10.9}$$

and the average torque is

$$I_{m1} I_{m2} \cos \phi \frac{dM}{d\theta} \tag{10.10}$$

Since the controlling torque is usually provided by a spring, when the moving coil system is at rest, the deflection is

$$\theta \alpha I_1 I_2 \cos \phi \frac{dM}{d\theta} \tag{10.11}$$

Where I_1 and I_2 are the r.m.s. values of the currents.

When used to measure power, the current in the fixed coils is the load current, or a known fraction of it if a current transformer is used to change the range of the instrument. The current in the moving coil is proportional to the voltage across the load, hence the deflection is proportional to $I_L V_L \cos\phi = P$ where I_L and V_L are the r.m.s. values of the load current and load voltage respectively and P is the power dissipated in the load.

A high-value non-inductive resistance is connected in series with the moving or voltage coil so that the resistance of the voltage coil circuit is very much greater than its reactance, thus ensuring that the current in the moving coil is very nearly in phase with the voltage across the load. The high resistance also results in a very small current being taken by the voltage coil circuit; the significance of this is apparent in the next paragraph.

There are two ways of connecting a wattmeter to a measure power; these are shown in figure 10.6.

In the connection of Fig.10.6a the current in the current coil is not the load current, but is the sum of the load current and current in the voltage coil; in the connection of Fig.10.6b the current in the current coil is the load current but the voltage across the voltage coil circuit is the sum of the load voltage and the voltage drop across the current coil.

In the connection of Fig.10.6a the current in the current coil is not the load current, but is the sum of the load current and current in the voltage coil; in the connection of Fig.10.6b the current in the current coil is the load current but the voltage across the voltage coil circuit is the sum of the load voltage and the voltage drop across the current coil.

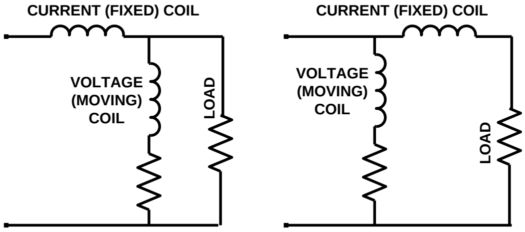


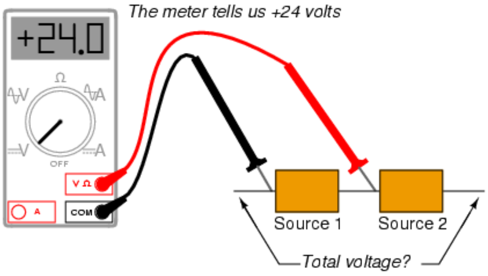
Fig.10.6

Both methods of connection therefore result in an error. If the load current is large, the connection shown in figure 10.6a is preferable and if the load current is small the connection shown in figure 10.6b results in a smaller error.

10.5 Digital Instruments

Instruments incorporating digital electronic systems are now being used in laboratories. The multimeter type of digital instrument is essentially for the measurement of d.c.; sinusoidal voltages can be measured by rectification before being applied to the measuring circuit, so that the average value is measured, but normally the r.m.s. value is presented by the digital read-out. The instrument is therefore subject to the same type of waveform error as was mentioned in section 10.2.3.

+) Reading indicates black is



10.6 The oscilloscope

The oscilloscope is the most powerful instrument in our arsenal of electronic instruments. It is widely used for measurement of time-varying signals. Any time you have a signal that varies with time - slowly or quickly - you can use an oscilloscope to measure it - to look at it, and to find any unexpected features in it.

The features you see in a signal when you use an oscilloscope to look at a signal are features you cannot see otherwise. Here's a photo of a **Hewlett-Packard (HP) 54601A**

Note the following features of the oscilloscope

- There is a CRT (Cathode Ray Tube) screen on which the signals will be presented. That's at the left.
- There are numerous controls to control things like:
  - The time scale of the presentation
  - A vertical scale
- A cable (IEEE-488) to connect the oscilloscope to a computer. That lets you:
  - Take measurements with the scope
  - Put the measurements in a computer file
  - Analyze the data with MathCAD, Matlab, Excel, etc.

- Notice that this oscilloscope has two input channels. The controls for the two channels are just to the right of the screen.

Now let us **explain how we use** an oscilloscope:

- Plug it in. That's not facetious.
- Turn it on. There is a push button at the lower right edge of the screen. It says "Line" and indicates a "0" and a "1" setting. Depress that button.
- Apply a signal to the input terminals.
  - Your oscilloscope may have provision for more than one signal input. Choose Channel 1 if that is the case.
- Make sure that the settings match the signal. For example:
  - If you have a signal at 1000 Hz, then the period of the signal is 1 millisecond (.001 sec) and you would not want the time scale set so that you only display a microsecond of data, and you also probably won't see much if you display 10 seconds worth of data.
  - If you have a signal that is 10 millivolts high, you won't see much if you set the oscilloscope to show you a signal at 20 volts full-scale. Conversely, you won't see much of a 20 volt signal if the scope is set for 10 millivolts full-scale.

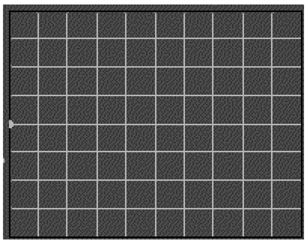
Showing a Simple Signal on the Scope

To get familiar with the scope, you can show a sine signal on the scope. We're going to ask that you show a signal with the following characteristics

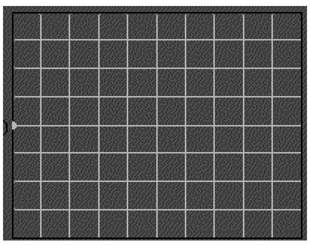
- 1 volt (2v peak-to-peak) signal. In other words, it has a peak of 1 volt and a negative "peak" at -1 volt.
- A frequency of 1000 Hz (i.e. 1 KHz).
- A sinusoidal signal. In other words, it looks like a familiar sine wave.

What will the signal look like?

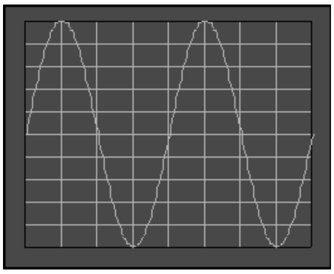
The oscilloscope has an illuminated dot that moves across the screen. With no signal, it would look like the following.



When a sinusoidal signal is applied, then the vertical position is proportional to the voltage at any instant. If you applied a low frequency sine signal, you would get a track like the one below.



If you have a sinusoidal signal that repeats every half millisecond - a frequency of 2 kHz - you would get a picture like this one. It would appear to be stationary on the oscilloscope screen, but it really isn't. It's just that it repeats so frequently that you see it as a constant image.



Simulation

In this simulation, a simulated function generator is connected to a simulated oscilloscope. Both are simplified versions of real instruments. Note the following.

- The function generator can produce a number of signals, including sine and cosine, square, triangular and sawtooth signals. You can choose which signal the function generator produces by clicking on the appropriate button.

Notice the following in this simulation.

- An oscilloscope displays a signal, and there is a unique time when the oscilloscope trace begins to move across the screen. There may be a unique event that triggers the start of the display - when the oscilloscope trace begins to move across the screen. In the simulation above, we have given you a button that starts the trace moving across the screen - a trigger button.Clearly you cannot trigger an oscilloscope by hitting a button every time you want to observe a new trace on an oscilloscope. Another alternative might be to let the oscilloscope free-run. In other words, let the oscilloscope start another trace as soon as a trace is finished. Here is a simulation of that situation.
-

Simulation - Free Running Oscilloscope

In this simulation, the signal trace begins anew as soon as it reaches the right hand side of the oscilloscope screen.

Notice the following about this situation.

- The value at which the trace starts is equal to the last value displayed at the end of the previous trace.
- That implies that the signal is displayed continuously, and that you see ever bit of the signal.
- If the sweep speed - the speed at which the trace moves across the screen - were much higher, the display would be a jumble.
- We can't speed up the sweep enough to really show you that. We can, however, speed it up just a bit, and here is the simulation.

- Use the buttons to change the sweep speed.
- Adjust the frequency so that you don't have an integral number of cycles in one sweep.

Note the following about what happens when the sweep speed changes.

When the sweep speed changes, the horizontal scale - the time scale - changes. Although this is a simulated oscilloscope and function generator, we have designed things so that it is real-time. In real oscilloscopes, everything is real time and when you change the time scale you change the sweep speed accordingly. On an oscilloscope, you can always adjust the sweep speed to "match" the time-scale of the signal you are displaying.

Example: If you have a 1.0 kilohertz signal, the period is one millisecond and you would probably want a scale than ran over 2 milliseconds or something like that.

In a real oscilloscope, the trigger signal can be generated when the signal value reaches some particular level - the trigger level. In most cases you can set the trigger level to a voltage value of your choosing.

Now that you have had a chance to experiment with the simulations above, it's time to define a few terms - and these are items you can control on most oscilloscope. You can control the sweep speed. Sweep speed is usually measured in units of time per distance, like milliseconds/centimeter. This might also be referred to as the horizontal sensitivity.You can control the vertical sensitivity. That's the measure of how sensitive the display dot is to voltage applied to the input terminals. It is usually measured in volts/centimeter.

11. Electronics

11.1. Diodes

Diodes are different and useful electrical components. Diodes are used in many applications like the following.

- Converting AC power from the 50 (or60) Hz line into DC power for radios, televisions, telephone answering machines, computers, and many other electronic devices.
- Converting radio frequency signals into audible signals in radios

Diode Properties : Diodes have the following characteristics.

Diodes are two terminal devices like resistors and capacitors. They don't have many terminals like transistors or integrated circuits.

- In diodes current is directly related to voltage, like in a resistor. They're not like capacitors where current is related to the time derivative of voltage or inductors where the derivative of current is related to voltage.
- In diodes the current is not linearly related to voltage, like in a resistor.
- Diodes only consume power. They don't produce power like a battery. They are said to be passive devices.

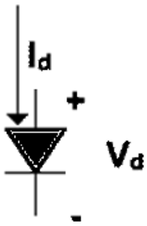
- Diodes are nonlinear, two terminal, passive electrical devices.

In general, diodes tend to permit current flow in one direction, but tend to inhibit current flow in the opposite direction. The graph below shows how current can depend upon voltage for a diode.

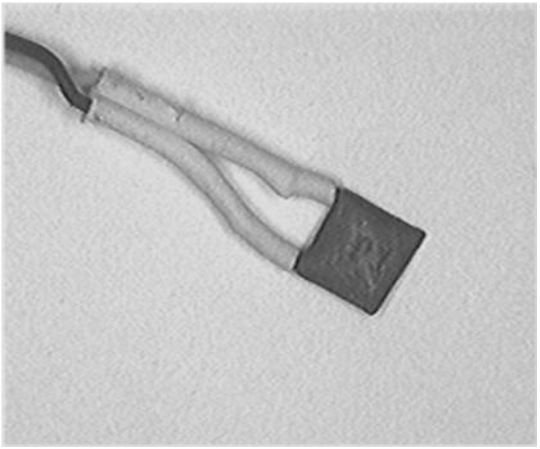
Note the following.

When the voltage across the diode is positive, a lot of current can flow once the voltage becomes large enough.
When the voltage across the diode is negative, virtually no current flows.

The circuit symbol for a diode is designed to remind you that current flows easily through a diode in one direction. The circuit symbol for a diode is shown below together with common conventions for current through the diode and voltage across the diode.



11.2 Temperature Sensor - The Thermistor



Thermistors are inexpensive, easily-obtainable temperature sensors. They are easy to use and adaptable. Circuits with thermistors can have reasonable output voltages - not the millivolt outputs thermocouples have. Because of these qualities, thermistors are widely used for simple temperature measurements. They're not used for high temperatures, but in the temperature ranges where they work they are widely used. Thermistors are temperature sensitive resistors. All resistors vary with temperature, but thermistors are constructed of semiconductor material with a resistivity that is especially sensitive to temperature. However, unlike most other resistive devices, the resistance of a thermistor decreases with increasing temperature. That's due to the properties of the semiconductor material that the thermistor is made from. For some, that may be counterintuitive, but it is correct. Why Use Thermistors To Measure Temperature?
They are inexpensive, rugged and reliable.
They respond quickly

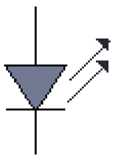
- What Does A Thermistor Look Like?
- Here it is.

- What Does A Thermistor Do?
    - A Thermistor is a temperature dependent resistor. When temperature changes, the resistance of the thermistor changes in a predictable way.
- How Does A Thermistor's Resistance Depend Upon Temperature?

11.3. Light Emitting Diodes (LEDs)

A light emitting diode - a.k.a. an LED - is precisely what the name implies.

- It emits light. Usually the light is very pure - being pretty much monochromatic - and it comes in a restricted range of colors. The most common LED color for the light is red.
- It is a diode. That means that current only flows through an LED in one direction. If you try to make current flow in the reverse direction, no current will flow, and you won't get any light either because you need current flowing in the LED to get any light.



The circuit symbol for an LED looks much like the symbol for a regular diode. There's usually an additional little arrow to indicate the light that comes from the diode. Here's the symbol.

Now, let's take a look at the LED in a simple circuit. Let's imagine that you have the following situation.

- You have a signal that can change. In one state it is 0 volts - a binary zero, and in the other state it is 5 volts - a binary one.
- When the signal is a one, you want an LED to light, and when the signal is a zero you want the LED off.

Here's a circuit that will do what you want when you choose the components correctly. The 5v source is what you have when the signal is a binary one. At that point, current should flow through the resistor and the diode should light.

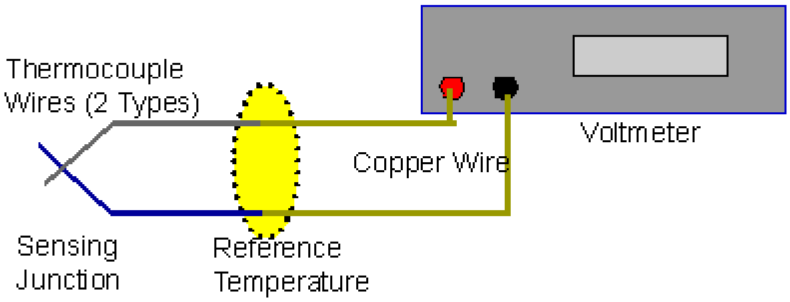
11.4. Temperature Sensor - The Thermocouple

A thermocouple is a junction formed from two dissimilar metals. Actually, it is a pair of junctions. One at a reference temperature (like 0 °C) and the other junction at the temperature to be measured. A temperature difference will cause a voltage to be developed that is temperature dependent. (That voltage is caused by something called the Seebeck effect.) Thermocouples are widely used for temperature measurement because they are inexpensive, rugged and reliable, and they can be used over a wide temperature range. In particular, other temperature sensors (like thermistors and LM35 sensors) are useful around room temperature, but the thermocouple can

The Thermocouple ,Why Use thermocouples To Measure Temperature?

- They are inexpensive.
- They are rugged and reliable.
- They can be used over a wide temperature range.
- **What Does A Thermocouple Look Like?**
- Here it is. Note the two wires (of two different metals) joined in the junction.

- **What does a thermocouple do? How does it work?**
- The junction of two dissimilar metals produces a temperature dependent voltage.
- **How Do You Use A Thermocouple?**
- You measure the voltage the thermocouple produces, and convert that voltage to a temperature reading.
- It may be best to do the conversion digitally because the conversion can be fairly nonlinear.
- **Things You Need To Know About Thermocouples**
- A junction between two dissimilar metals produces a voltage.
- In the thermocouple, the sensing junction - produces a voltage that depends upon temperature.
- Where the thermocouple connects to instrumentation - copper wires? - you have two more junctions and they also produce a temperature dependent voltage. Those junctions are shown inside the yellow oval.



- When you use a thermocouple, you need to ensure that the connections are at some standard temperature, or you need to use an electronically compensated system that takes those voltages into account. If your thermocouple is connected to a data acquisition system, then chances are good that you have an electronically compensated system.
- Once we obtain a reading from a voltmeter, the measured voltage has to be converted to temperature. The temperature is usually expressed as a polynomial function of the measured voltage. Sometimes it is possible to get a decent linear approximation over a limited temperature range.
- There are two ways to convert the measured voltage to a temperature reading.

- Measure the voltage and let the operator do the calculations.
- Use the measured voltage as an input to a conversion circuit - either analog or digital.

12. Problems and solutions.

A. WORKED PROBLEMS INVOLVING ELECTRICAL QUANTITES

Problem I. A mass of 5000 g is accelerated at $2m/s^2$ by of force. Determine the force needed

$$Force = mass \times acceleration = 5kg \times \frac{2m}{s^2} = 10\frac{kgm}{s^2} = 10N$$

Problem 2. Find the force acting vertically downwards on a mass of 200 g attached to a wire.

Mass= 200g = 0.2kg; Acceleration due to gravity = $9.81m/s^2$

Force acting downwards=weight= mass X acceleration $= 0.2kg \times 9.81m/s^2 = 1.962N$

Problem 3. A portable machine requires a force of 200 N to move it, how much work is done if the machine is moved 20m and what average power is utilized if the movement takes 25 s?

$$Work\ done = force \times distance = 200N \times 20m = 4000Nm\ \ or 4Kj$$

$$power = \frac{workdone}{timetaken} = \frac{4000}{25} = \frac{J}{s} = 160W$$

Problem 4. A mass of 1000 kg is raised through a height of 10 m in 20 s.

What is (a) the work done and (b) the power developed?

(a) $Work\ done = force \times distance\ \ and\ \ force = mass \times acceleration..Hence$

$$Work\ done = (1000kg \times 9.81m/s^2) \times (10m) =$$

$= 98100Nm = 98.1kNm\ or 98.1kj$

$$(b)\ \ Power = \frac{work\ done}{time\ taken} = \frac{98\ 100}{20}\ \frac{J}{s}\ (or\ W) = 4905\ W = 4.905\ kW$$

Problem 5. What current must flow if 0.24 C is to be transferred in 15 ms?

Since thequantity of electricity $Q = It$

$$I = \frac{Q}{t} = \frac{0.24}{15 \times 10^{-3}} = \frac{240}{15} = 16A$$

Problem 6. If a current of 10 A flows for 4 minutes, find the quantity of electricity transferred.

Quantity of electricity $Q = It$ coulombs

$I = 10A; t = 4 \times 60 = 240\ S$

$HenceQ = 10 \times 240 = 2400C$

Problem 7. Find the conductance of a conductor of resistance (a) 10Ω, (b) 5 kΩ and (c) 100 mΩ.

$$(a)\ Conductance\ \ G = \frac{1}{R} = \frac{1}{10}\ \ siemen = 0.1\ S$$

$$(b)\ \ G = \frac{1}{R} = \frac{1}{5 \times 10^3}\ \ s = 0.2 \times 10^{-3}\ \ S = 0.2\ mS$$

$$(c)\ G = \frac{1}{R} = \frac{1}{100 \times 10^{-3}}\ S = \frac{10^3}{100}\ S = 10\ S$$

Problem 8. A source emf of 15 V supplies a current of 2 A for 6 minutes.
How much energy is provided in this time?
Energy = power X time and power = voltage X current .Hence
Energy $= VIt = 15 \times 2 \times (6 \times 60) = 10800 Ws\ or\ \ J = 10.8kJ$

Problem 9. Electrical equipment in an office takes current of 13 A from a 240 V supply. Estimate , to the nearest pence, the cost per week of electricity if the equipment is used for 30 hours each week and I k W h of energy costs 0.13 L

$Power = VIwatts = 240 \times 13 = 3120W = 3.12kW$

Energy used per week = $Power \times time = (3.12kW) \times (30h) = 93.6kWh$

Cost at 0.13 L. per kW h 93.6 X 0.13 = 12.16
Hence, weekly cost of electricity =12.16 L.

Problem 10. An electric heater consumes 3.6 MJ when connected to a 250 V, supply for 40 minutes. Find the power rating of the heater and the current taken from the supply.

$$Power = \frac{Energy}{time} = \frac{3.6 \times 10^6}{40 \times 60}\ \frac{J}{s}(or\ W) = 1500\ W$$

i.e. Power rating of heater = 1.5 kW

$$Power = P = VI, Thus I = \frac{P}{V} = \frac{1500}{250} = 6A.$$

Hence the current taken from the supply is 6 A.

B. FURTHER PROBLEMS INVOLVING ELECTRICAL QUANTITIES

(a) SHORT ANSWER PROBLEMS

1. What does 'SI units' mean?
2. Complete the following: Force =………… X…………………
3. What do you understand by the term 'potential difference'?
4. Define electric current in terms of charge and time.
5. Name the units used to measure (a) the quantity of electricity, (b) resistance and (c) conductance.
1. Define the coulomb.
2. Define electrical energy and name its unit.
3. Define electrical power and name its unit.
4. What is electromotive force?
5. Write down three formulae for calculating the power in a dc circuit.
6. Write down the symbols for the following quantities: (a) electrical charge; (b) work; (c) emf; d pd
7. State to which units the following abbreviations refer to :

(a) A ; (b) C; (c) J ; (d) N; (e) m.

(b) MULTI –CHOICE PROBLEMS

1. Which of the following formulae for electrical power is incorrect?

   $(a)\ VI;\ \ (b)\ \frac{V}{I};\ \ (c) I^2;\ \ (d)\ \frac{V^2}{R}$

2. A resistance of 50 kΩ has a conductance of

(a) 20 S; (b) 0.02 S; (c) 0.02 mS; (d) 20 kS.

3. State which of the following is incorrect;

(a) 1 N= 1kg m/s^2; (b) 1V= 1 J/A; (c) 30Ma =0.03 A; (d) 1J =N/m.

4. The power dissipated by a resistor of 4Ω when a current of 5 A passes through it is

(a) W; (b) 20W; (c) 80W; (d) 100 W.

5. 60μs is equivalent to: (a) 0.06s; (b) 0.000 06 s; (c) 1000 minutes: (d) 0.6 s.

6. A mass of 1200 g is accelerated at 200 cm/s^2 by a force. The value of the force required is :

(a) N; (b) 2400 N; (c) 240 kN; (d) 0.24 N.

7. A current of 3 A flows for 50 h through a 6Ω resistor. The energy consumed by the resistor is:

(a) 0.9 kWh; (b) 2.7 kWh; (c) 9 kWh; (d) 27 kWh.

8. What must be known in order to calculate the energy used by an electrical appliance?

(a) voltage and current; (b) current and time of operation ; (c) power and time of operation; (d) current and quantity of electricity used.

C. CONVENTIONAL PROBLEMS

(Take g= 9.81 m/s^2 where appropriate.)

1. What force is required to give a mass of 20 kg on acceleration of 30 m/s^2? (600 N)
2. Find the accelerating force when a car having a mass of 1.7 Mg increases its speed with a constant acceleration of 3 m/s^2. (5.1 kN)
3. A force of 40 N accelerates a mass at 5 m/s^2. Determine the mass. (8 kg)
4. Determine the force acting downwards on a mass of 1500 g suspended on a string. (14.72N)

   amount of work is done? (8 J)

6. A force of 2.5 k N is required to lift a load. How much work is done if the load is lifted through 500 cm? (12.5kJ)
7. An electromagnet exert s a force of 12 N and moves a soft iron armature through a distance of 1.5 cm in 40 ms . Find the power consumed. (4.5W)

8. A mass of 500 kg is raised to a height of 6 m in 30 s. Find(a) the work done and (b) the power developed. [(a) 29.43 k N m ; (b)981W]
9. What quantity of electricity is carried by 6.24 X 10^{21} electrons? (1000C)
10. in what time would a current of 10 A transfer a charge of 50 C?
(5s)
11. A current of 6 A flows for 10 minutes. What charge is transferred?
(3600C)
12. How long must a current of 100 m A flow so as to transfer a charge of 50 C?
(8 min 20 s)
13. Find the conductance of a resistor of resistance (a) 10Ω; (b) 2kΩ; (c) 2mΩ.
[(a) 0.1 S ;(b) 0.5 mS; (c) 500 S]
14. A conductor has a conductance of 50μS .What is its resistance?
(20kΩ)
15. An emf of 250 V is connected across a resistance and the current flowing through the resistance is 4A.What is the power developed?
(1 kW)
16. 85.5 J of energy are converted into heat in 9s. What power is dissipated? (9.5 W)
17. A current of 4 A flows through a conductor and 10 W is dissipated. What pd exists across the ends of the conductor? (2.5V)
18. Find the power dissipated when:
(a) a current of 5 mA flows through a resistance of 20kΩ;
(a) a voltage of 400 V is applied across a 120 kΩ resistor;
(a) a voltage applied to a resistor is 10 kV and the current flow is 4 mA.
[(a) 0.5 W; (b) 4/3 W; (c) 40 W]
19. A battery of emf 15 V supplies a current of 2 A for 5 minutes. How much energy is supplied in this time? (9kJ)
20. An electric heater takes 7.5 A from a 250 V supply. Find the annual cost if the heater is used an average of 25 hours per week for 48 weeks. Assume that 1 kWh of energy costs 4 p. (£90.00)
21. A dc electric motor consumes 72 MJ when connected to a 400 V supply for 2 h 30 min. Find the power rating of the motor and the current taken from the supply. (8 kW; 20 A)

D .WORKED PROBLEMS ON DC CIRCUIT THEORY

(a) OHM'S LAW

Problem I. A coil has a current of 50 mA flowing through it when the applied voltage is 12V. What is the resistance of the coil?

Resistance, $R = \frac{V}{t} = \frac{12}{50 \times 10^{-3}} = \frac{12 \times 10^{3}}{50} = 240\Omega$

Problem 2. An electric kettle has a resistance of 30 Ω. What current will flow when it is connected to a 240V supply? Find also the power rating of the kettle.

Current $I = \frac{V}{R} = \frac{240}{30} = 8A$

power $P = VI = 240 \times 8 = 1920kW = Power\ rating\ of\ kettle$

(b) RESISTANCES IN SERIES AND IN PARALLEL

Problem 3. Find the equivalent resistance for the circuit shown in Fig 12.1.

R_3 R_4 and R_5 are connected in parallel and the equivalent resistance R is given by:

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} + \frac{1}{18} = \frac{6+3+1}{18} = \frac{10}{18}$$

$$Hence R = \frac{18}{10} = 1.8\Omega$$

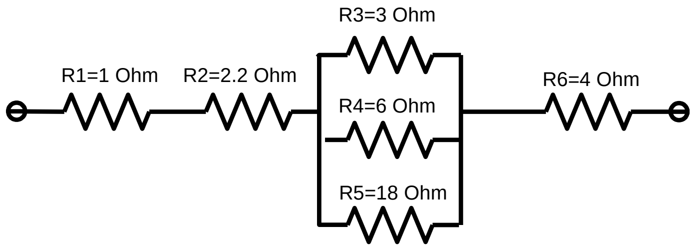


Fig.12.1

The circuit is now equivalent to four resistors in series and the equivalent circuit resistance = $1 + 2.2 + 1.8 + 4 = 9\Omega$

Problem 4. Calculate the equivalent resistance, between the points A and B for the circuit shown in Fig 12.2.

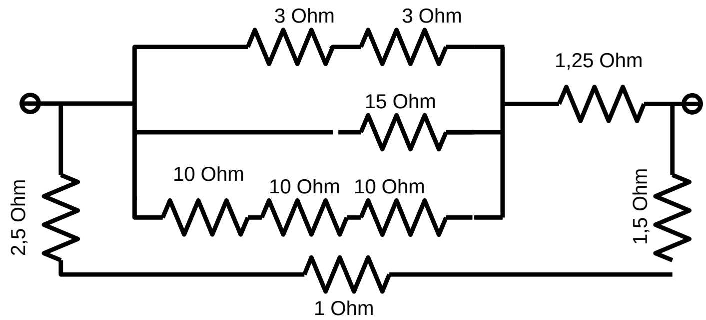


Fig.12.2

Combining the two 3Ω resistors in series , the three 10Ω resistors in series and the 2.5Ω , 1Ω and 1.5Ω resistors in series gives the simplified equivalent circuit of Fig 10. The equivalent resistance R of 6 Ω , 15Ω and 30 Ω in parallel is given by:

$$\frac{1}{R}=\frac{1}{6}+\frac{1}{15}+\frac{1}{30}=\frac{5+2+1}{30}=\frac{8}{30}$$

Thus, $$R=\frac{30}{8}=3.75\Omega$$

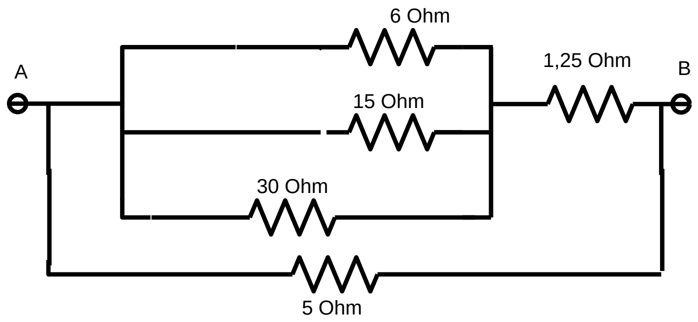


Fig.12.3

The equivalent circuit is now as shown in Fig 12.3 Combining the 3.75 Ω and 1.25Ω is series gives an equivalent resistance of 5Ω. The equivalent resistance R_t of 5Ω in parallel with another 5Ω resistor is given by:

$$R_T=\frac{5\times 5}{5+5}=\frac{25}{10}=2.5$$

(Note that when two resistors having the same value are connected in parallel the equivalent resistance will always be half value of one of the resistors

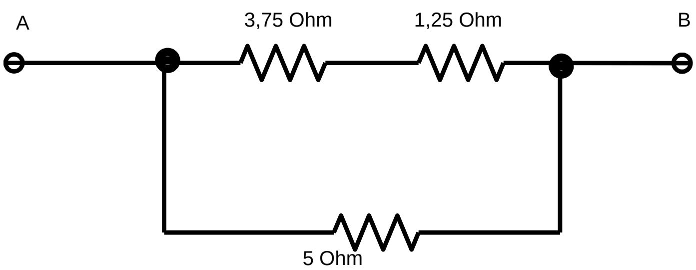


Fig.12.4

The circuit of Fig.12.2 can thus be replaced by a 2.5 Ω Resistor placed between points A and B.

Problem 5. Determine the equivalent resistance for the series –parallel arrangement shown in Fig 12.5, correct to 2 decimal places

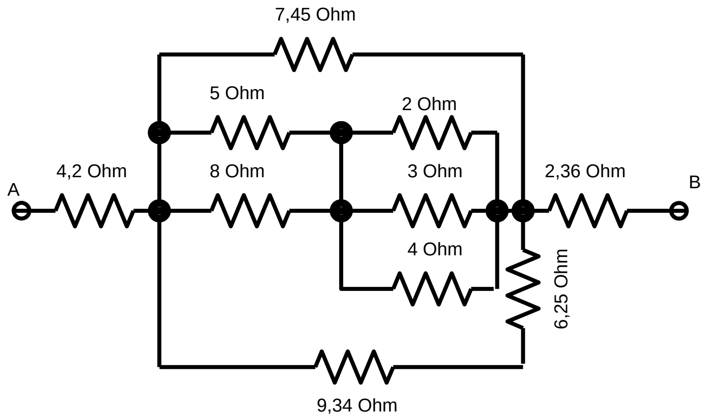


Fig.12.5

The equivalent resistance of 5Ω in parallel with $8\Omega is \frac{5\times 8}{5+8}=\frac{40}{13}, i.e. 3.077\Omega$

The equivalent resistance R of 2Ω, 3Ω and 4Ω in parallel is given by:

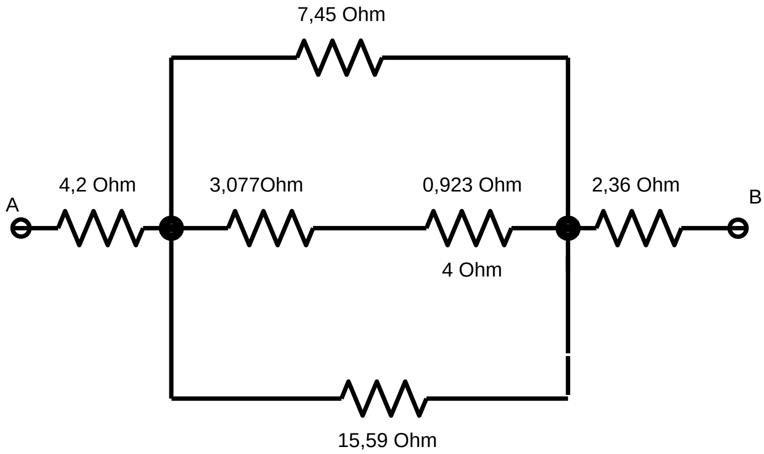


Fig.12.6

$$\frac{1}{R}=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{6+4+3}{12}=\frac{13}{12}.$$

Hence $R=\frac{12}{13}=0.923\Omega$

The equivalent resistance of 9.34Ω and 6.25Ω in series is $9.34+6.25=15.59\Omega$

Thus a simplified circuit diagram is shown in Fig 12.6.

3.077 Ω in series with 0.923 Ω gives an equivalent resistance of 4.00Ω. The equivalent resistance R_x of 7.45 Ω, 4.00Ω and 19.59Ω in parallel is given by :

$$\frac{1}{R_x}=\frac{1}{7.45}+\frac{1}{4.00}+\frac{1}{15.59}$$

i.e. conductance G_x = 0.134+0.250+ 0.064 =0.448 Siemens

$$\text{Since } G = \frac{1}{R_x} \text{ then} R_x = \frac{1}{G_x} = \frac{1}{0.448} = 2.23\Omega$$

The circuit is now equivalent to three resistors of 4.20Ω, 2.23Ω and 2.36Ω connected in series , which gives an equivalent resistance of $4.20 + 2.23 + 2.36 = 8.79\Omega$

(a) CURRENTS AND PD'S IN SERIES–PARALLEL CIRCUIT ARRANGMENTS

Problem 6. Resistance of 10 Ω, 20 Ω and 30 Ω are connected (a) in series and (b) in parallel to a 240 V supply. Calculate the supply current in each case.

(a) The series circuit is shown in Fig 12.7.

The equivalent resistance

$R_T = 10\Omega + 20\Omega + 30\Omega = 60\Omega$

$$\textit{Supply curent } I = \frac{V}{R_T} = \frac{240}{60} = 4A$$

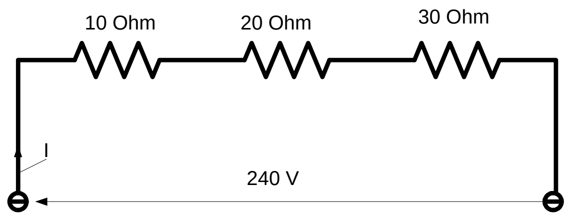


Fig.12.7

(a) The parallel circuit is shown in Fig 12.8. The equivalent resistance R_T of 10Ω , 20Ω and 30Ω resistances connected in parallel is given by:

$$\frac{1}{R_T} = \frac{1}{10} + \frac{1}{20} + \frac{1}{30} = \frac{6+3+2}{60} = \frac{11}{60}$$

$$\textit{Hence} R_T = \frac{60}{11}\Omega$$

$$\textit{Supply curent } I = \frac{V}{R_T} = \frac{240}{60/11} = \frac{240 \times 11}{60} = 44A$$

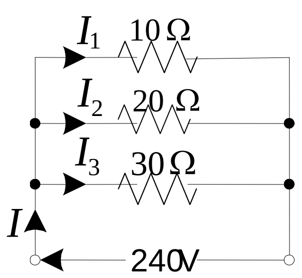


Fig.12.8

[*Check*: $I_1 = \frac{V}{R_1} = \frac{240}{10} = 24\ A$

$I_2 = \frac{V}{R_2} = \frac{240}{20} = 12A$

$I_3 = \frac{V}{R_3} = \frac{240}{30} = 8\ A$

$$\textit{For a parallel circuit} I = I_1 + I_2 + I_3 = 24 + 12 + 8 = 44A, \textit{as above}$$

Problem 7. For the series–parallel arrangement shown in Fig 12.9 find (a) the supply current, (b) the current flowing through each resistor and(c) the pd across each resistor.

(a) The equivalent resistance R_x OF R_2 and R_3 in parallel is

$$R_x = \frac{6 \times 2}{6+2} = \frac{12}{8} = 1.5\Omega$$

The equivalent resistance R_t of R_1 , R_x and R_4 in series is:

$$R_T = 2.5 + 1.5 + 4 = 8\Omega$$

$$Supply \quad current \ I = \frac{V}{R_T} = \frac{200}{8} = 25\ A$$

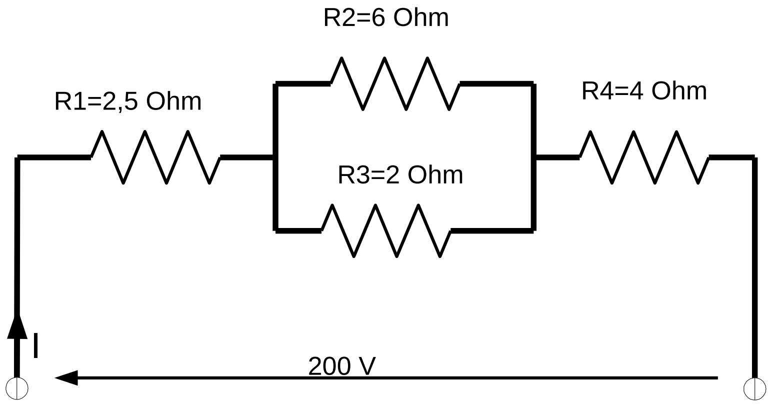


Fig. 12.9

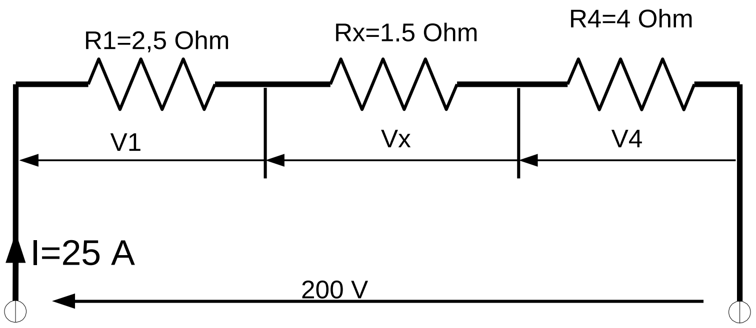


Fig.12.10

(b) The current flowing through R_1 and R_4 is 25 A

The current flowing through R_2 $R_2 = \left(\frac{R_3}{R_2 + R_3}\right) I = \left(\frac{2}{6+2}\right) 25 = 6.25A$

The current flowing through R_3 $R_3 = \left(\frac{R_2}{R_2 + R_3}\right) I = \left(\frac{2}{6+2}\right) 25 = 18.75A$

(Note that the currents flowing through R_2 and R_3 must add up to the total current flowing into the parallel arrangement, i.e. 25 A)

(c) The equivalent circuit of Fig 12.9 is shown in Fig 12.10.

pd across R_1,i.e. $V_1 = IR_1 = (25)(2.5) = 62.5V$

pd across $R_x i.e. V_x = IR_x = (25)(15) = 37.5V$

pd across , R_4 i.e. $R_4 i.e. V_4 = IR_4 = (25)(4) = 100V$

Hence the pd across $R_2 = pd$ across $R_3 = 37.5V$

The Problem 8. For the circuit shown in Fig.12.11 calculate (a) the value of resistor R_x such that the total power dissipated in the circuit is 2.5 kW and (b) the current flowing in each of the four resistors.

(a) Power dissipated $P = VI$ watts $P = VI$
Hence, 2500 = (250)(I)

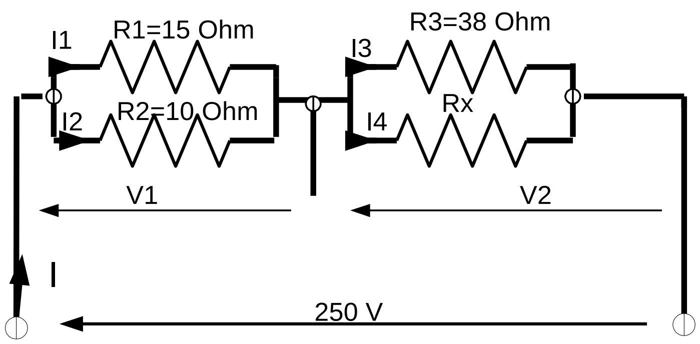


Fig.12.11

$$I = \frac{2500}{250} = 10A$$

From Ohm's law,

$$R_T = \frac{V}{I} = \frac{25}{10} = 25\Omega$$

Where R_T is the equivalent circuit resistance.

The equivalent resistance of R_1 and R_2 in parallel is $\frac{15 \times 10}{15 + 10} = \frac{150}{25} = 6\Omega$

The equivalent resistance of R_3 and R_x in parallel is equal to $25\Omega - 6\Omega$
i.e. 19Ω

There are three methods whereby R_x may be determined.

Method 1

The voltage V_1 =IR, where R is 6Ω from above .

i.e. V_1 = (10) (6)= 60 V.

Hence, V_2 =250V -60V =190 V =pd across R_3=pd across R_x

$$I_3 = \frac{V_2}{R_3} = \frac{190}{38} = 5A.\ Thus I_4 = 5A, also, \sin ce I = 10A$$

Thus, $R_X = \frac{V_2}{I_4} = \frac{190}{5} = 38\Omega$

Method 2

Since the equivalent resistance of R_3 and R_x in parallel is 19Ω

Then, $19 = \frac{38R_X}{38 + R_X}\left(i.e. \frac{product}{sum} \right)$

Hence, $19(38R_X) = 38R_X$

$$722 + 19R_X = 38R_X$$

$$722 = 38R_X - 19R_X = 19R_X$$

Thus $R_x = \frac{722}{19} = 38\Omega$

Method 3

When two resistors having the same value are connected in parallel the equivalent is always half the value of one of the resistors. Thus, in this case , since R_T =19Ω and R_3 = 38Ω, then R_x = 38Ω could have been Deduced on sight.

$$(b) Current I_1 = \left(\frac{R_2}{R_1 + R_2} \right) I = \left(\frac{10}{15+10} \right) 10 = \left(\frac{2}{5} \right) 10 = 4A$$

$$Current I_2 = \left(\frac{R_1}{R_1 + R_2} \right) I = \left(\frac{15}{15+10} \right) 10 = \left(\frac{3}{5} \right) 10 = 6A$$

From part (a), method1. $I_3 = I_5 = 5A$

Problem 9. For the arrangement shown in Fig 12.12, find the current I_x

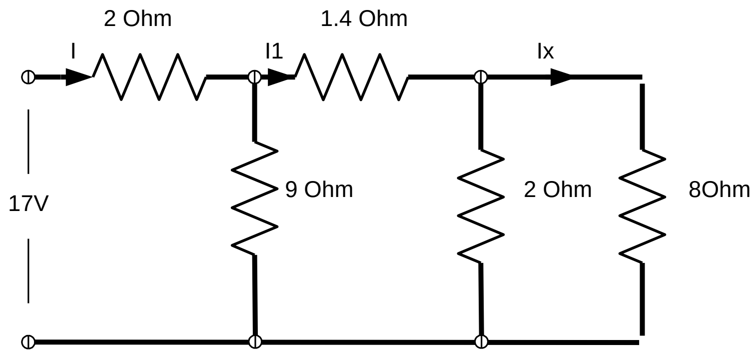


Fig.12.12

Commencing at the right hand side of the arrangement shown in Fig 19, the circuit is gradually reduced in stages as shown in Fig.12.13 (a)-(d)

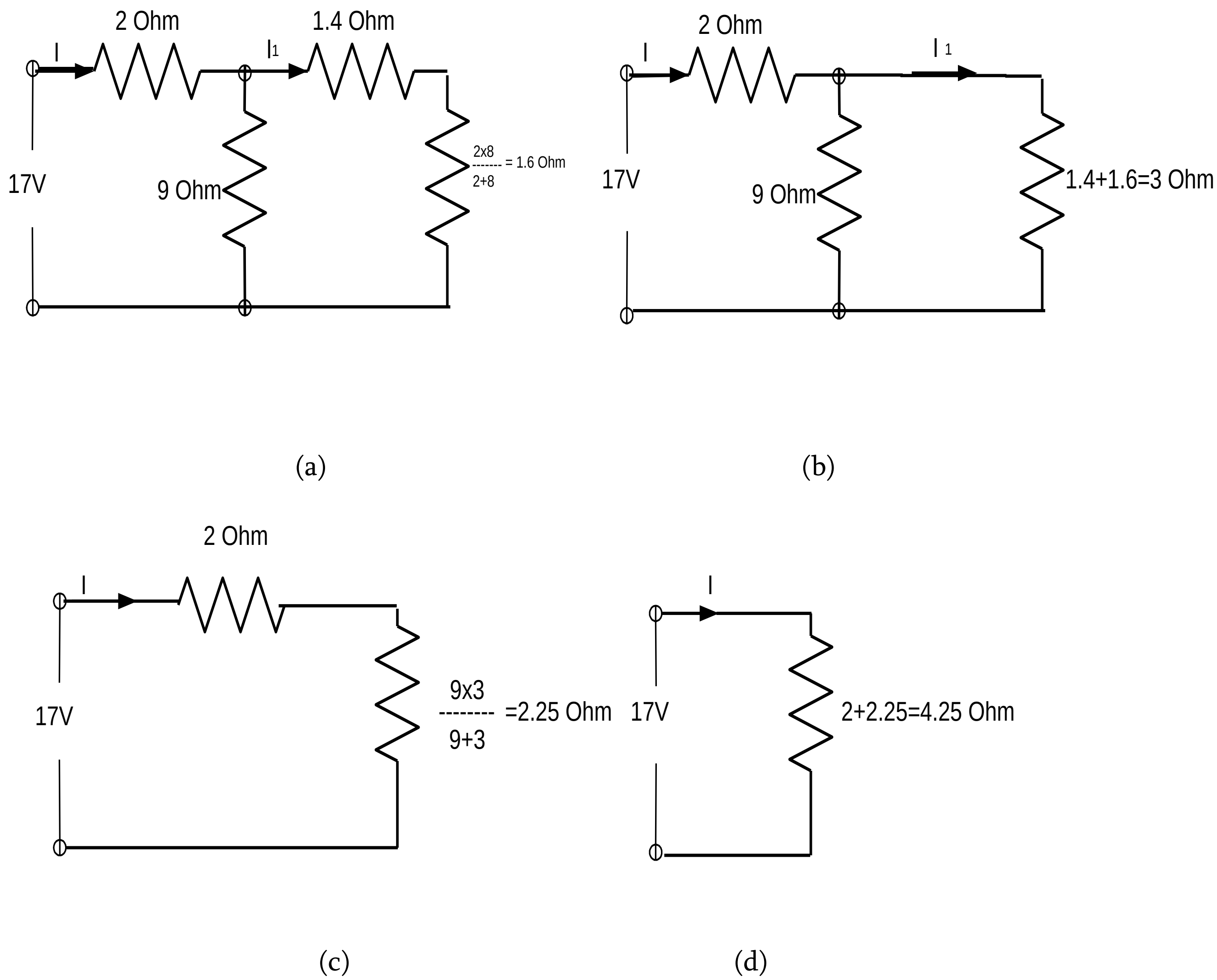


Fig.12.13

From Fig.12.13,d

$$I = \frac{17}{4.25} = 4\ A$$

From Fig.12.13b

$$I_1 = \left(\frac{9}{9+3}\right) I = \left(\frac{9}{12}\right) 4 = 3\ A$$

From Fig.12.12

$$I_x = \left(\frac{2}{2+8}\right) I_1 = \left(\frac{2}{10}\right) 3 = 0.6\ A$$

(d) INTERNAL RESISTANCE

Problem 10. A cell has an internal resistance of 0.03Ω and an emf of 2.20 V. Calculate its terminal pd if it delivers (a) 1A; (b) 10A; (c) 40 A.

(a) For 1 A, terminal pd, V=E-Ir=2.20-(1)(0.03) =2.17 V

(b) For 1 A, terminal pd, V=E-Ir=2.20-(10)(0.03) =1.90V

(c) For 1 A, terminal pd, V=E-Ir=2.20-(40)(0.03) =1.00 V

(d)

Problem 11. The voltage at the terminals of a battery is 75 When no load is connected and 72 V when a load of 60 A is connected .Find the internal resistance of the battery. What would be the terminal voltage when a load taking 40A is connected?

When no load is connected E=V Hence, the emf E of the battery is 75 V. When a load is connected, the terminal voltage, V is given by V=E-I$_r$

Hence 72= 75 –(60)(r) 60r = 75-72=3

$$r = \frac{3}{60} = \frac{1}{20} = 0.05\Omega$$

When a current of 40 A is flowing then V= 75-(40)(0.05)=75-2=73 V

Problem 12. A battery consists of 10 cells connected in series each cell having an emf of 2 V and an internal resistance of 0.05 Ω. The battery supplies a load R taking 4 A. Find the and the value of the load R.

For cells connected in series , total emf= sum of individual emfs=20V. Total internal resistance =sum of individual internal resistances = 0.5Ω The circuit diagram is shown in Fig 12.14.

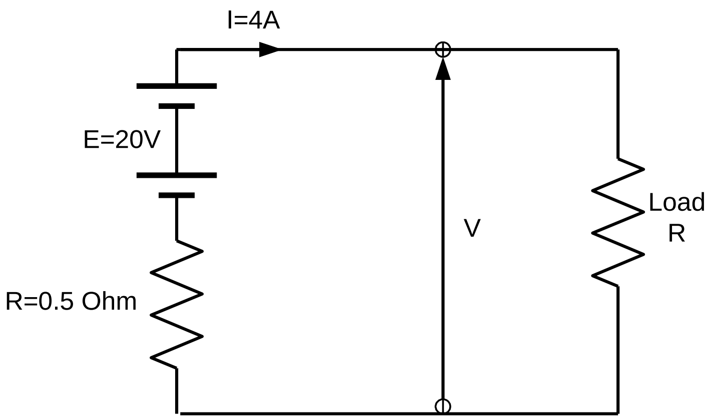


Fig.12.14

Voltage at battery terminals V= E-Ir = 20 – (4) (0.

i.e. V= 18 V Resistance of load $R = \frac{V}{I} = \frac{18}{4} = 4.5\Omega$

Problem 13. Determine the equivalent resistance of thenetwork shown in Fig.12.15. Hence determine the current taken from the supply when a battery of emf 12 V and internal resistance 0.2 Ω is connected across the terminals PQ .Find also the current flowing through the 2.9 Ω resistor and the pd across the 5.1 Ω resistor.

R_2 in series with R_3 is equivalent to 5.1Ω+2.9Ω, i.e.8Ω

R_1 in parallel with 8Ω gives an equivalent resistance of $\frac{2 \times 8}{2+8} = 1.6\Omega$

1.6 Ω in series with 1.2 Ω gives an equivalent resistance of 2.8Ω. Hence the equivalent resistance of the network shown in Fig.12.15 is 2.8 Ω. Fig.12.16 shows the equivalent resistance connected to the battery.

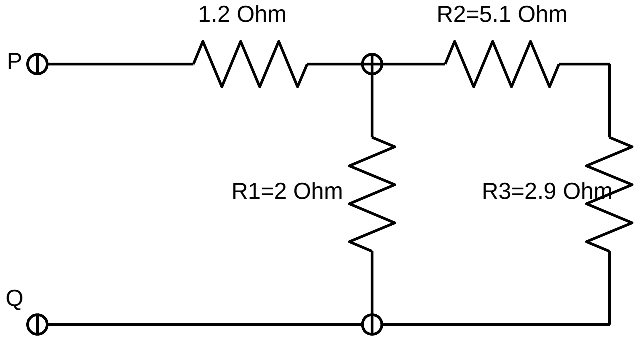


Fig.12.15

Current $I = \frac{E}{R_T}$, where R_T is the total circuit resistance (i.e. including the internal resistance r of the battery). Hence $I = \frac{12}{2.8+0.2} = \frac{12}{3.0} = 4A$

(Note that in Fig.12.16 the resistances 2.8 Ω and 0.2Ω are connected in series with each other and not in parallel).

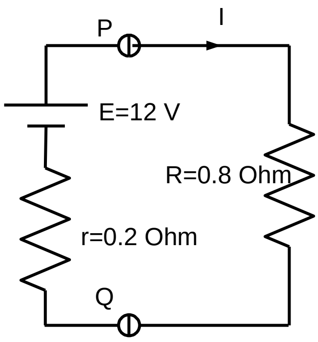


Fig.12.16

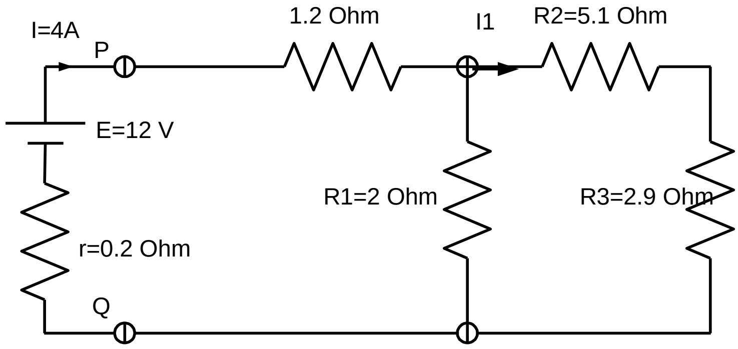


Fig.12.17

From Fig.12.17 , the current flowing through the 2.9Ω resistor , i.e. I_1 is given by

$$I_1 = \left(\frac{2}{2+5.1+2.9}\right)(4) = 0.8A$$

The p.d. across the 5.1Ω resistor is given by

$$V = I_1\,(5.1) = (0.8)(5.1) = 4.08V$$

(E) SUPERPOSITION THEOREM

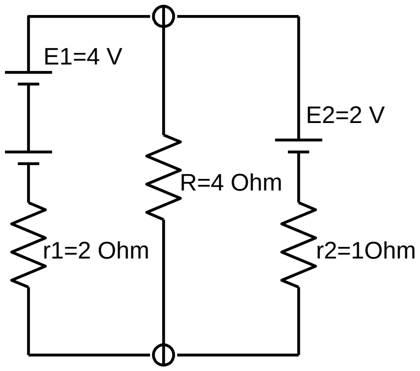


Fig.12.18

Problem 14. Fig.12.18 shows a circuit with their internal resistances. Determine the current in each branch of the network by using the superposition theorem.

Procedure:

1. Redraw the original circuit with source E_2 removed, being replaced by r_2 only, as shown in Fig.12.19(a)
2. Label the currents in each branch and their directions as shown in Fig.12.19(a) and determine their values.(Note that the choice of current directions depends on the battery polarity , which , by convention is taken as flowing from the positive battery terminal as shown.) R in parallel with r_2 gives an equivalent resistance of $\frac{4\times 1}{4+1} = 0.8\Omega$

From the equivalent circuit of Fig.12.19(b)

$$I_1 = \frac{E_1}{r_1+0.8} = \frac{4}{2+0.8} = 1.429A$$

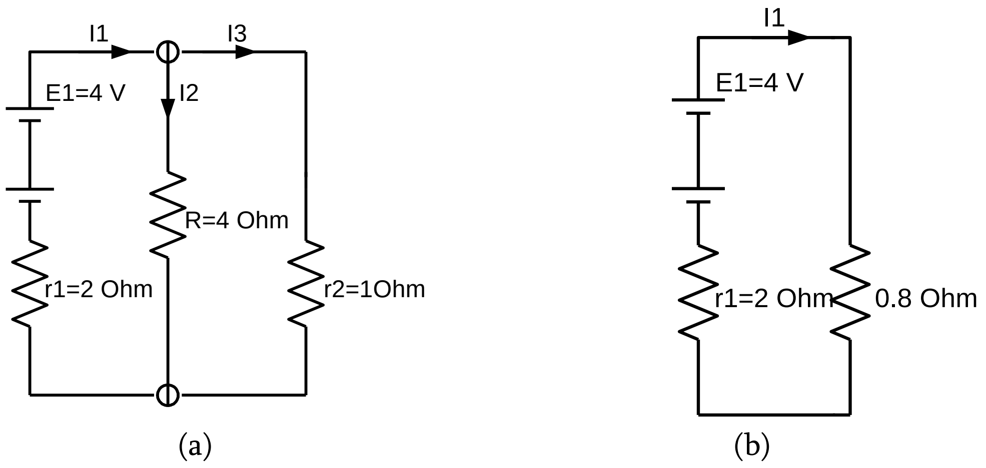


Fig.12.19

From Fig.12.19(a)

$$I_2 = \left(\frac{1}{4+1}\right) I_1 = \frac{1}{5}(1.429) = 0.286A$$

and

$$I_3 = \left(\frac{4}{4+1}\right) I_1 = \frac{4}{5}(1.429) = 1.143A$$

3. Redraw the original circuit with source E_1 removed, being replaced by r_1 only, as shown in Fig.12.20 (a)

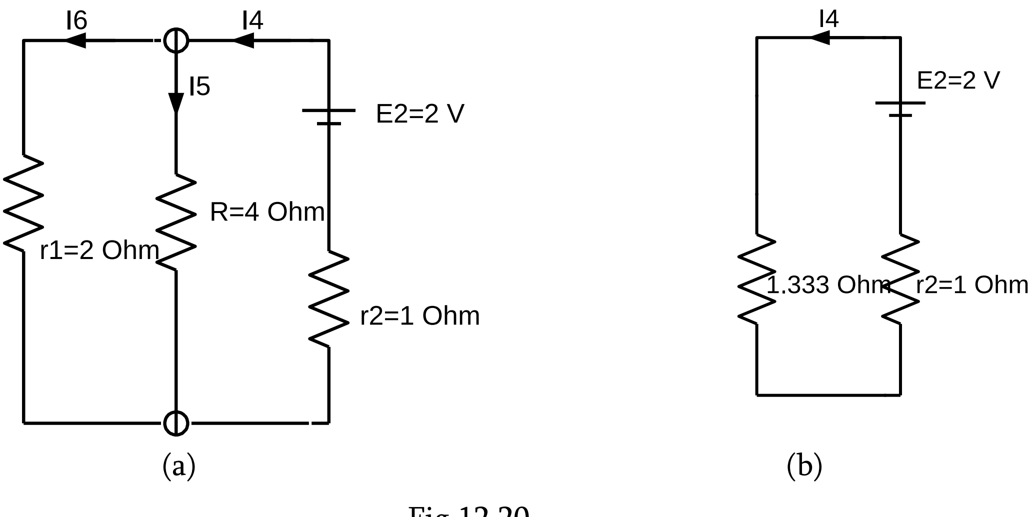


Fig.12.20

4. Label the currents in each branch and their directions as shown in Fig.12.20(a) and determine their values. r_1 in parallel with R gives an equivalent resistance of

$$\frac{2\times 4}{2+4} = \frac{8}{6} = 1.333$$

From the equivalent circuit of Fig.12.20b)

$$I_4 = \frac{E}{1.333 + r_2} = \frac{2}{1.333+1} = 0.857A$$

From Fig.12.20a)

$$I_5 = \left(\frac{2}{2+4}\right) I_4 = \frac{2}{6}(0.857) = 0.286A$$

$$I \; = \left(\frac{4}{2+4}\right) I_4 = (0.857) = 0.571A$$

5. Superimpose Fig.12.20(a) on to Fig.12.19(a) as shown in Fig.12.21

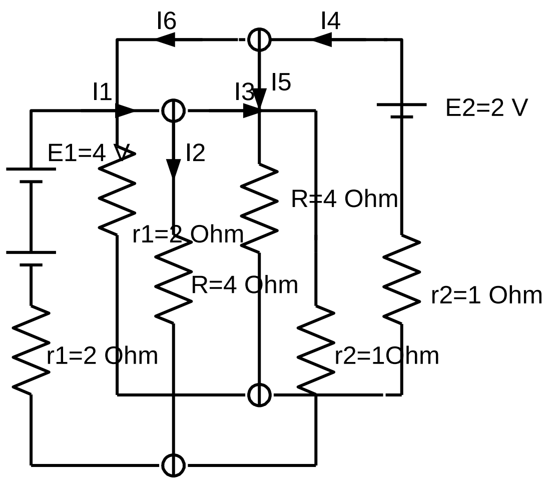


Fig.12.21

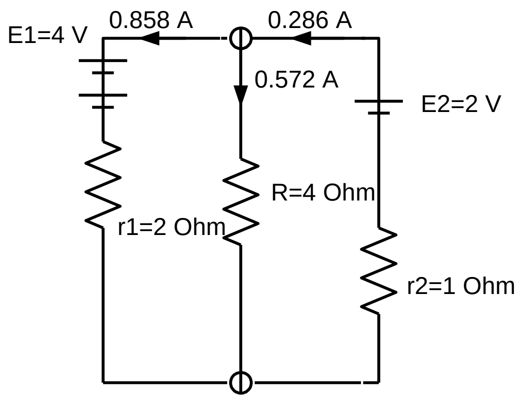


Fig.12.22

6. Determine the algebraic sum of the currents flowing in each branch. Resultant current flowing through source 1, i.e.

$$I_1 - I_6 = 1{,}429 - 0.571 = 0.858A \quad \text{(discharging)}$$

Resultant current flowing through source 2.i.e.

$$I_4 - I_3 = 0.857 - 1.143 = -0.286A \quad \text{(charging)}$$

Resultant current flowing through resistor R, i.e.

$$I_2 + I_5 = 0.286 + 0.286 = 0.572A$$

The resultant currents with their directions are shown in Fig.12.22.

Problem 15. For the circuit shown in Fig.12.23, find, using the superposition theorem, (a) the current flowing in and the pd across the 18Ω resistor, (b) the current in the 8 V battery and (c) the current in the 3 V battery.

1. Removing source E_2 gives the circuit of Fig.12.24 (a)
2. The current directions are labeled as shown in Fig.12.24 (a) , I_1 flowing from the positive terminal of E_1.From Fig.12.24(b)

$$I_1 = \frac{E_1}{3+1.8} = \frac{8}{4.8} = 1.667A$$

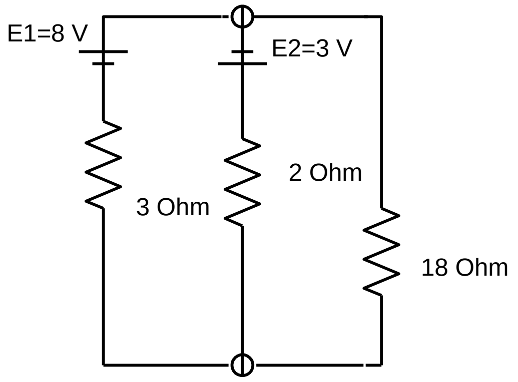


Fig. 12.23

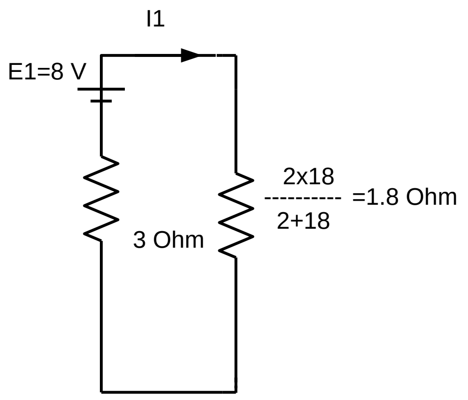


Fig.12.24

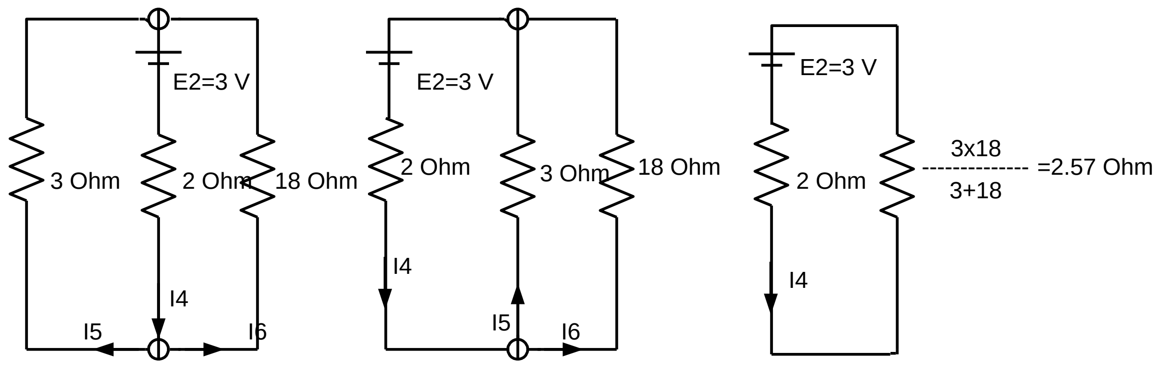
From Fig. 12.24 (a)

$$I_2 = \left(\frac{18}{2+18}\right) = I_1 = \frac{18}{20}(1.667) = 1.500A$$

and

$$I_2 = \left(\frac{2}{2+18}\right) I_1 = \frac{18}{20}(1.667) = 0.167A$$

3. Removing source E1 gives the circuit of Fig.12.25(a) (which is the same as Fig .12.25.(b)).
4. The current directions are labeled as shown in Figs.12.25 (a) and 12.25 (b), I_4 flowing from the positive terminal of E_2



(a) (b) (c)

Fig.12.25

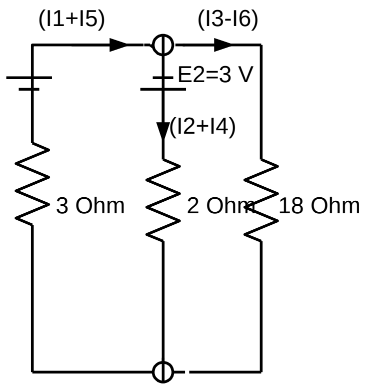


Fig.12.26

From Fig.2.25(c)

$$I_4 = \frac{E_2}{2+2.572} = \frac{3}{4.571} = 0.656A$$

From Fig.12.25(b)

$$I_5 = \left(\frac{18}{3+18}\right) I_4 = \frac{18}{21}(0.656) = 0.562A$$

$$I_6 = \left(\frac{3}{3+18}\right) = I_4 = \frac{3}{21}(0.656) = 0.094A$$

5. Superimposing Fig.12.25(a) on to Fig.12.24(a) gives the circuit shown in Fig.12.26.
6. (a) Resultant current in the 18Ωresistor $= I_3 - I_6 = 0.167 - 0.094 = 0.073$

   Pd across the 18Ω resistor = 0.073×18=1.314 V

(d) Resultant current in the 8 V battery= $I_1 + I_5 =$ 1.667+0.562=2.29 A (discharging)

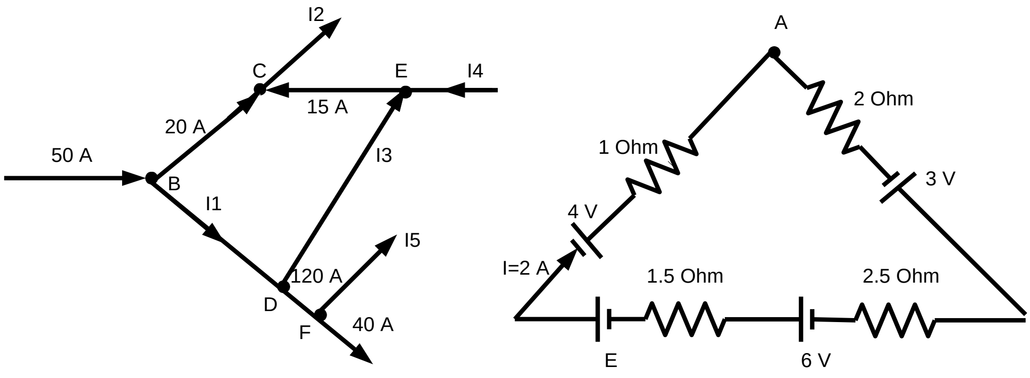
(c) Resultant current in the 3 V battery= $I_2 + I_4 =$ 1.500 +0.656= 2.156 A (discharging)

(f) KIRCHHOFF'S LAWS

Problem 16. (a) Find the unknown currents marked in Fig.12.27 (a), (b) Determine the value of emf E in Fig.12.27(b)

(a) Applying Kirchhoff's current law:

For junction B: 50= 20+I_1.Hence I_1 = 30A For junction C: 20+15=I_2.Hence I_2=35A For junction D: I_1=I_3+120. i.e. 30=I_3+120 . Hence I_3= -90A (i.e. in the opposite direction to that shown in Fig 34 (a)) For junction E: I_4=15-I_3 i.e. I_4=15 –(-90). Hence I_4 =105 A For junction F: 120= I_5+40. Hence I_5 =80 A



a) b)

Fig.12.27

(b) Applying Kirchhoff's voltage law and moving clockwise around the loop of Fig.12.27(b) starting at point A:

$$3+6+E-4=(I)(2)+(I)(2.5)+(I)(1.5)+(I)(1)=I(2+2.5+1.5+1)$$

i.e. 5+E =2(7) $5+E=2[7]$

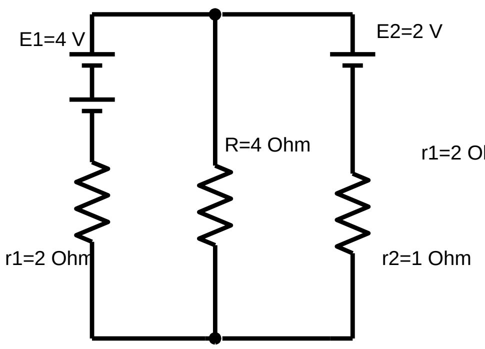
Hence $E=14-5=9V$

Problem 17 .Use Kirchhoff's laws to determine the currents flowing in each branch of the network shown in Fig.12.28 .

(Note that this is the same problem as Problem 15 and a comparison of methods may be made.)

Procedure

1. Use Kirchhoff's current law and label current directions on the original circuit diagram The directions chosen are arbitrary, but it is usual ,as a starting point, to assume that current flows from the positive terminals of the batteries. This is shown in Fig.12.29 where the three branch currents are expressed in terms of I_1 and I_2 only , since the current through R is $I_1 + I_2$.



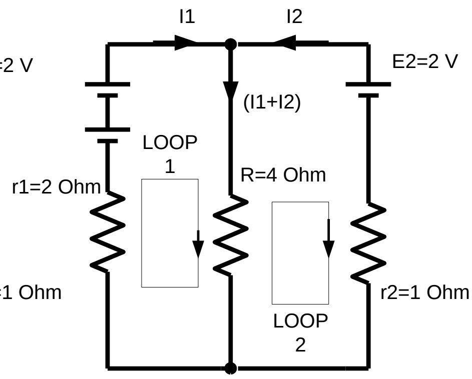


Fig.12.28 Fig.12.29

1. Divide the circuit into two loops and apply Kirchhoff's voltage law to each. From loop 1 of Fig.12.29, and moving in a clockwise direction as indicated (the direction chosen does not matter), gives

$$E_1 = I_1 r_1 + (I \ \ + I_2)R, i.e. 4 = 2l_1 + 4(I_1 + I_2) i.e. 6I_1 + 4I_2 = 4$$

(1)

From loop 2 of Fig.12.29, and moving In an anticlockwise direction as indicated (once again, the choice of direction does not matter ; it does not have to be in the same direction as that chosen for the first loop), gives

$$E_2 = I_2 r_2 + (I_1 + I_2)R. i.e. 2 = I_2 + 4(I_1 + I_2), i.e. 4I_1 + 5I_2 = 2 \qquad (2)$$

2. Solve equations (1) and (2) for I_1 and I_2

$2\times(1)$ gives: $\quad 12I_1 + 8I_2 = 8 \qquad (3)$

$3\times(2)$ gives: $\quad 12I_1 + 15I_2 = 6 \qquad (4)$

(3)-(4) gives $\quad -7I_2 = 2 \quad I_2 = \frac{2}{7} = -0.286A$

(i.e. I_2 is flowing in the opposite direction to that shown in Fig .12.29.)

From $\qquad$ (1) $\quad 6I_1 + 4(-0.286) = 4$

$6I_1 = 4 + 1.144$

Hence $\qquad I_1 = \frac{5.144}{6} = 0.857A$

Current flowing through resistance

$R = I_1 + I_1 = 0.857 + (-0.286) = 0.571A$

(The values of the currents in each branch are seen to be the same as in Problem15 when taken correct to two decimal places.) Note that a third loop is possible , as shown in Fig.12.30 giving a third equation which can be used as a cheek:

$$E_1 - E_2 = I_1 r_1 - I_2 - r_2 = 4 - 2 = 2I_1 - I_2$$

$$2 = 2I_1 - I_2 \text{Check: } \ 2I_1 - I_2 = 2(0.857) - (-0.286) = 21$$

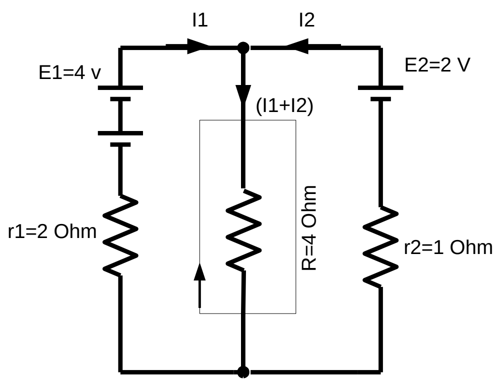


Fig.12.30

Problem 18. Determine , using Kirchhoff's laws , each branch current for the network shown in Fig.12.31

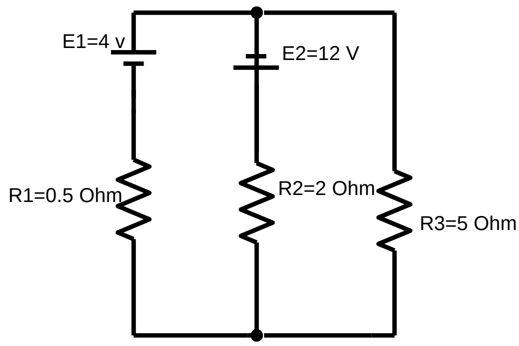


Fig.12.31

1. Currents, and their directions are shown labeled in Fig.12.32 following Kirchhoff's current law. It is usual, although not essential, to follow conventional current flow with current flowing from the positive terminal of the source.
2. The network is divided into two loops as shown in Fig12.32. Applying Kirchhoff's voltage law gives:

For loop 1:

$$E_1 + E_2 = I_1 R_1 + I_2 R_2$$

i.e. $$16 = 0.5 I_1 + 2 I_2 \qquad (1)$$

For loop 2:

$$E_2 = I_2 R_2 - (I_1 - I_2) R_2$$

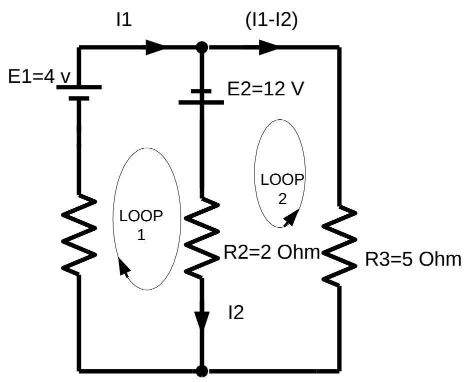


Fig.12.32

Note that since loop 2 is in the opposite direction to current $(I_1 - I_2)$the volt drop across R3 (i.e$(I_1 - I_2)\ R_3$) is by convention negative.

Thus

$$12 = 2I - 5(I_1 - I_2) i.e. 12 = -5I_1 + 7I_2 \qquad (2)$$

3 Solving equations (1) and (2) to find I_1 and I_2

$$10 \times (1) \text{ gives } 160 = 5I_1 + 20I_2 \qquad (3)$$

$$(2)+(3) \text{ gives } 172 = 27I_2 \qquad I_2 = \frac{172}{27} = 6.37A$$

From (1): 16 = 0.5 I_1 +2 (6.37)

$$I_1 = \frac{16 - 2(6.37)}{0.5} = 6.52A$$

Current flowing in $R_3 = I_1 - I_2$ =6.52 -6.37= 0.15 A

Problem 19. For the bridge network shown in Fig.12.33 determine the currents in each of the resistors.

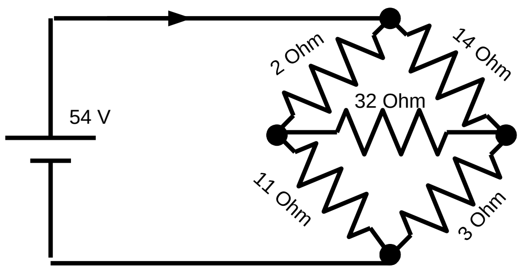


Fig.12.33

Let the current in the 2 Ω resistor be I_1, Then by Kirchhoff's current law, the current in the 14Ω resistor is $(I-I_1)$. Let the current in the 32Ω resistor be I_2 as shown in Fig.12.34.Then the current in the 11Ω resistor is (I_1-I_2).and that in the 3Ω resistor is $(I - I_1 + I_2)$ Applying Kirchhoff's voltage law to loop 1 and moving in a clockwise direction as shown in Fig 41 gives:

$$54= 2 I_1 +11(I_1-I_2) \quad \text{i.e.} \quad 13 I_1 - 11 I_2 =54 \tag{1}$$

Applying Kirchhoff's voltage law to loop 2 and moving in a clockwise direction as shown in Fig.12.34 gives:

$$0 = 2I_1 + 32I_2 - 14(I - I_1)$$

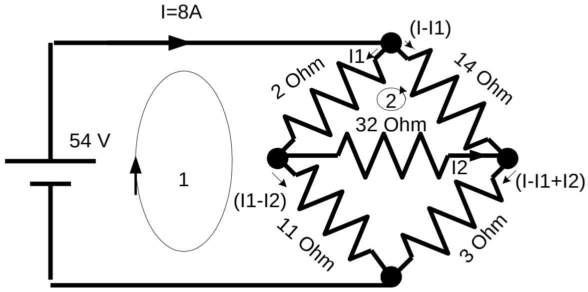


Fig.12.34

However $I = 8A$

Hence $\quad 0 = 2I_1 + 32I_2 - 14(8 - I_1) \quad$ i.e. $\quad 16I_1 - I_2 + 32I_2 = 112 \qquad (2)$

Equations (1) and (2) are simultaneous equations with two unknowns, I_1 and I_2.

$$16 \times (1)\ gives \qquad 208I_1 - 176I_2 = 864 \tag{3}$$

$$13 \times (2)\ gives \qquad 208I_1 + 416I_2 = 1456 \tag{4}$$

$$(4)-(3)\ gives \qquad 592I_2 = 592$$

$$I_2 = 1A$$

Substituting for I_2 in (1) gives:

$$13I_1 - 11 = 54$$

$$I_1 = \frac{65}{13} = 5$$

Hence,

the current flowing in the 2Ω resistor= $I_1 = 5$

the current flowing in the 14Ω resistor= $I - I_1 = 8 - 5 = 3A$

the current flowing in the 32Ω resistor= $I_2 = 1A$

the current flowing in the 11Ω resistor= $I_1 - I_2 = 5 - 1 = 4A.and$

the current flowing in the 3Ω resistor= $I - I_1 + I_2 = 8 - 5 + 1 = 4A$

C. FURTHER PROBLEMS ON DC CIRCUIT THEORY

(a) SHORT ANSWER PROBLEMS

1. State Ohm 's law.
2. What is a passive network?
3. What is an active network?
4. Name three characteristics of a series circuit.
5. Name three characteristics of a parallel circuit.
6. What is a potentiometer?
7. Define internal resistance and terminal pd as applied to a voltage source.
8. State Kirchhoff's current law.
9. State Kirchhoff's voltage law.
10. State, in your own words, the superposition theorem.

(a) MULTI-CHOICE PROBLEMS

1. If two 4Ω resistors are placed in series the effective resistance of the circuit is (a) 8Ω; (b) 4Ω; (c) 2Ω; (d) 1Ω
2. If two 4Ω resistors are placed in parallel the effective resistance of the circuit is (a) 8Ω; (b) 4Ω; (c) 2Ω; (d) 1Ω
3. With the switch in Fig12.35 closed the ammeter reading will indicate a) 108A; (b) $\frac{1}{3}A$ (c) A; (d) $4\frac{3}{5}$ A.

4. A 6 Ω resistor is connected in parallel with the three resistors of Fig 12.35 With the switch closed the ammeter reading will indicate:
   a) $\frac{3}{4}$ A; (b) 4 A; (c) $\frac{1}{4}$ A; (d) $1\frac{1}{3}$ A.
5. A 10 Ω resistor is connected in parallel with a 15 Resistor and the combination is connected in series with a 12 Ω resistor. The equivalent resistance of the circus is (a) 37Ω; (b) 18Ω; (c) 27Ω; (d) 4Ω

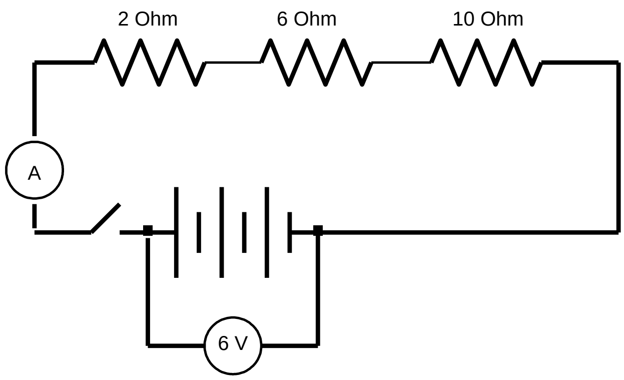


Fig.12.35

6. The terminal voltage of a cell of emf 2 V and internal resistance 0.1 Ω when supplying a current of 5 A will be (a) 1.5V; (b) 2V; (c) 1.9 V; (d) 2.5V.
7. The effect of connecting an additional parallel load to an electrical supply source is to iincreas the (a) resistance of the load; (b) voltage of the source; (c) current taken from source; (d) pd across the load.
8. The equivalent resistance when a resistor of ¼ Ω is connected in parallel with a 1/5 Ω resistor is (a) 1/9Ω ; (b) 9Ω.
9. Which of the following statements is true?
   For the junction in the network shown in Fig12.36:

(a) $I_5 - I_4 = I_3 - I_2 + I_1$

(b) $I_1 + I_2 + I_3 = I_4 + I_5$

(c) $I_2 + I_3 + I_5 = I_1 + I_4$

(d) $I_1 - I_2 - I_3 - I_4 + I_5 = 0$

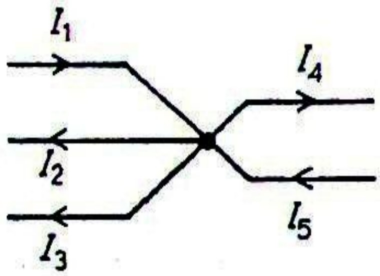
10. Which of the following statements is true?
    For the circuit shown in Fig.12.37:

(a) $E_1 + E_2 + E_3 = Ir_1 + Ir_2 + Ir_3$

(b) $E_2 + E_3 - E_1 = I(r_1 + r_2 + r_3) = 0$

(c) $I(r_1 + r_2 + r_3) = E_1 - E_2 - E_3$

(d) $E_2 + E_3 - E_1 = Ir_1 + Ir_2 + Ir_3$



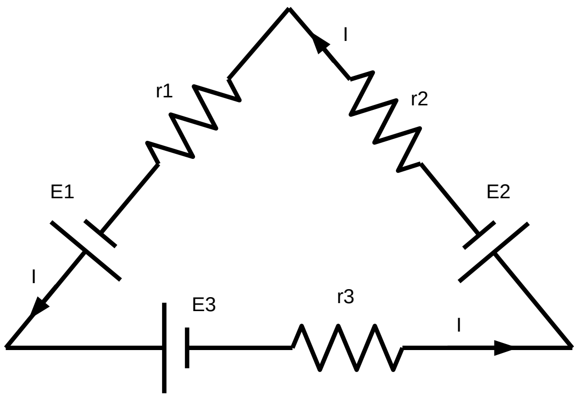


Fig. 2.36 Fig.12.37

(c) CONVENTIONAL PROBLEMS

Ohm's law

1. Determine what voltage must be applied to a 2 kΩ resistor in order that a current of 10 mA may flow. [20V]
2. The hot resistance of a 240 V filament lamp is 960 Ω. Find the current taken by the lamp and its power rating. [0.25A; 60W]
3. Determine the pd across a 240Ω resistance when 12.5 mA is flowing through it. [3V]
4. Find the resistance of an electric fire which takes a maximum current of 13 A from a 240 V supply. Find also the power rating of the fire. [18.46Ω]; 3.12kW]
5. What is the resistance of a coil which draws a current of 80 Ma from a 120 V supply? 1.5kΩ]
6. Find the equivalent resistance when the following resistances are connected in series, and (b) in parallel.

(i) 3Ω and 2Ω; (ii) 20kΩ and 40kΩ;

(iii) 4Ω, 8Ω and 16Ω; (iv) 800kΩ; 4kΩ and 1500kΩ

$$\begin{bmatrix} (a)(i)5\Omega;(ii)60K\Omega;(iii)28\Omega;(iv);6.3K\Omega \\ (b)(i)1.2\Omega;(ii)(40/3)K\Omega;(iii)(16/7)\Omega;(iv)461.5\Omega \end{bmatrix}$$

7. if four similar lamps are connected in parallel and the total resistance of the circuit is 150Ω, find the resistance of one lamp. [600Ω]

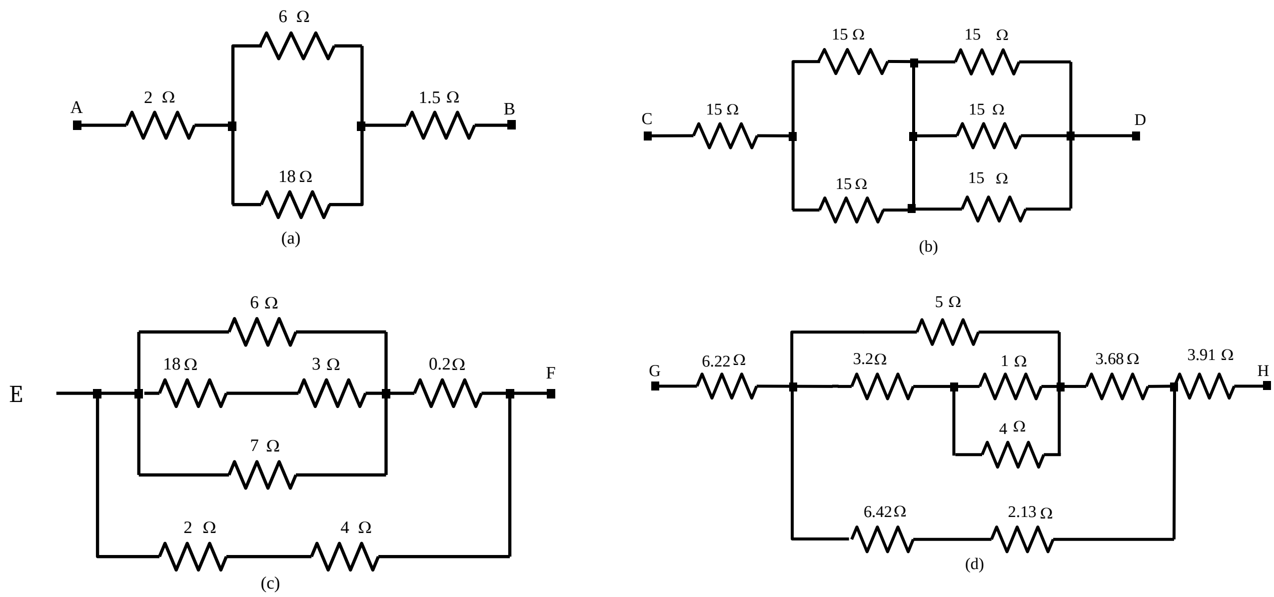


Fig.12.38

8. An electric circuit has resistances of 2.41 Ω, 3,57Ω and 5.82Ω connected in parallel. Find (a) the total circuit conductance, and (b) the total circuit resistance.[(a) 0.867S; (b) 1.154Ω]
9. Find the total resistance between terminals A and B of the circuit shown in the Fig.12.38 (a) [8Ω]
10. Find the equivalent resistance between terminals C and D of the circuit shown in the Fig. 12.38 (b) [27.5Ω]
11. Determine the equivalent resistance between terminals E and F of the circuit shown in the Fig. 12.38 (c) [2Ω]
12. Find the equivalent resistance between terminals G and H of the circuit shown in the Fig. 12.38 (d) [13.62Ω]
13. State how four 1Ω resistors must be connected to give an overall resistance of:

a $\frac{1}{4}\Omega$ (b) $1\frac{1}{3}\Omega$; (c) 1Ω; (d) $2\frac{1}{2}\Omega$

(a) Four in parallel, (b) Three in parallel, in series with one
(c) Two in parallel, in series with another two in parallel
(or two in series, in parallel with another two in series)
(d) Two in parallel, in series with two in series.

Currents and pd's in series-parallel arrangements

14. Resistors of 20Ω , 20Ω and 30 Ω are connected in parallel. What resistance must be added in series with the combinations to obtain a total resistance of 10Ω if the complete circuit expends a power of 0.36 kW, find the total current flowing? [2.5Ω; 6A]
15. (a)Calculate the current flowing in the 30Ω resistor shown in Fig. 12.39.

(A) What additional value of resistance would have to be placed in parallel with the 20Ω and 30Ω resistors to change the supply current to 8A, the supply voltage remaining constant. [(a) 1.6 A; (b) 6Ω]

16. For the circuit shown in Fig.12.40 find (a) V_1 ; (B) V_2; without calculating the current flowing. [(a) 30 V; (b) 42 V]

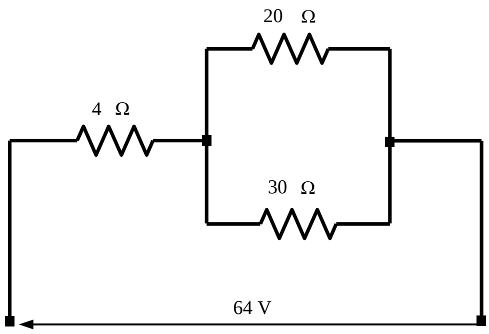


Fig.12.39

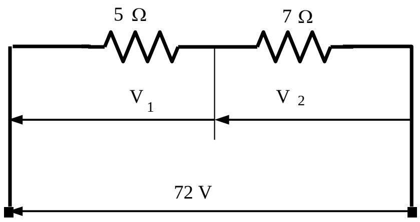
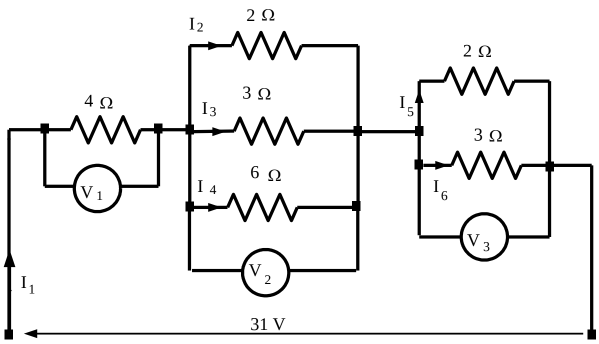


Fig.12.40

17. Determine the currents and voltage indicated in the circuit shown in Fig.12.41.
$I_1 = 5A; I_2 = 2.5A I_3 = 1\frac{2}{3}A, I_4 = \frac{5}{6}A$
$I_5 = 3A, I_6 = 2A, V_1 = 20V, V_2 = 5V, V_3 = 6V$



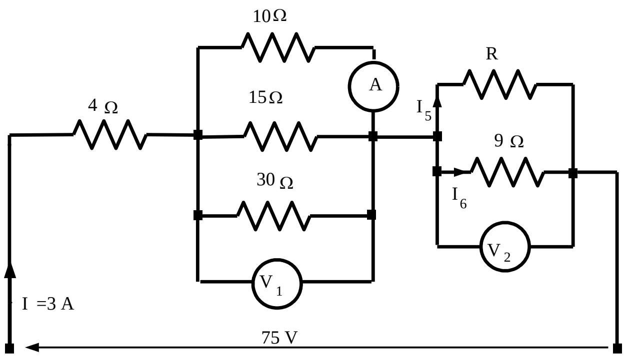


Fig. 12.41 Fig.12.42

18. Fig.12.42 shows part of an electric circuit. Find the value of resistor R and the reading on the ammeter and voltmeters. [R=18 Ω; 1.5A; V_1=15V; V_2=18V]

19. A resistor R_x ohms is connected in series with two parallel connected resistors each of resistance 8Ω. When the combination is connected across a 280 V supply the power taken by each of the 8Ω resistors is 392 W. Calculate (a) the resistance of R_x and (b) the single resistance which would take the same power as the series parallel arrangement. [(a) 16 Ω; (b) 20Ω] [1.8 A]
20. Find current I in Fig.12.43.

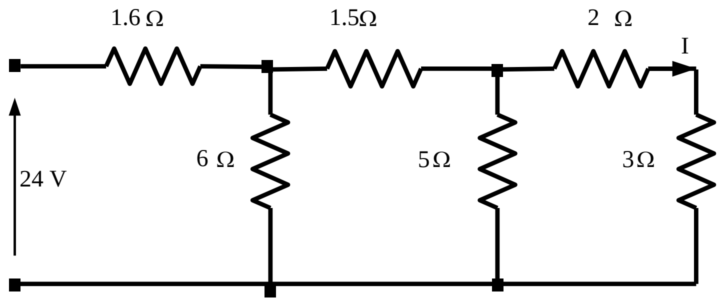


Fig.12.43

Internal resistance

21. A cell has an internal resistance of 0.06 Ω and emf of 2.18V.Find the terminal voltage if it delivers (a) 0.5 A, (b) 1A, (c) 20 A. [(a) 2.15 V; (b) 2.12 V; (c) 0.98 V]
22. A battery of emf 18 V and internal resistance 0.8Ω supplies a load of 4 A. Find the voltage at the battery terminals and the resistance of the load. [14.8 V; 3.7Ω]

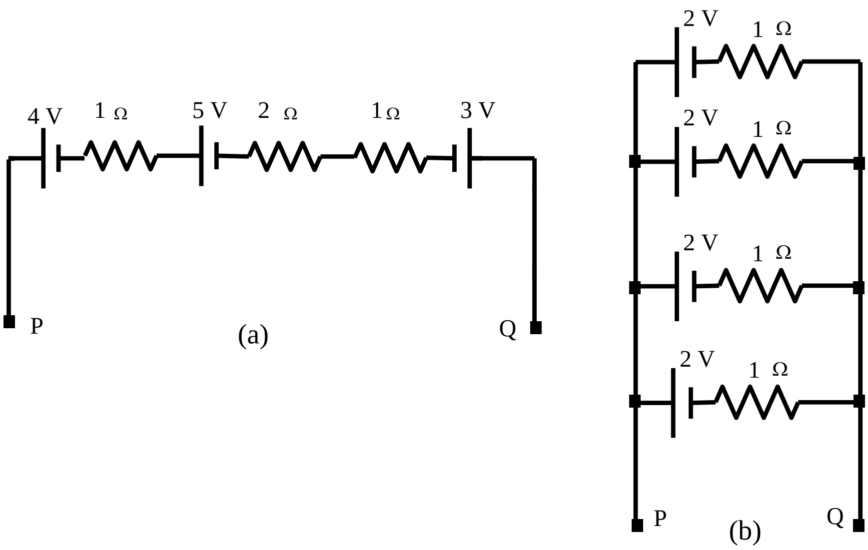


Fig.12.44

23. For the circuit shown Fig.12.44 the resistors represent of the batteries. Find, in each case ,(a) the total emf across PQ and (b) the total equivalent internal resistances of the batteries. [(a) (i) 6V, (ii) 2V; (b) (i) 4Ω, (ii)0.25 Ω]
24. The voltage at the terminals of a battery is 52 V when no load is connected and 48.8 V when a load taking 80A is connected. Find the internal resistance of the battery. What would be the terminal voltage when a load taking 20 A is connected? [0.04Ω; 51.2V]
25. A battery of emf 36.9 V and internal resistance 0.6Ω is connected to a circuit consisting of a resistance of 1.5 Ω in series with two resistors of 3Ω and 6Ω in parallel. Calculate the total current in the circuit, the current flowing through the 6Ω resistor, the battery terminal pd and the volt drop across each resistor.
[9A; 3A; 31.5 V ; 13.5V; 18V; 18 V]
26. A battery consists of four cells connected in series , each having an emf of 1.28 V and an internal resistance of 0.1Ω. Across the terminals of the battery are two parallel resistors, R_1 = 8Ω and R2= 24Ω . Calculate the current taken by each of the resistors and the energy dissipated in the resistances, in joules , if the current flows for 3 1/2min. [I_1 = 0.6 A; I_2 = 0.2A; W= 806.4J]
27. In Fig.12.45, find the total resistance measured between the points A and B .If a battery of emf 80 V and internal resistance 1Ω is connected across AB , find the current in each resistor and the pd across R_3.

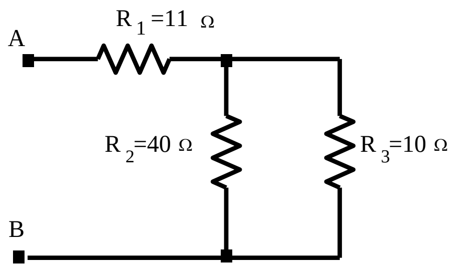
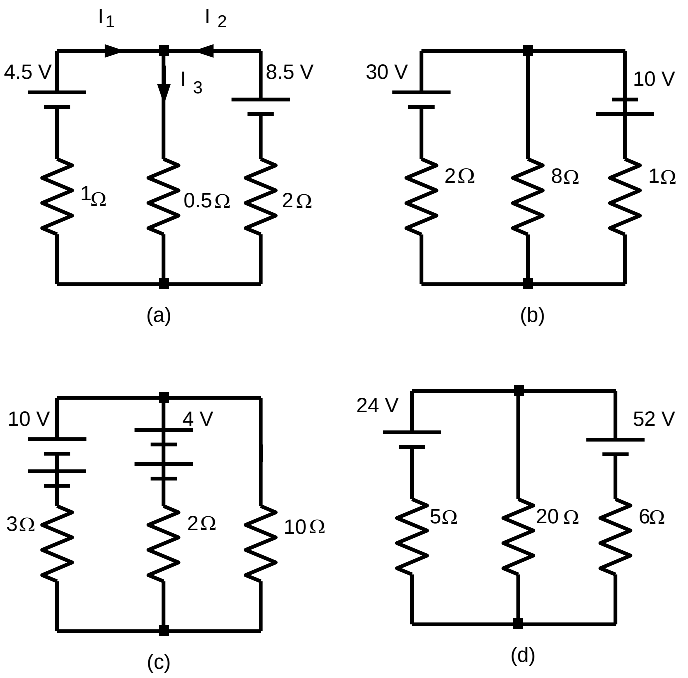


Fig.12.45

19 Ω; I_1 = 4A; I_2 = 0.8A; I_3= 3.2 A; 32V

Superposition theorem

28. Use the superposition theorem to currents I_1, I_2 and I_3 of Fig.12.46(a).
[I_1 = 2A; I_2 = 3A; I_3= 5A;]
29. Use the superposition theorem to current in the 8Ωresistor of Fig.12.46(b)
[0. 385 A]



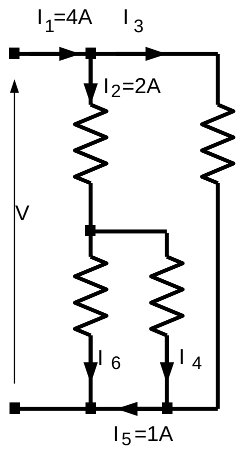


Fig. 12.46 Fig.12.47

30. Use the superposition theorem to find the current in each branch of the network showin in Fig.12.46(c) 10V battery discharges at 1. 429 A; 4V battery charges at 0. 857 A; Current through 10Ω resistor is 0.572 A
31. Use the superposition theorem to determine the current in each branch of the arrangement shown in Fig.12.46(d) . 24 V battery charges at 1. 664A; 4V battery discharges at 3. 280 A; Current through 20Ω resistor is 1.616A.

Kirchhoff's laws

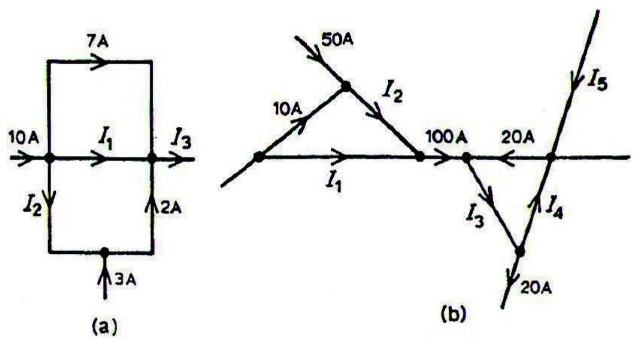
32. Find currents I_3 I_4 and I_6 in Fig.12.47

[I_3 = 2A; I_4 = -1A; I_6 =3A]

33. For the networks shown in Fig.12.48, find the values of the currents marked.

(a) I_1 = 4A; I_2 = -1A; I_3=13A.

(b) I_1 = 40A; I_2 = 60A; I_3=120A
I_4 = 100A; I_5= -80A.



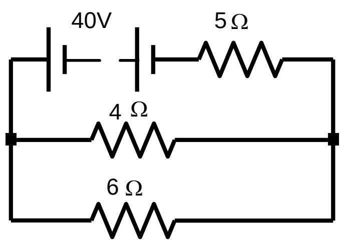


Fig. 12.48 Fig.12.49

34. Use Kirchhoff's laws to find the current flowing in the 6Ω resistor of Fig.12.49 and the power dissipated in the 4 Ω resistor. [2.162 A; 42.07 W]
35. Repeat Problem 28 to 31 using Kirchhoff's laws instead of the superposition theorem.
36. Find the current flowing in the 3Ω resistor for the network shown in Fig.12.50 (a). Find also the pd across the 10 Ω and 2Ω resistors.

[2.715A; 7.410 V; 3, 948V]

37. For the network shown in Fig.12.50 (b) find: (a) the current in the battery; (b) the current in

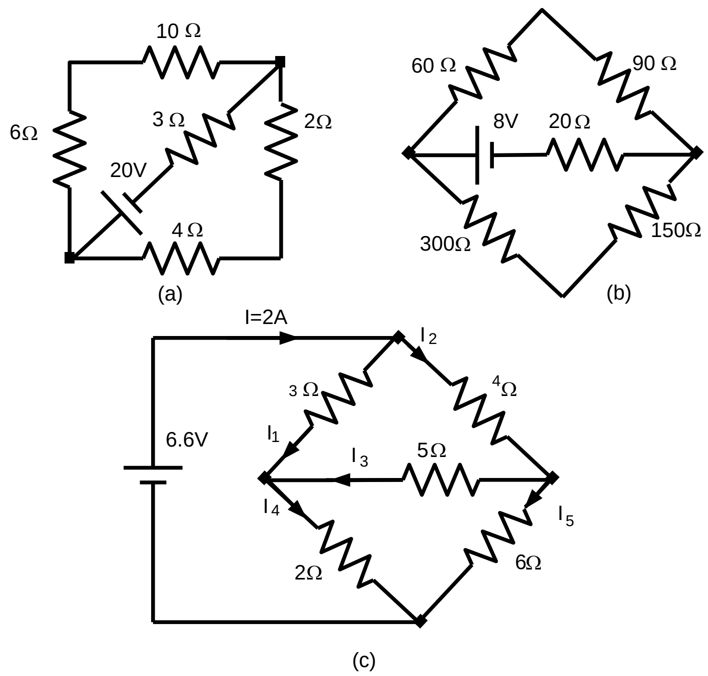


Fig.12.50

the 300Ω resistor; (c) the current in the 90Ω resistor; (d) the power dissipated in the 150Ω resistor

(a) 60.38 mA;(b) 15.10 mA:
(c) 45.28 mA; (d) 35.20 Mw

38. For the bridge network shown in Fig 57 (c) , find the current I_1 to I_5

I_1 = 1.25 A; I_2 = 0.75 A; I_3=0.15 A
I_4 =1. 40A; I_5 = 0.60A;

3 Capacitors and Capacitance

1. WORKED PROBLEMS ON CAPACITORS AND CAPACITANCE

(a) ELECTRIC FLUX DENSITY AND ELECTRIC FIELD STRENGTH PROBLEMS

Problem 1. two parallel rectangular plates measuring 20cm by 40 cm carry an electric charge of 0.2 μC. Calculate the electric flux density. If the plates are spaced 5mm apart and the voltage between them is 0.25 kV determine the electric fielded strength

Charge Q= 0.2 μC = 0.2 X 10^{-6} C;

Area A =20 cm x 40cm =800 cm^2= 800 X 10^{-4} m2

Electric flux density $D = \frac{Q}{A} = \frac{0.2\times10^{-6}}{800\times10^{-4}} = \frac{0.2\times10^{4}}{800\times10^{6}} = \frac{2000}{800}\times10^{-6}$

= 2.5 μC/m^2

Voltage V = 0.25 kV=250V; Platw spacing , d = 5mm = 5 x 10^{-3}m.

Electric field strength $E = \frac{V}{d} = \frac{250}{5\times10^{-3}} = 50Kv/m$

problem 2. The flux density between two plates separated by mica of relative permittivity 5 is 2 μC/m^2 . Find the voltage gradient between the plates.

Flux density D= 2 μC/m^2 = 2x 10^{-6} μC/m^2 ; ε_0= 8.85 x 10^{-12} F/m ;. $\varepsilon_r = 5$

$\frac{D}{E} = \varepsilon_0\varepsilon_r$

Hence voltage gradient $E = \frac{D}{\varepsilon_0\varepsilon_r}$

$= \frac{2\times10^{-6}}{8.85\times10^{-12}\times5}$ V/m = 45.2 kV/m

problem 3. Two parallel plates having a pd of 200 V between them are spaced 0.8 mm apart. What is the electric field strength? Find also the flux density when the dielectric between the plates is (a) air

and (b) polythene of relative permittivity 2.3.

Electric field strength $E = \frac{V}{d} = \frac{200}{0.8\times10^{-3}}$ = 250 kV/m

(a) For air $= \varepsilon_r = 1$

$$\frac{D}{E} = \varepsilon_0 \varepsilon_r$$

Hence electric flux density $D = E\varepsilon_0\varepsilon_r$

$250 \times 10^3 \times 8.85 \times 10^{-12} \times 1G/m^2$

$$= 2.213\mu G/m^2$$

(b) For polythene $\varepsilon_r = 2.3$.

electric flux density $D = E\varepsilon_0\varepsilon_r$ $250 \times 10^3 \times 8.85 \times 10^{-12} \times 2.3C/m^2$

$$= = 2.213\mu G/m^2$$

Further problems on electric flux density and electric field strength may be found in section C (c) , Problems 1 to 6 , page 44

(b) $a = cv$ problems

problem 4 . (a) Determine the pd across a 4μF capacitor when charged with 5 mC . (b) Find the charge on a 50 pF capacitor when the voltage applied to it is 2 kV.

(a) $C = 4\mu F = 4 \times 10^{-6} F; Q = 5mC = 4 \times 10^{-3} C$

Since $C = \frac{Q}{V} then V = \frac{Q}{C} = \frac{5 \times 10^{-3}}{4 \times 10^{-6}} = \frac{5 \times 10^{6}}{4 \times 10^{3}} = \frac{5000}{4}$

Hence pd $= 1250 V or 1.25 Kv$

$C = 50pF = 50 \times 10^{-12} F; V = 2kV = 2000V$

$$Q = CV = 50 \times 10^{-12} \times 2000 = \frac{5 \times 2}{10^8} = 0.1 \times 10^{-6}$$

Hence charge = 0.1 μC

Problem 5. A direct current of 4 A flows into a previously uncharged 20 μF capacitor for 3 ms. Determine the pd between the plates.

$I = 4A; G = 20\mu F = 20 \times 10^{-6} F; t = 3ms = 3 \times 10^{-3} s$

$Q = It = 4 \times 3 \times 10^{-3} C$

$$\frac{Q}{C} = \frac{4 \times 3 \times 10^{-3}}{20 \times 10^{-6}} = 0.6 \times 10^3 = 600V$$

Hence, the pd between the plates is 600V

Problem 6. A 5 μF capacitor is charged so that the pd between its plates is 800V. Calculate how long the capacitor can provide an average discharge current of 2Ma

$C = 5\mu F = 5 \times 10^{-6} F; V = 800V; I = 2Ma = 2 \times 10^{-3} A$

$Q = CV = 5\times10^{-6}\times800 = 4\times10^{-3}C$

Also,

Q= it Thus, $t = \frac{Q}{I} = \frac{4\times10^{-3}}{2\times10^{-3}} = 2S$

Hence the capacitor can provide an average discharge current of 2 mA for 2s

(c) PARALLEL PLATE CAPACITOR PROBLEMA

Problem 7. (a) A ceramic capacitor has an effective plate area of 4 cm^2 separated by 0.1 mm of ceramic of relative permittivity 100. Calculate the capacitance of the capacitor in picofarads. (b) If the capacitor in part (a) is given a charge of 1.2 μC what will be the pd between the plates?

(a) Area A=4 cm^2=4 x $10^{-4}m^2$; d =0.1mm = 0.1 x 10^{-3}m;

$\varepsilon_0 = 8.95\times10^{-12} F/m; \varepsilon_r = 100$

Capacitance $C = \frac{\varepsilon_0\varepsilon_r A}{d}$ = FARADS $= \frac{8.85\times10^{-12}\times100\times4\times10^{-4}}{0.1\times10^{-3}}F$

$\frac{8.85\times4}{10^{10}}F = \frac{8.85\times4\times10^{12}}{10^{10}}pF = 3540\,pF$

(b) Q =CV thus $V = \frac{Q}{C} = \frac{1.2\times10^{-6}}{3540\times10^{-12}}V = 339V$

Problem 8. A waxed paper capacitor has two parall plates
each of effective area 800 cm^2 .If the capacitance of the capacitor is 4425 pF determine the effective thickness of the paper if its relative permittivity is 2.5.

A= 800 cm^2= 800 x$10^{-4}m^2$ = 0.08 m^2; C= 4425 X 10^{-12} F;

ε_0 =8.85 x 10^{-12} F/m; ε_r =2.5.

Since $C = \frac{\varepsilon_0\varepsilon_r A}{d}$

Then $d = \frac{\varepsilon_0\varepsilon_r A}{C}$

Hence, d = $\frac{8.85\times10^{-12}\times2.5\times0.08}{4425\times10^{-12}}$ =0.0004m

Hence the thickness of the paper IS 0.4 mm

Problem 9. A parallel plate capacitor has nineteen interleaved plates each
75mm by 75mm separated by mica sheets 0.2 mm thick. Assuming the relative
permittivity of the mica is 5

calculate the capacitance of the capacitor.

n=19 ; n-1=18; A= 75 x75 = 5625mm^2= 5625 x 10^{-6} m^2;
ε_r = 5; ε_0 = 8.85 x 10^{-12} F/m ; d = 0.2 mm = 0.2 x 10^{-3}m.

Capacitance $C = \dfrac{\varepsilon_0 \varepsilon_r A(n-1)}{d} = \dfrac{8.85 \times 10^{-12} \times 5 \times 5625 \times 10^{-6} \times 18}{0.2 \times 10^{-3}} F$

= 0.0224 μF or 22.4 Nf

Problem 10 . A capacitor is to be constructed so that its capacitance is 0.2 μF and to take a pd of 1.25 kV across its terminals. The dielectric is to be mica which after allowing a safety factor
has a dielectric strength of 50 MV/m .Find (a) the thickness of the mica needed and (b) meth area of a plate assuming a two plate construction . (Assume ε_r for mica to be 6.)

(a) Dilectric strength $E = \dfrac{V}{d}$ i.e. d $= \dfrac{V}{E} = \dfrac{1.25 \times 10^3}{50 \times 10^{16}}$ m =0.025 mm

(b) Capacitance $C = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$

Hence area A = $\dfrac{cd}{\varepsilon_0 \varepsilon_r} = \dfrac{0.2 \times 10^{-6} \times 0.025 \times 10^{-3}}{8.85 \times 10^{-12} \times 6} m^2$

= 0.094 16 m^2 = 941.6cm^2

(c) PROBLEMS ON CAPACITORS CONNECTED IN PARALLEL AND IN SERIES

Problem 11. Calculate the equivalent capacitance of two capacitors of 6 μF and 4 μF connected (a) in parallel and (b) in series.

(a) In parallel , equivalent capacitance C= $C_1 + C_2$ = 6 μF+4 μF=10 μF.

(b) In series , equivalent capacitance C is given by $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{C_2 + C_1}{C_1 C_2}$

i.e. $C = \dfrac{C_1 C_2}{C_1 + C_2} i.e. \dfrac{product}{sum}$

This formula is esed for the special case of two capacitors in series (which is similar to two resistors in parallel).

Thus $C = \dfrac{6 \times 4}{6+4} = \dfrac{24}{10} = 2.4 \mu F$

Problem 12. what capacitance must be connected in series with a 30 μF capacitor for the equivalent capacitance to be 12 μF?

Let C= 12 μF(the equivalent capacitance), C_1=30 μF and C_2 be the unknown capacitance.

For two capacitors in series $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$

Hence $$\frac{1}{C_2}=\frac{1}{C}+\frac{1}{C_1}=\frac{C_1-C}{C\,C}$$

$$C_2=\frac{CC_1}{C_1-C}=\frac{12\times 30}{30-12}=\frac{360}{18}=20\mu F$$

Problem 13. Capacitances of 1 μF 3 μF 5 μF and 6 μF are connected in parallel to a direct voltage supply of 100V. Determine (a) the equivalent circuit capacitance(b) the total charge and (c) the charge on each capacitor.

(a) The equivalent capacitance C for 4 capacitors in parallel is given by:

$C=C_1+C_2+C_3+C_4$

i.e. $C=1+3+5+6=15\ \mu F$

(b) Total charge $Q_T=CV$ where C is the equivalent circuit capacitance.

i.e. $Q_T=15\times 10^{-16}\times 100=1.5\times 10^{-3}C=1.5mC$

(c) (The charge on the 1 μF capacitor $Q_1=C_1V=1\times 10^{-6}\times 100=0.1mC$

The charge on the 3 μF capacitor $Q_2=C_2V=3\times 10^{-6}\times 100=0.3mC$

The charge on the 5 μF capacitor $Q_3=C_3V=5\times 10^{-6}\times 100=0.5mC$

The charge on the 6 μF capacitor $Q_4=C_4V=6\times 10^{-6}\times 100=0.6mC$

[Check : In a parallel circuit $Q_T=Q_1+Q_2+Q_3+Q_4$

$$Q_1+Q_2+Q_3+Q_4=0.1+0.3+0.5+0.6=1.5mC=Q_T$$

Problem 14. Capacitances of 3 μF 6 μF and 12 μF are connected in a series across a 350 V supply. Calculate (a) the equivalent circuit capacitance (b) the charge on each capacitor (c) the pd across each capacitor

The circuit diagram is shown in Fig 12.51

(a) The equivalent circuit capacitance C for three capacitors in series is given by:

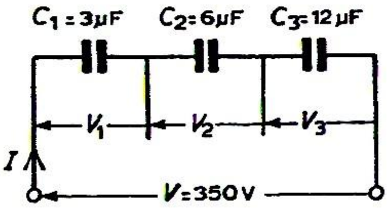


Fig.12.51

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{4+2+1}{12} = \frac{7}{12}$$

Hence the equivalent circuit capacitance

$C == \frac{12}{7} = 1\frac{5}{7}\,\mu F$

(a) Totan charge $Q_T = CV$

Hence $Q_T = \frac{12}{7} \times 10^{-6} \times 350 = 600\mu C$ or $0.6mC$

Since the capacitors are connected in series $0.6mC$ is the charge on each of them.

(b) The voltage across the 3 μF capacitor, $V_1 = \frac{Q}{C_1} = \frac{0.6\times10^{-3}}{3\times10^{-6}} = 200V$

The voltage across the 6 μF capacitor, $V_2 = \frac{Q}{C_2} = \frac{0.6\times10^{-3}}{3\times10^{-6}} = 100V$

The voltage across the 8 μF capacitor, $V_3 = \frac{Q}{C_3} = \frac{0.6\times10^{-3}}{12\times10^{-6}} = 50V$

[Check : In a series circuit $V = V_1 + V_2 + V_3$

$V_1 + V_2 + V_3 \quad 200 + 100 + 50 = 350V$ = supply voltage.]

In practice , capacitors are rarely connected in series unless they are of the same capacitance. The reason for this can be seen from the above problem where the lowest valued capacitor (I.e. 3 μF) has the highest pd across it (i.e. 200V) which means that if all the capacitors have an identical construction they must all be rated at the highest voltage.

Problem 15. For the arrangement shown in Fig.12.52 find (a) the equivalent capacitance of the circuit, (b) the voltage across QR and (c) the charge on each capacitor.

i. 2 μF in parallel with 3 g μF ives an equivalent capacitance of 2 μF +3 μF =5. μF The circuit is now as shown in Fig.12.53. The equivalent capacitance 0f 5 μF in series with 15 μF is given by

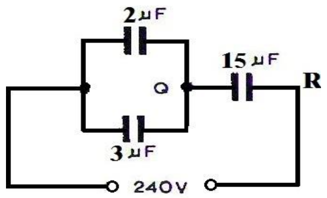


Fig.12.52

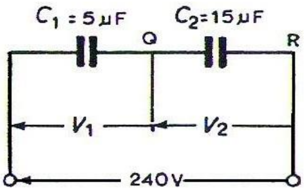


Fig.12.53

$\frac{5\times 15}{5+15}\ \mu F$ i.e. $\frac{75}{20} or 3.75$

(b)The charge on each of the capacitors shown in Fig.12.53 will be the same since they are connected in series. Let this charge be Q coulombs. Then $Q = C_1 + V_1 = C_2 V_2$ i.e $5V_1 = 15$, $V_1 = 3V_2$

Also $V_1 + V_2$=240V $V_1 + V_2 = 240V$ Hence $3V_2 + V_2 =$

Thus $V_2 = 60V and V_1 = 180V$

Hence the voltage across QR is 60V (c) The charge on the 15 μF capacitor is $C_2 V_2 = 15\times 10^{-6} \times 60 = 0.9mC$. The charge on the 2 μF capacitor is $2\times 10^{-6} \times 180 = 0.36mC$ The charge on the 3 μF capacitor is 3x 10^{-6}x 180= 0.54 m C $3\times 10^{-6} \times 180 = 0..54mC$

(e) PROBLEMS ON ENERGY STORED IN CAPACITORS

Problem 16. (a) Determine the energy stored in a 3 μF capacitor when charged to 400 V.

(b) Find also the average power developed if this energy is dissipated in a time of 10 μs.

(a) Energy stored $W = \frac{1}{2} CV^2$

$= \frac{1}{2} \times 3\times 10^{-6} \times 400^2 = \frac{3}{2} \times 16\times 10^{-2} = 0.24J$ Power = $\frac{energy}{time} = \frac{0.24}{10\times 10^{-6}} W = 24kW$

Problem 17. A 12 μF capacitor is required to store 4 J of energy . Find the pd to which the capacitor must be charged

Energy stored ² $W = \frac{1}{2} CV^2$ hence $V^2 = \frac{2W}{C}$

and V $V = \sqrt{\left(\frac{2W}{C}\right)} = \sqrt{\left(\frac{2\times 4}{12\times 10^{-6}}\right)} = \sqrt{\left(\frac{2\times 10^6}{3}\right)} = 816.5$

Problem 18. A capacitor is charged with 10 m C. If the energy stored is 1.2 J. Find (a) the voltage and (b) the capacitance.

Energy stored $W = \frac{1}{2}CV^2$ and $C = \frac{Q}{V}$

Hence $W = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV$

from which $V = \frac{2W}{Q}$

$$Q = 10mC = 10 \times 10^{-3} C and W = 1.2 j$$

Voltage $V = \frac{2W}{Q} = \frac{2 \times 1.2}{10 \times 10^{-3}} 0.24kV \quad or 240V$

Capacitance $C = \frac{Q}{V} == \frac{10 \times 10^{-3}}{240} F = \frac{10 \times 10}{240 \times 10^{3}} \mu F = 41.67 \mu F$

Problem 19. Compare briefly standard types of capacitor.

1.**Variable air capacitors.** These usually consist of two sets of metal plates (such as aluminium) one fixed , the other variable . The set of moving plates

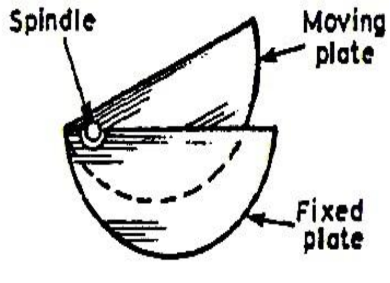


Fig.12.54

rotate on a spindle as shown by the end view in Fig.12.54. As the moving plates are rotated through half a revolution, the meshing , and therefore the capacitance , varies from minimum to a maximum value. Variable air capacitors are used in radio and electronic circuits where very low losses are required , or where a variable capacitance is needed.The maximum value of such capacitors is between 500 pF and 1000 pF.

2.Mica capacitors. A typical older type construction is shown in Fig.12.55.

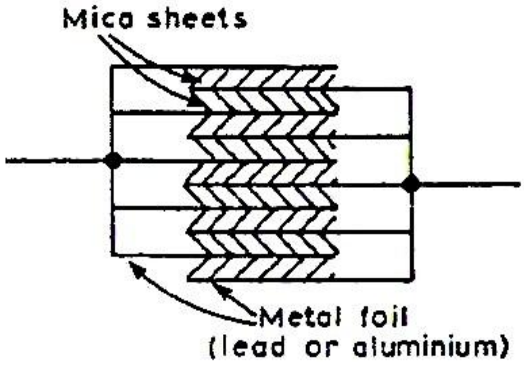


Fig.12.55

Usually the whole capacitor is Impregnated with wax and placed in a Bakelite case. Mica is easily obtained in thin sheets and is a good insulator.However , mica is expensive and is not used in capacitors above about 0.1 μF .A modified form of mica capacitor is the silvered mica type. The mica is coated on both sides with a thin layer of silver which forms the plates . Capacitance is stable and less likely to change with age.

Such capacitors have a constant capacitance with change of temperature, a high working voltage rating and a long service life and are used in high frequency circuits with fixed values of capacitance up to about 1000 pF.

3.Paper capacitors. Atypical paper capacitor is shown in Fig.12.56 where the length of the roll corresponds to the capacitance required. The whole is usually impregnated with oil or wax to exclude moisture, and then placed

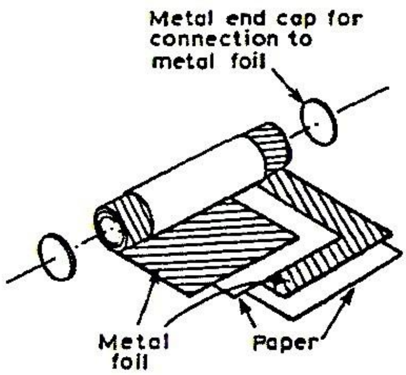


Fig.12.56

in a plastic or a luminium container for protection.Paper capacitors are made in various working voltages up to about 1 μF. Disadvantages of paper capacitor include variation in capacitance with temperature change and a shorter service life than most other types of capacitor

4. **Ceramic capacitors.** These are made in various forms, each type of construction depending on the value of capacitance required. For high values , a tube of ceramic material is used as shown in the cross section of Fig.12.57. For smaller values

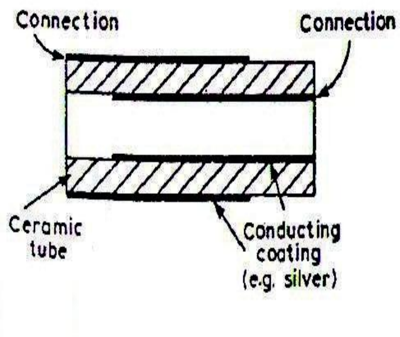


Fig.12.57

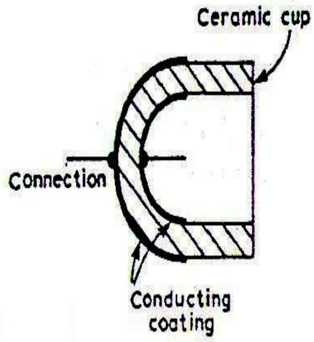


Fig.12.58

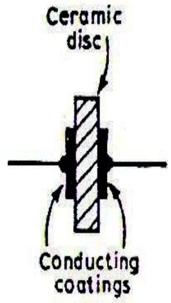


Fig.12.59

the cup construction is used as shown in Fig.12.58, and for still smaller values the disc construction shown in Fig.12.59 is used .Certain ceramic materials have a very high permittivity and this enables capacitors of high capacitance to be made which are of small physical size with a high working voltage rating . Ceramic capacitors are available in the range 1p F to 0.1 μF and may be used in high frequency electronic circuits subject to a wide range of temperatures.

5 **Plastic capacitors**. Some Plastic materials such as polystyrene and Teflon can be used as dielectrics. Construction is similar to the paper capacitor but using a plastic film instead of paper. Plastic capacitors operate well and conditions of high temperature provide a precise value of capacitance, Avery long service life and high reliability.

6. Elecrolytic capacitors. Construction is similar to the paper capacitor with aluminums foil used for the plates and with a thick absorbent material, such as paper, impregnated with an electrolyte (ammonium borate), separating the plates. The finished capacitor is usually assembled in an aluminum container and hermetically sealed. Its operation depends on the formation of a thin aluminum oxide layer on the positive plate by electrolytic action when a suitable direct potential is maintained between the plates. The oxide layer is very thin and forms the dielectric.(The absorbent paper between the plates is a conductor and does not act as a dielectric.) Such capacitors must always be used on dc and must be connected with the correct polarity; if this is not done the capacitor will be destroyed since the oxide layer will be destroyed. Electrolytic capacitors are manufactured with working voltage from 6 V to 500 V , although accuracy is generally not very high. These capacitors posses a much larger capacitance than other types of capacitors of similar dimensions due to the oxide film being only a few microns thick. The fact that they can be used only on dc supplies limit their usefulness.

C. FURTHER PROBLEMS ON CAPACITORS AND CAPACITANCE
(a) SHORT ANSWER PROBLEMS

1. Explain the term ‚electrostatistic'.
2. Complete the statements : Like charges……………..; unlike charges……….

3. How can an ‘ electric field ‘ be established between two parallel metal plates?
4. What is capacitance?
5. State the unit of capacitance.
6. Complete the statement : Capacitance= $\frac{\ldots}{\ldots}$
7. Complete the statement: (a) 1 μF = . . . F

   (b) 1 p F = . . . F

8. Complete the statement: Electric field strength E= $\frac{\ldots}{\ldots}$
9. Complete the statement: Electric flux density D= $\frac{\ldots}{\ldots}$
10. Draw the electrical circuit diagram symbol for a capacitor.
11. Name two practical examples where capacitance is present , although undesirable.
12. The insulating material separating the plates of a capacitor is called the…….
13. 10 volts applied to a capacitor results in a charge of 5 coulombs . What is the capacitance of the capacitor?
14. There 3 μF capacitors are connected in parallel. The equivalent capacitance is ……
15. There 3 μF capacitors are connected in series. The equivalent capacitance is ……
16. State an advantage of series connected capacitors.
17. Name three factors upon which capacitance depends.
18. What does ,relative permittivity , mean?
19. Define , permittivity of free space’
20. Name five types of capacitor commonly used.
21. Sketch a typical rolled paper capacitor.
22. Explain briefly the construction of a variable air capacitor.
23. State three advantages and one disadvantage of mica capacitors.
24. Name two disadvantages of paper capacitors.
25. Between what values of capacitance are ceramic capacitors normally available.
26. What main advantages to plastic capacitors posses?
27. Explain briefly the construction of an electrolytic capacitor.
28. What in the main disadvantage of electrolytic capacitors?
29. Name an important advantage of electrolytic capacitor.
30. What safety precautions should be taken when a capacitor is disconnected from a supply?
31. What in the meant by the ,dielectric strength ‘ of a material?
32. State the formula used to determine the energy stored by a capacitor.

(a) MULTI –CHOICE PROBLEMS

1. Electrostatics is a branch of electricity concerned with
   (a) energy flowing across a gap between conductors;

(b) charges at rest;
(c) charges in motion
(d) energy in the form of changes.

1. The capacitance of a capacitor is the ratio
   (a) charge to pd between plates;
   (b) pd between plates to plate spacing;
   (c) pd between plates to thickness o dielectric;
   (d) pd between plates to charge.
3. The pd across a 10 μF capacitor to charge it with 10 mC is
   (a) 100V; (b) 1k V; (c) 1V; (d) 10V.
4. The charge on a 10 μF capacitor when the voltage applied to it is 10 KV is:
   (a) 100 μC; (b) 0.1C; (c) 0.1 μC; (d) 0.01 μC.
5. Four 2 μF capacitors are connected in parallel. The equivalent capacitance is
   (a) 8 μF; (b) 0.5 μF.
6. Four 2 μF capacitors are connected in series. The equivalent capacitance is
   (a) (a)8 μF; (b) 0.5 μF.
7. State which of the following is false.
   The capacitance of a capacitor
   (a) is proportional to the cross sectional area of the plates;
   (b) is proportional to the distance between the plates;
   (c) depends on the number of plates;
   (d) is proportional to the relative permittivity of the dielectric .
8. State which of the following statements is false.
   (a) An air capacitor is normally a variable type.
   (b) A paper capacitor generally has a shorter service life than most other types of capacitor.
   (c) An electrolytic capacitor must be used only a,c.suppllies.
   (d) Plastic capacitors generally operate satisfactorily under conditions of high temperature.
9. The energy stored in a 10 μF capacitor when charged to 500 V is
   (a) mJ (b) 0.025 μ J; (c) 1.25 J ;(d) 1.25 C.
10. The capacitance of a variable air capacitor is a maximum when
    (a) the movable plates half overlap the fixed plates ;
    (b) the movable plates are most widely separated from the fixed plates;
    (c) both sets of plates are exactly meshed;
    (d) the movable plates are closer to one side of the fixed plate than to the other.

(c) CONVETIONAL PROBLEMS

(Where appropriate take ε_0 as 8.85×10^{-12} F/m)

Electric flux density and electric field strength

1. A capacitor uses a dielectric 0.04 mm thick and operates at 30 V . What is the electric field strength across the dielectric at this voltage?

[750 kV/m].

2. A two plate capacitor has a charge of 25 C. If the effective area of each plate is 5 cm^2 find the electric flux density of the electric field.

[50 kC/ m^2].

3. A charge of 1.5 μC is carried on two parallel rectangular plates each measuring 60mm by 80mm. Calculate the electric flux density.If the plates are spaced 10mm apart and the voltage between them is 0.5 k V Determine the electric field strength.

[312,5 μC/m^2; 50 k V/m].

4. Two parallel plates are separated by a dielectric and charged with 10 μC. Given that the area of each plate is 50 cm^2, calculate the electric flux density in the dielectric separating the plates.

[2mC/ m^2].

5. The flux density between two plates separated by polystyrene of relative permittivity 2.5 is 5 μC/ m^2. Find the voltage gradient between the plates.

[226kV/m].

6. Two parallel plates having a pd of 250 V between them are spaced 1 mm apart. Determine the electric field strength. Find also the density when the dielectric between the plates is (a) air and (b) mica of relative permittivity 5.

$$\begin{bmatrix} 250\ kV/m (a) 2.213\ \mu C/m2 \\ (b) 11.063 \mu C/m2 \end{bmatrix}.$$

Q=CV problems

7. Find the charge on a 10 μF capacitor when the applied voltage is 250V.

[2.5mC].

8. Determine the voltage across a 1000 pF capacitor to charge it with 2 μC.

[2 kV].

9. The charge on the plates of a capacitor is 6mC when the potential between them is 2.4 k V .Determine the capacitance of the capacitor.

[2.5 μF].

10. For how long must a charging current of 2 A be fed to a 5 μF capacitor to raise the pd between its plates by 500V.

[1.25ms].

11. A direct current of A flows into a previously uncharged 5 μF capacitor for 1 ms. Determine the pd between the plates.

[2kV].

12. A 16 μF capacitor is charged at a constant current of 4 μA for 2 min. Calculate the final pd across the capacitor and the corresponding charge in coulombs.

[30 V; 480 μC].

13. A steady current of 10 A flows into a previously uncharged capacitor for 1.5 ms when the pd between the plates is 2 k V. Find the capacitance of the capacitor.

[7.5 μF].

Prallel plate capacitor

14. A capacitor consists of two parallel plates, each of area 0.01 m^2, spaced 0.1 mm in air Calculate the capacitance in picofarads.

[885pF].

15. A waxed paper capacitor has two parallel plates , each of effective area 0.2 m^2 .If the capacitance is 4000 pF determine the effective thickness of the paper if its relative permittivity is 2.

[0.885mm]

16. Calculate the capacitor of a parallel plate capacitor having 5 plates , each 30 mm by 20 mm and separated by a dielectric 0.75 mm thick having a relative permittivity of 2.3.

[65.14 pF].

17. How many plates has a parallel plate capacitor having a capacitance of 5 nF, if each plate is 40 mm aquare and each dielectric is 0,10 mm thick with a relative permittivity of 6.

[7].

18. A parallel plate capacitor is made from 25 plates , each 70mm by 120mm interleaved with mica of relative permittivity 5. If the capacitance of the capacitor is 3000 pF determine the thickness of the mica sheet.

[2.97mm].

19. A capacitor is constructed with parallel plates and has a value of 50 pF . What would be the capacitane of the capacitor if the plate area is doubled and the plate spacing is doubled?

[200 pF].

20. The capacitance of a parallel plate capacitor is 1000 pF. It has 19 plates , each 50mm by 30mm separated by a dielectric of thickness 0.40 mm. Determine the relative permittivity of the dielectric.

[1.67].

21. The charge on the square plates of a multiplate capacitor is 80 μC when the potential between them is 5 kV. If the capacitor has twenty –five plates separated by a dielectric of thickness 0.102 mm and relative permittivity 4.8, determine the width of a plate.

[40mm].

22. A capacitor is to be constructed so that its capacitance is 4250 pF and to operate at a pd of 100 V across its terminals. The dielectric is to be polythene (ε_r= 2.3)which, after allowing a safety factor , has a dielectric strength of 20 MV/m .Find (a) the thickness of the polythene needed, and (c) the area of a plate.

[(a) 0.005 mm; (b) 10.44cm^2].

Capacitors connected in parallel and in series

23. Capacitors of 2 μF and 6 μF are connected (a) in parallel and (b) in series. Determine the equivalent capacitance in each case.

[(a) 8 μF; (b) 1.5 μF].

24. Find the capacitance to be connected in series with a 10 μF capacitor for the equivalent capacitance to be 6 μF

[15 μF].

25. What value of capacitance would be obtained if capacitors of 0.15 μF and 0.1 μF are connected (a) in series and (b) in parallel.

[(a) 0.06 μF; (b)0.25 μF].

26. Two 6 μF capacitors are connected in series with one having a capacitance of 12 μF . Find the total equivalent circuit capacitance. What capacitance must be added in series to obtain a capacitance of 1.2 μF?

[2.4μF; 2.4μF].

27. Determine the equivalent capacitance when the following capacitors are connected (a) in parallel and (b) in series:

(i) 2μF,4μF and 8μF

(ii) 0.02μF, 0,05μF and 0.1`μF

(iii) 50 pF and 450 PF

(iv) 0.01μF and 200 PF

$$\begin{bmatrix} (a)\ (i)14\mu F, (ii)0.17\mu F, (\ iii)500pF, & (iV)0.0102\mu F \\ (b)(i)1\frac{1}{7}, (ii)0.0125\mu F, \quad (iii)45pF, & (iV)196.1pF \end{bmatrix}$$

28. For the arrangement shown in Fig.12.60 find (a) the equivalent circuit capacitance and (b) the voltage across a 4.5 μF capacitor.

[(a) 1.2 μF; (b)100 V].

29. Three 12 μF capacitors are connected in series across a 750 V supply. Calculate (a) the equivalent capacitance , (b) the charge on each capacitor and (c) the pd across each capacitor.

[(a) 4 μF; (b) 3 mC (c)250 V].

30. If two capacitors having capacitances 3 μF and 5 μF respectively are connected in series across a 240 V supply determine (a) the pd across each capacitor and (b) the charge on each capacitor.

[(a)150 V, 90 V;(b) 0.45 mC on each].

31. In Fig.12.61 capacitors P, Q , and R are identical and the total equivalent capacitance of the circuit is 3 μF . Determine the values of P, Q , and R.

[4.2 μF].

32. Capacitances of 4 μF ,8 μF and 16 μF are connected in parallel across a 200V supply. Determine (a) the equivalent capacitance, (b) the total charge and (c) the charge on each capacitor.

[(a) 28 μF, (b)5.6 mC, (c)0.8 mC, 1.6 mC, 3.2 mC].

33. A circuit consists of two capacitors P and Q in parallel , connected in series with another capacitor R. The capacitances od P, Q are 4 μF, 12 μF, and 8μF respectively. When the circuit in connected across a 300Vdc supply find (a) the

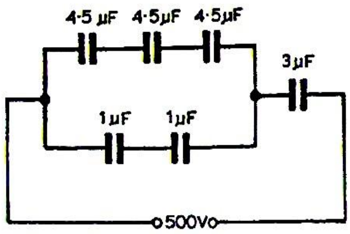


Fig. 12.60

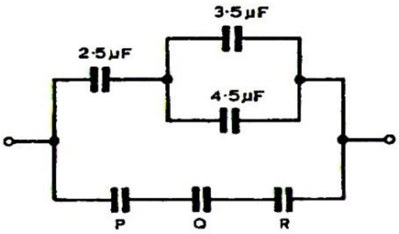


Fig.12.61

total capacitance of the circuit , (b) the pd across each capacitor and (c) the charge on each capacitor.

$$\begin{bmatrix} (a)5\frac{1}{3}\mu\text{F}, (\text{b})100 \text{ V across P}, & 100\ Vacross\ R, \\ (c)0{,}4\ mC\ on\ P, 1.2\ mC\ on\ Q & 1{,}6\ mC\ on\ R \end{bmatrix}$$

Energy stored in capacitors

34. When a capacitor is connected across a 200 V supply the charge is 4 μC. Find (a) the capacitance and (b) the energy stored.

[(a) 0.02 μF; (b)0.4 mJ].

35. Find the energy stored in a 10 μF capacitor when charged to 2 kV

[20J].

36. A3300 pF capacitor is required to store 0.5 mJ of energy. Find the pd to which the capacitor must be charged.

[550 V].

37. A capacitor is charged with 8 mC .If the energy stored is 0.4Jfind (a) the voltage and (b) the capacitance.

[(a) 100V, (b) 80 μF]

38. A capacitor ,consisting of two metal plates each of area 50cm^{2}and spaced 0.2 mm apart I air, is connected across a 120 V supply .Calculate (a) the energy stored, (b) the electric flux density and (c) the potential gradient.

[(a) 1.593 μJ; (b)5.31μC/m^2; (c) 600 kV/m]

39. A bakelite capacitor is to be constructed to have a capacitance of 0.04 31μF and to have a steady working potential of 1 kV maximum. Allowing a safe value of field stress of 25 MV/m find (a) the thickness of Bakelite required , (b) the area of plate required if the relative permittivity of Bakelite is 5, (c) the maximum energy stored by the capacitor and (d) the average power developed if this energy is dissipated in a time of 20 μs.

[(a) 0.04 mm; (b) 361 ,6cm^2; (c) 0.02 J; (d) 1 kW]

A. FORMULAE AND DEFINITIONS ASSOCIATED WITH MAGNETIC CIRCUITS

1. **A permanent magnet** is a piece of ferromagnetic material (such as iron ,nickel or cobalt) which has properties of attracting other pieces if these materials.
2. The area around a magnet is called the **magnetic field** and it is in this area that the effect of the **magnetic force** produced by the magnet can be detected.
3. **Magnetic fields** can be established by electric currents as well as by permanent magnets.
4. The **magnetic flux φ** is the amount of magnetic field (or the number of lines of force) produced by a magnetic source.
5. The **unit of magnetic flux** is the weber, **Wb**
6. **magnetic flux density *B*** is the amount of flux passing through a defined area that is perpendicular to the direction of the flux.
7. Magnetic flux density =, $\frac{magniflux}{area}$ i.e. B= φ/A
8. The **unit of flux density** is the tesla T , where 1 T= 1 Wb /m^2
9. **Magnetomotive force** (mmf)

F_m = NI ampere –turns (At)

where N= numbers of conductors (or turns)

I= current in ampers.

Since ‘turns has n units , the SI unit of mmf is the ampere , but to avoid any possible confusion ampere-turns’, (At), are used in this chapter.

10. **Magnetic fields strength,** or **magnetizing force**

$H = \frac{NI}{l}$ At / m where l mean length of flux path in metres.

Hence ,mmf NI= Nl At

11 For air , or any non- magnetic medium ,the ratio of magnetic flux density to magnetizing force is a constant , i.e. B/H =a constant. This constant is μ_0 , the **permeability of free space** (or the magnetic space constant) and is equal to

4π x 10^{-7} H/m.

Hence $\frac{B}{H} = \mu_0$

12. For **ferromagnetic mediums**:

$$\frac{B}{H} = \mu_0 \mu_x$$

where μ_r is the relative permeability , and is defined as

$$\frac{\textit{flux density in material}}{\textit{flux density in air}}$$

its value varies with the type of magnetic material and since μ_r is a ratio of flux densites, it has no units, From its definition , μ_r for air is 1.

13. $\mu_0\mu_r = \mu$, called the **absolute permeability.**

14. By plotting measured values of flux density B against magnetic field strength H , **a magnetization curve** (or B- H curve) is produced. For non- magnetic materials this is a straight line. Typical curves for four magnetic materials are shown on Fig.12.63.

15. The **relative permeability** of a ferromagnetic material is proportional to the slope of the B-H curve and thus varies with magnetic field strength. The approximate range of values of relative permeability μ_r for some common magnetic materials are: Cast iron μ_r = 100-250; Mild steel μ_r = 200- 800 Silicon iron μ_r = 1000- 5000; Cast steel μ_r = 300-900 Mumetal μ_r = 200-5000; Stalloy μ_r = 500-6000

16. The ,magnetic resistance ' of a magnetic circuit to the presence of magnetic flux is called **reluctance**. The symbol for reluctance is S (or R_m).

17. Reluctance $S = \frac{mmf}{\varphi} = \frac{Nl}{\varphi} = \frac{l}{\frac{B}{H}A} = \frac{l}{u_0 u_r A}$

18. The unit of **reluctance** is 1/ H (or H^{-1})or At /Wb .

19. For a series magnetic circuit having n parts, the total reluctance S is given by:

$S = S_1 + S_2 + \ldots + S_n$

(This is similar to resistors connected in series in an electrical circuit)

20. **Comparison between electrical and magnetic quantities**.

Electric circuit	Magnetic circuit
emf E (V)	mmf F_m (A)
current I (A)	flux φ (Wb)
resistance R (Ω)	reluctances (H^{-1})
$I = \frac{E}{R}$	$\varphi = \frac{mmf}{S}$
$R = \frac{pl}{A}$	$S = \frac{l}{u_0 u_r A}$

21. **Ferromagnetic materials** have a low reluctance and can be used as magnetic screens to prevent magnetic fields affecting materials within the screen.

22. **Hysteresis** is the 'lagging 'effect of flux density B whenever there are changes in the magnetic field strength H.

23. when an initially unmagnetised ferromagnetic material is subjected to a varying magnetic field strength H, the flux density B produced in the material varies as shown in Fig 1, the arrows indicating the direction of the cycle .Fig.12.62 is known as a hysteresis loop.

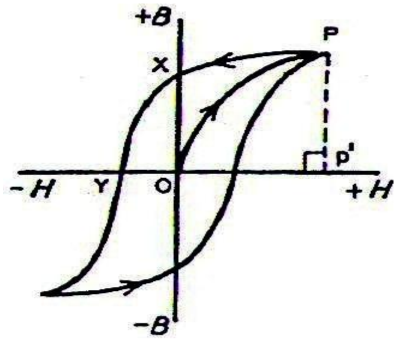


Fig.12.62

OX=residual flux density or remanence

OY=coercive force

PP= saturation flux density.

24. Hysteresis results in **a dissipation of energy** which appears as a heating of the magnetic material. The energy loss associated with hysteresis is proportional to the area of the hysteresis loop.

25. The area of a hysteresis loop varies with the type of material. The area, and thus the energy loss, is much greater for hard materials than for soft materials.

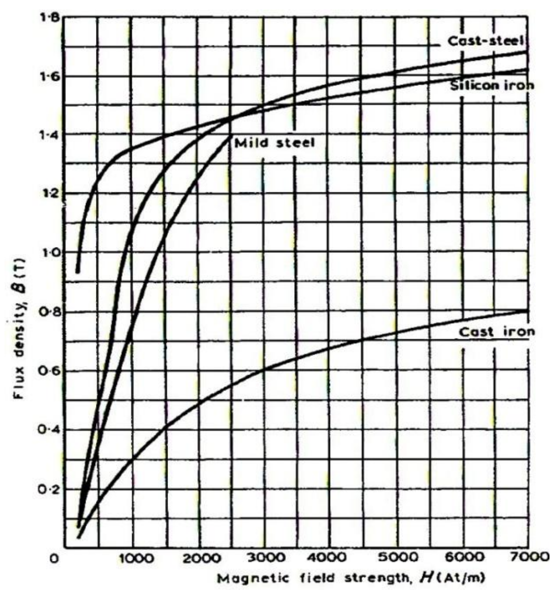


Fig.12.63

B. WORKED PROBLEMS ON MAGNETIC CIRCUITS

(a) MAGNETIC CIRCUITS QUANTITIES

Problem 1. A magnetic pole face has a rectangular section having dimensions 20 cm by by 10 cm . If the total flux emerging from the pole is 150μWb, calculate the flux density.

$$flux\,\varphi = 150\mu Wb = 150\times10^{-6}Wb$$

Cross sectional area $A = 20\times10 = 200cm^2 = 200\times10^{-4}m^2$

Flux density $B = \dfrac{\Phi}{A} = \dfrac{150\times10^{-6}}{200\times10^{-4}}$ =0.0075 T or 7.5 m T

Problem 2. A flux density of 1.2 T is produced in a piece of cast steel by a magnetizing force of 1250 At/m. Find the relative permeability of the steel under these conditions

For magnetic material :

$$B = \mu_0\mu_r\ \mathrm{H}$$

$$\mu_r = \frac{B}{\mu_0 H} = \frac{1.2}{(4\pi\times10^{-7})(1250)} = 764$$

Problem 3. Determine the magnetic field strength and the mmf required to produce a flux density of 0.25 T in an air gap of length 12 mm.

For air : $B = \mu_0 H$ (since $\mu_r = 1$)

Magnetic field strength $H = \dfrac{B}{\mu_0} = \dfrac{0.25}{4\pi\times10^{-7}} = 198940$ At/m

$$mmf = HI = 198940\times12\times10^{-3} = 2387At$$

Problem 4. A magnetic force of 8000 At /m is applied to a circular magnetic circuit of mean diameter 30 cm by passing a current through a coil wound on the circuit. If the coil is uniformly wound around the circuit and has 750 turns, find the current in the coil.

$$H = 8000At/m; 1 = \pi d\times30\times10^{-2}m; N = 750turns$$

Since $H = \dfrac{Nl}{N}$

then , $I = \dfrac{Hl}{N} = \dfrac{800\times30\times10^{-2}}{750}$

Thus, current I= 10.05 A

Problem 5. A coil of 300 turns is wound uniformly on a ring of non- magnetic material. The ring has a mean circumference of 40 cm and a uniform cross sectional area 4 cm². If the current in the coil is 5A,calculate (a) magnetic field strength ,(b) the flux density and (c) the total magnetic flux in the ring.

(a) Magnetic field strength $H = \frac{NI}{l} = \frac{300 \times 5}{40 \times 10^{-2}} = 3750 At/m$

(b) For a non- magnetic material $u_x = 1$, thus flux density $B = \mu_0 H$

$$4\pi \times 10^{-7} \times 3750 = 4.712 mT$$

(c) Flux $\Phi = BA = (4.712 \times 10^{-3})(4 \times 10^{-4}) = 1.885 \mu Wb$

Problem 6. Determine the reluctance of a piece of mumetal of length 150 mm , cross-sectional area 1800 mm² when the relative permeability is 4000. Find also the absolute permeability of the mumetal.

Reluctance $S = \frac{l}{\mu_0 \mu_r} = \frac{150 \times 10^{-3}}{(4\pi \times 10^{-7})(4000)(1800 \times 10^{-6})} = 16580/H$

Absolute permeability, $\mu = \mu_0 \mu_r (4\pi \times 10^{-7})(4000) = 5.027 \times 10^{-3} H/m$

Problem 7. An iron ring of mean diameter 10 cm is uniformly wound with 2000turns of wire. When a current of 0.25 A is passed through the coil a flux density of 0.4 T is set up in the iron.

Find (a) the magnetizing force and (b) the relative permeability of the iron under these conditions.

$1 = \pi d = \pi \times 10 cm = \pi \times 10 \times 10^{-2} m; N 2000 turns; I = 0.25A; B = 0.4T$.

(a) $H = \frac{NI}{l} = \frac{2000 \times 0.25}{\pi \times 10 \times 10^{-2}} = \frac{5000}{\pi}$ 1592 At/m

$$= \mu_0 \mu_r H. Hence \mu_r = \frac{B}{\mu_0 H} = \frac{0.4}{(4\pi \times 10^{-7})(1592)} = 200$$

Problem 8. A mild steel ring has a radius of 50 mm and a cross – sectional area of 400mm². A current of 0.5A flows in a coil wound uniformly around the ring and the flux produced is 0.1 m Wb . If the relative permeability at this value of current is 200 find (a) reluctance of the mild steel and (b) the number of turns on the coil.

$1 = 2\pi r = 2 \times \pi \times 50 \times 10^{-3} m; A = 400 \times 10^{-6} m^2; I = 0,5A$

$\Phi = 0.1 \times 10^{-3} Wb, \mu_x = 2000$

(a) Recultance

$$S = \frac{l}{\mu_0 \mu_r A} = \frac{2 \times \pi \times 50 \times 10^{-3}}{(4\pi \times 10^{-7})(200)(400 \times 10^{-6})} = 3.125 \times 10^6 / H$$

(b) $S = \frac{mmf}{\Phi}$

i.e $$mmf = S_\Phi$$
$$NI = S_\Phi$$

Hence $$N = \frac{S_\Phi}{I} = \frac{3.125\times10^6 \times 0.1\times10^{-3}}{0.5} = 625turns$$

Problem 9. A uniform ring of cast iron has a cross sectional area of 10 cm^2 and a mean circumference of 20 cm. Determine the mmf necessary to produce a flux of 0.3 m Wb in the ring .The magnetization curve for cast iron is shown on page 00.

$$A = 10cm^2 = 10\times10^{-4}m^2; 1 = 20cm = 0.2m; \Phi = 0.3\times10^{-3}Wb$$

flux density $B = \frac{\Phi}{A} = \frac{0.3\times10^{-3}}{10\times10^{-4}} = 0.3T$

From the magnetization curve for cast iron on page 50, when B = 0,3 T ,

H = 1000 At/ m

Hence mmf = Hl = 1000 x 0.2 = 20 At

A tabular method could have been used in this problem. Such a solution is shown below.

part of circuit	Material	ϕ Wb	A m²	$B=\frac{\phi}{A}$T	H from graph	l m	mmf Hl At
Ring	Cast iron	0.3×10^{-3}	10×10^{-4}	0.3	1000	0.2	200

Problem 10. From the magnetization curve for cast iron, shown on page 50, derive the curve of μ_r against H.

$$B = \mu_0\mu_r H, hence. \mu_r = \frac{B}{\mu_0 H} = \frac{1}{\mu_0}\times\frac{B}{H} = \frac{10^7}{4\pi}\times\frac{B}{H}$$

A number of co- ordinates are selected from the B-H curve and μ_r is calculated for each as shown in the following table.

B(T)	0.04	0.13	0.17	0.30	0.41	0.49	0.60	0. 68	0.73	0.76	0.79
H (At/m)	200	400	500	1000	1500	2000	30000	4000	5000	6000	7000
$\mu_r=\frac{10^7}{4\pi}\times\frac{B}{H}$	159	259	271	239	218	195	159	135	116	101	90

μ_r is plotted against H as shown in Fig12.64 . The curve demonstrates the change that occurs in the relative permeability as the magnetizing force increases.

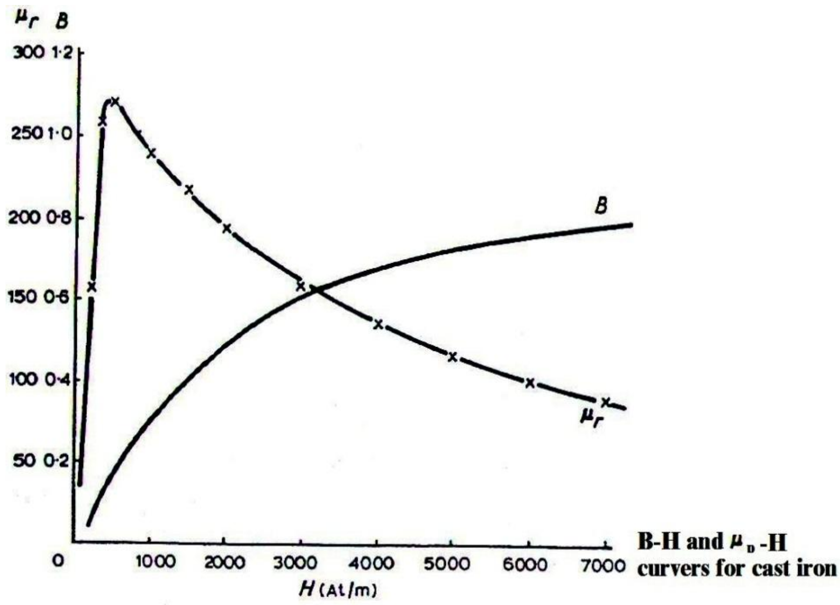


Fig. 12.64 **B-H and μp–H curves for cast iron**

(b) COMPOSITE SERIES MAGNETIC CIRCUITS

Problem 11. A closed magnetic circuit of cast steel contains a 6 cm long path of cross sectional area 1 cm^2 and 2 cm path of cross –sectional area 0,5 cm^2 .A coil of 200 turns is wound around the 6 cm length of the circuit and a current of 0.4 A flows. Determine the flux density in the 2cm path , if the relative permeability of the cast steel is 750.

For the 6 cm long path :

$$\text{Reluctance } S_1 = \frac{l_2}{\mu_0 \mu_r A} = \frac{6\times10^{-2}}{(4\pi\times10^{-7})(750)(1\times10^{-4})} = 6.366\times10^5 / H$$

For the 2 cm long path:

$$\text{Reluctance } \quad S_2 = \frac{l_2}{\mu_0 \mu_r A_2} = \frac{2\times10^{-2}}{(4\pi\times10^{-7})(750)(0.5\times10^{-4})} = 4.244\times10^5 / H$$

$$\text{Total circuit reluctance } S = S_1 + S_2 = (6.366+4.244)\times10^5 = 10.61\times10^5 / H$$

$$S = \frac{mmf}{\Phi}, \Phi = \frac{mmf}{S} = \frac{NI}{S} = \frac{200\times0{,}4}{10.61\times10^5} = 7.54\times10^5 Wb$$

$$\text{Flux density in the 2 cm path , } B = \frac{\Phi}{A} = \frac{7.45\times10^{-5}}{0.5\times10^{-4}} = 1.51T$$

Problem 12. A silicon iron ring of cross –sectional area 5 cm^2 has a radial air gap of 2 mm cut into it. If the mean length of the silicon iron path is 40 cm , calculate the magneto motive force to produce a flux of 0.7 m Wb. The magnetization curve for silicon iron is shown on page 50.

There are two parts to the circuit – the silicon iron and the air gap. The total mmf will the sum of the mmf's of each part.

For the silicon iron : $B = \frac{\Phi}{A} = \frac{0.7\times10^{-3}}{5\times10^{-4}} = 1{,}4T$

From the B-H curve for silicon iron on page , when $B = 1.4T, H = 1650At/M$

Hence the mmf for the iron path $= Hl = 1650\times0.4 = 660At$

For the air-gap:

The flux density will be the same in the air gap as in the iron , i.e 1.4 T (This assumes no leakage or fringing occurring.)

For air $H = \frac{B}{\mu_0} = \frac{1.4}{4\pi\times10^{-7}} = 1114000At/m$

Hence the mmf for the air gap = At= $Hl = 114000\times10^{-3} = 228$

Total mmf to produce a flux of 0.6 m $Wb = 660 + 2228 = 2228At$

A tabular method could have been used as shown below.

part of circuit	Material	ϕ Wb	A m^2	B T	H At/m	lm	mmf=Hl At
Ring	Silicon	0.7×10^{-3}	5×10^{-4}	1.4	1650 (from graph)	0.4	660
Air-gap	Air	0.7×10^{-3}	5×10^{-4}	1.4	$\frac{1.4}{4\pi \times 10^{-7}}$ = 1 114 000	2×10^{-3}	2228

Total: 2888At

Problem 13. Fig.12.65 shows a ring formed with two different materials – cast steel and mild steel. The dimensions are:

	mean length	cross sectional
M ild steel	400mm	$500mm^2$
Castel steel	300mm	$312.5\ mm^2$

Find the total mmf required tocause a flux of 500 μWb in the magnetic circuit. Determine also the total circuit reluctance.

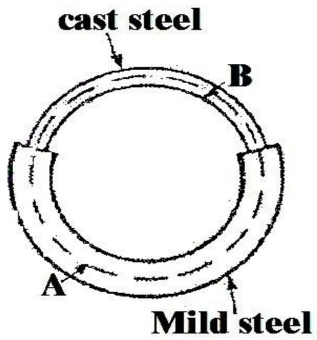


Fig.**12.65**

A tabular solution is shown below.

part of circuit	Material	ϕ Wb	A m^2	B T	H At/m	lm	mmf=Hl At
A	Mild steel	$500 x10^{-6}$	500×10^{-6}	1.0	1400	$400x 10^{-3}$	560
B	Castel st.	$500x10^{-6}$	$5312.5x 10^{-6}$	1.6	4800	$300x10^{-3}$	1440

Problem 14 . A section trough a magnetic circuit of uniform cross-section area 2 cm^2 is shown in Fig.12.66. The cast steel core has a mean length of 25 cm .The air gap is 1 mm wide and the coil has 5000 turns. The B-H curve for cast steel is shown on page 50. Determine the current in the coil to produce a flux density of 0.80 T in the air gap, assuming that all the flux passes through both parts of the magnetic circuit
For the cast steel core : When B = 0.80 T,H = 750 At/m (Fig.12.63)

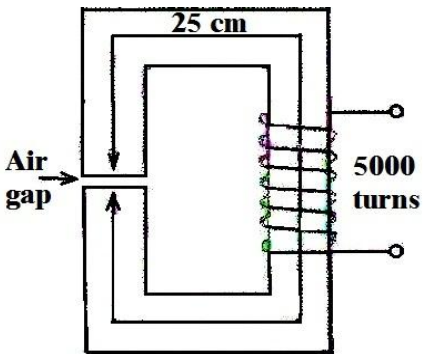


Fig.**12.66**

Reluctance $S_1 = \dfrac{I_1}{\mu_0 \mu_r A_1}$ and , since $B = \mu_0 \mu_{r\,A} H$

then $\mu_r = \dfrac{B}{\mu_0 H}$ $S_1 = \dfrac{l_1}{\dfrac{B}{\mu_0 \mu_0 H}} = \dfrac{l_1 H}{BA} = \dfrac{(25\times10^{-2})(750)}{(0.8)(2\times10^{-4})} = 1172000/H$

For the air-gap : Reluctance $S_2 = \frac{I_2}{\mu_0 \mu_r A_2}$ (since μ_2= 1 for air)

$$\frac{1\times10^{-3}}{(4\pi\times10^{-7})(2\times10^{-4})} = 3979000/H$$

Total circuit reluctance $S=S_1+S_2=1172000+3979000 = 5151000/H$

$$Flux - \Phi = 0.80\times2\times10^{-4} = 1.6\times\times10^{-4}Wb$$

$$S = \frac{mmf}{\phi}$$

$$Thus - mmf = S\phi$$

$$Hence - NI = S\phi$$

Hence current $I = \frac{S}{\phi} = \frac{(5151000)(1.6\times10^{-4})}{5000} = 0.165A$

C. FURTHER PROBLEMS ON MAGNETIC CIRCUITS

(a) SHORT ANSWER PROBLEMS

1. Define magnetic flux.
2. The symbol for magnetic flux is …………… and the unit of flux is the………..
3. Define magnetic flux density.
4. The symbol for magnetic flux density is …………… and the unit of flux density is ……
5. The symbol for mmf is …… and the unit of mmf is the ……
6. Another name for the magnetizing force is …….. ; its symbol is ……… and its unit is…….
7. Complete the statement : For magnetic materials $\frac{fluxdensity}{magneticfi\,eldstrength}$ = ….
8. What is absolute permeability?
9. The value of the permeability of free space is…..
10. What is a magnetization curve?
11. The symbol for reluctance flux is …………… and the unit of reluctance is ……
12. Make a comparison between magnetic and electrical quantities.
13. What is hysteresis?
14. Draw a typical hysteresis loop and identify (a) saturation flux density, (b) remanence and (c) coercive force.
15. State the units of (a) remanence , (b) coercive force.
16. How is magnetic screening achived?
17. Complete the statement :Magnetic materials have a ….. reluctance; non-magnetic materials have a ……… reluctance.
18. What loss is associated with hysteresis?

(b)MULTI –CHOICE PROBLEMS

1. The unit of magnetic flux density is the:
   (a) weber; (b) weber per metre; (c) ampere per metre; (d) tesla
2. The total flux in the core of an electrical machine is 20 m Wb and its flux density is 1 T. The cross- sectional area of the core is (a) 0.05 m^2; (b) 0.02 m^2; (c) 20 m^2; (d) 50m^2.
   A coil of 100 turns is wound uniformly on a wooden ring. The ring has a mean circumference of 1 m a uniform cross- sectional area 10 cm^2. The current in the xoil is 1 A.
   In **problems 3 to 7** select the correct answer for each of the required quantities from the following list.
   (a) $40\pi\, mT$; (b) $100\, A$; (c) $4\pi \times 10^{-1} / H$; (d) 100 At; (e) $0.01 At/m$;
   (g) (f) 40 π μT; 40π $m\, Wb$ (h) $\frac{2.5}{\pi} \times 10^{9} / H$ (i) 100 $100 At/m$; (j) 0.04 π Wb.
3. Magnetomotive force.
4. Magnetic field strength.
5. Magnetic flux density.
6. Magnetic flux.
7. Reluctance.
8. Which of the following statements is false?

(a) For non- magnetic materials reluctance is high.
(b) Energy loss due to hysteresis is greater for harder magnetic materials than for softer magnetic materials.
(c) The remanence of a ferrous material is measured in ampere – turns/metre.
(d) Absolute permeability is measured in henrys per metre.

9. The current flowing in a 500 turn coil wound on an iron ring is 4A. The reluctance of the circuit is $2 \times 10^{6} H$. The flux produced is (a) 1 Wb; (b) 1000 ; Wb (c) 1 m Wb;

(e) $62.5 \mu Wb$.

10. A comparison can be made between magnetic and electrical quantities. From the following list, match the magnetic quantities with their equivalent electrical quantities.

(a) Current ; (b) reluctance; (c) emf ; (d) flux; (e) mmf; (f) resistance.

(b) CONVENTIONAL PROBLEMS

(Where appropriate, assume $\mu_2 = 4\pi \times 10^{-7} H/m$)

Magnetic circuit quantities

1.What is the flux density in a magnetic field of cross –sectional area 20 cm^2 having a flux of 3 mWb? [1.5 T]

2.(a) Determine the flux density produced in an air- cored solenoid due to a uniform magnetic field strength of 8000 At/M.

(b) Iron having a relative permeability 150 at 8000 At/m is inserted into the solenoid of part(a). Find the flux density now in the solenoid.

[(a) 10.05 mT; (b) 1.508 T]

3. Find the relative permeability of a material if the absolute permeability is 4. 084 x 10^{-4} H/m. [325]

4. Find the relative permeability of a piece of silicon iron if a flux density of 1.3 T is produced by a magnetic field strength of 700 At / m.

[1478]

5. Determine the total flux emerging from a magnetic pole face having dimensions 5 cm by 6 cm , if the flux density is 0.9 T

[2.7 m Wb]

6.The maximum working flux density of a lifting electromagnet is 1.8 T and the effective area of a pole face is circular in cross-section. If the total magnetic flux produced is 353 mWb determine the radius of the pole face.

[25cm]

7. A solenoid 20 cm long is wound with 500 turns of wire. Find the current required to establish a magnetizing force of 2500 At/m inside the solenoid.

[1 A]

8. An electromagnet of square cross-section produces a flux density of 0.45 T. if the magnetic flux is 720 μWb find the dimensions of the electromagnet cross-section.

[4cm by 4cm]

9.Find the magnetic field strength and the magnetomotive force needed to produce a flux density of 0.33 T in an air-gap of length 15 mm.

[(a) 262 600 At/m; (b) 3939At]

10. An air-gap between two pole pieces is 20 mm in length and thearea of the flux path across the gap is 5 cm^2. If the flux required in the air-gap is 0.75 m Wb find the mmf necessary. [23870At]

11.Find the magnetic field strength applied to a magnetic circuit of mean length 50 cm when a coil of 400 turns ia applied to it carrying a current of 1.2 A.

[960At/m]

12. A magnetic field strength of 5000 At/m is applied to a circular magnetic circuit of mean diameter 250 mm. If the coil has 500 turns find the current in the coil.

[7.85A]

13. Part of a magnetic circuit is made from steel of length 120mm, cross-sectional area 15 cm^2 and relative permeability 800. Calculate (a) the reluctance and (b) the absolute permeability of the steel.

[(a) 79 580 / H ; (b) 1mH/m]

14. A steel ring of mean diameter 120 mm is uniformly wound with 1500 turns of wire. When a current of 0.30 A is passed through the coil a flux density of 1.5 T is set up in the steel. Find the relative permeability of the steel under these conditions.

[1000]

15. A mild steel closed magnetic circuit has a mean length of 75 mm and a cross- sectional area of 320.0 mm^2.A current of 0.40 A flows in a coil wound uniformly around the circuit and the flux produced is 200 μ Wb.If the relative permeability of the steel at this value of current is 400 find (a) the reluctance of the material and (b) the number of turns of the coil.

[(a) 466 000 /H; (b) 233]

16.A uniform ring of cast steel has a cross –sectional area of 5 $cm^2$and a mean circumference of 15 cm. Find the current required in a coil of 1200 turns wound on the ring to produce a flux of 0.8 mWb . (Use the magnetization curve for cast steel shown on page 50.)

[0.60A]

17. (a) A uniform mild steel ring has a diameter of 50 mm and a cross-sectional area of 1 cm^2. Determine the mmf necessary to produce a flux of 50 μ Wb in the ring. Use the B-H curve for mild steel shown on page

(b) If a coil of 440 turns is wound uniformly around the ring in part (a) what current would be required to produce the flux?

[(a) 110 At ; (b) 0.25 A]

18.From the magnetization curve for mild steel shown on page 50, derive the curve of relative permeability against magnetic field strength.Fromyour graph determine (a) the value of μ_x when the magnetic field strength is 1200 At/m , and (b) the value of the magnetic field strength when is μ_x 500. [(a) 590-600; (b) 2000]

Composite series magnetic circuits

1. A magnetic circuit of cross –sectional area 0.4 cm^2 consists of one part 3 cm long of material having relative permeability 1200, and a second part 2 1cm ongmaterial having relative permeability 750. With a 100 turn coil carrying 2A , find the value of flux existing in the circuit.

[0.195 mWb]

20. (a) A cast steel ring has a cross –sectional are 600 mm^2 and a radios of 25 mm.Determine the mmf necessary to establish a flux of 0.8 m Wb in the ring. Use the B-H curve for cast steel shown on page 50.

(b) If a radial air gap 1.5 mmwide is cut in the ring of part (a) find the mmf now necessary to maintain the same flux in the ring.

[(a) 267 At; (b) 1859 At]

21.A closed magnetic circuit made of silicon iron consists of a 40 mmlong path of cross sectional area 90 mm^2 and 15 mm long path of cross sectional area 70 mm^2.A coil of 50 turns is wound around the 40 mmlength of the circuit and a current of 0.39 A flows. Find the flux density in the 15 mm length path if the relative permeability of the silicon iron at this value of magnetizing force is 3000 .

[1.59T]

22. For the magnetic circuit shown in Fig.12.67 find the current **I** in the coil needed to produce a flux of 0.45 mWb in the air- gap. The silicon iron magnetic circuit has a uniform cross- sectional area of 3 cm^2 and its magnetization curve as shown on page 50.

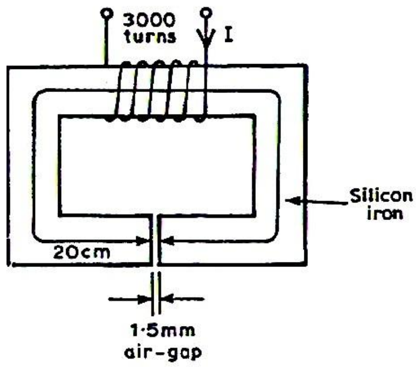


Fig.12.67

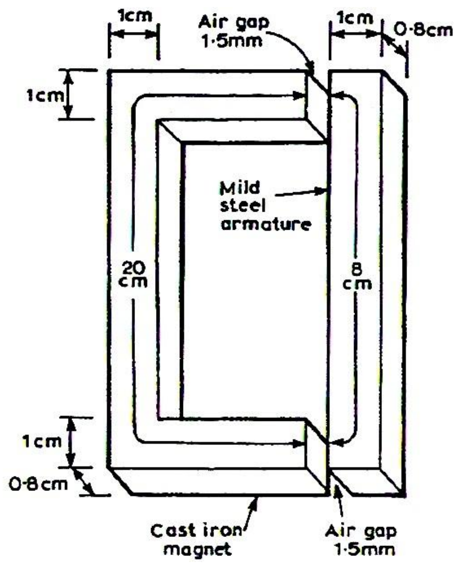


Fig.12.68

23. A ring forming a magnetic circuit is made from two materials; one part is mild steel of mean length 25 cm and cross-sectional area 4 cm^2 , and the remainder is cast iron of mean length 20 cm and cross- sectional area 7.5 cm^2.Use a tabular approach to determine the total mmf required to cause a flux of 0.30 mWbin the magnetic circuit.Find also the total reluctance of the circuit .Use the magnetization curves shown on page 50.

[540 At; 18 x 10^5/H]

24. Fig.12.68 shows the magnetic circuit of a relay. When each of the air gaps are 1.5 mm wide find the mmf required to produce a flux density of 0.75 T in the air gaps. Use the B-H curves shown on page 50.

[2990 At]

Electromagnetic induction

A. FORMULAE AND DEFINITIONS ASSOCIATED WITH ELECTROMAGNETIC INUCTION

1. When a conductor is moved across a magnetic field , an electromotive force (emf) is produced in the conductor. If the conductor forms part of a closed circuit then the emf produced causes an electric current to flow round the circuit. Hence an emf(and thus

current) is „induced" in the conductor as a result of its movement across the magnetic field. This effect is known as, electromagnetic induction' .

2. Faraday 's laws of electromagnetic induction state:

(i)' An induced emf is set up whenever the magnetic field linking that circuit changes'.
(ii) ' The magnitude of the induced emf in any circuit is proportional to the rate of change of the magnetic flux linking the circuit'.

3. Lenz's law states:

'The direction of an induced emf is always such as to opposite the effect producing it'.

4. An alternative method to Lenz's law of determining relative directions is given by Fleming's Right-hand rule (often called the generator rule) which states:

'Let the thumb, first finger and second finger of the right hand be extended such that they are all at right angles to each other, as shown in Fig.12.69. If the first finger points in the direction of the magnetic field , the thumb points in the direction of motion of thre conductor relative to the magnetic field , then the second finger will point in the direction of the induced emf'.

Summarizing: First finger –Field, Thumb_ Motion, Second finger __ Emf.

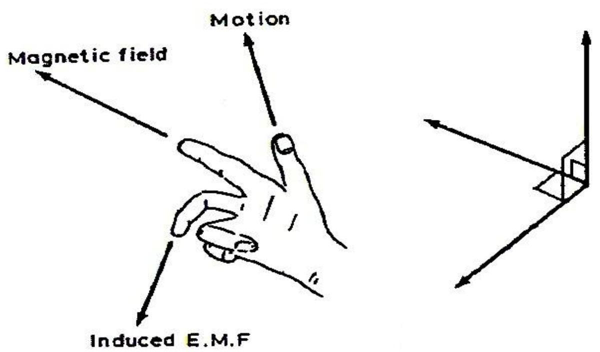


Fig.12.69

5. In a generator, conductors forming an electric circuit are made to move through a magnetic field. By Faraday's law an emf is induced in the conductors and thus a source of emf is created .A generator converts mechanical energy into electrical energy.

6. The induced emf E set between the ends of the conductors shown in Fig.12.70 is given by : E= Blv volts, where B, the flux density, is measured in teslas, l , the length of conductor in the magnetic field , is measured in metres , and v ,the conductor elocity,is measured in metres per second. If the conductor moves at an angle θ^0 to the magnetic field (instead of at 90^0as assumed above) then

$$E = Blv \sin \theta$$

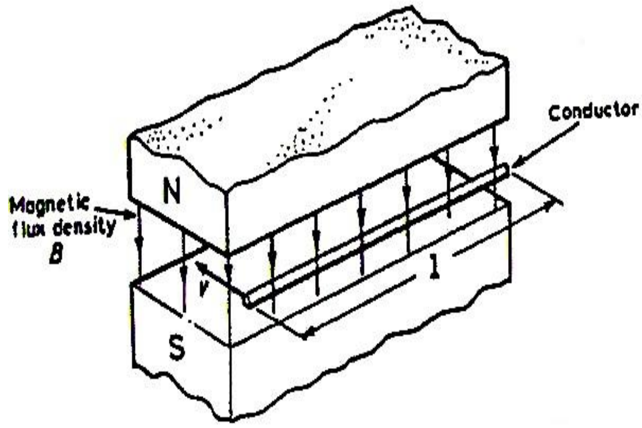


Fig.12.70

7. If B teslas is the magnetic flux density , I amperes the current in the conductor and l metres the length of conductor in the magnetic field, then the force F on the current carrying conductor tying at right angles to the direction of the magnetic field is given by:

$$F = BIl \quad newtons$$

If the conductor and field are at an angle θ^0 to each other then:

$$F = BIl \ \sin \ \theta^0 \ \ newtons$$

8. The flow of current in a conductor results in a magnetic field around the conductor , the direction of the magnetic field being given by the screw rule, which states : 'If a normal right-hand thread screw is screwed along the conductor in the direction of the current , the direction of rotation of the screw is in the direction of the magnetic field'. This rule is illustrated in Fig.12.71.

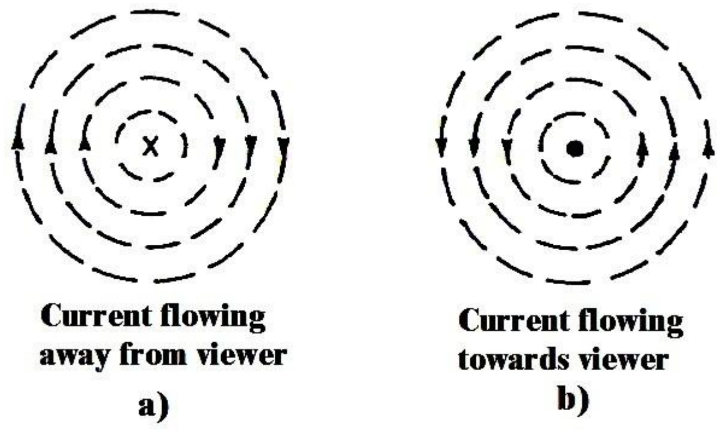


Fig.12.71

9. If the current- carrying conductor shown in Fig 3 (a) is placed in the magnetic field shown in Fig 4(a) then the two fields interact and cause a force to be exerted on the conductor as shown in Fig.12.72(b). The field is strengthened above the conductor and weakened below,

thus tending to move the conductor downwards. This is the basic principle of operation of the electric motor. The forces experienced by a number of such conductors can produce motion. A motor is a device that takes in electrical energy and transfers is into mechanical energy.

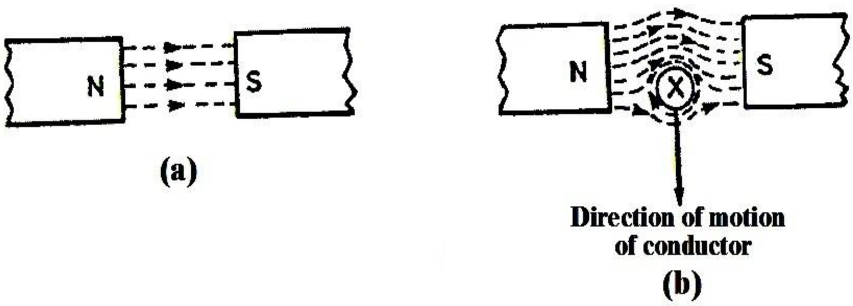


Fig.12.72

10. The direction of the force exerted on a conductor can be predetermined by using **Fleming's left –hand rule** (often called the motor rule), which states: Let the thumb , first finger and second finger of the left-hand be extended such that they are all at right angles to each other, as shown in Fig 5.If the first finger points in the direction of the magnetic field, the second finger points in the direction of the current , then the thumb will point in the direction of the motion of the conductor. Summarising:
First finger- Field, Second finger- Current, Thumb - Motion.

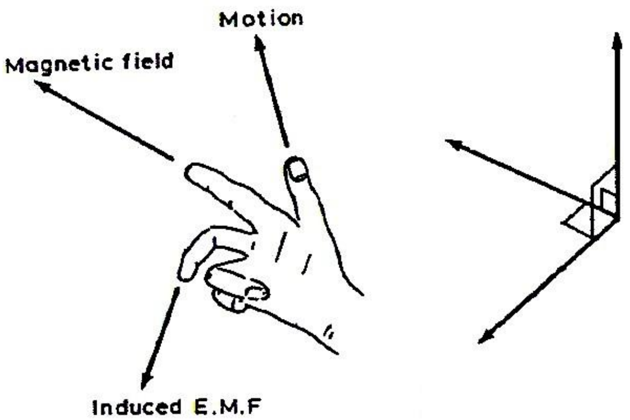


Fig.12.73

11. **Inductance** is the name given to the property of a circuit whereby there is an emf induced into the circuit by the change of flux linkages produced by a current change.
   (i) When the emf is induced in the same circuit as that in wich the current is changing , the property is called self inductance, L.
   (ii) When the emf is induced in a circuit in circuit by a change of flux due to current changing in an adjacent circuit , the property is called mutual Induced emf in a coil of N turns ,M

12. The unit of **inductance** is the henry ,H. A circuit has an inductance of one henry when an emf of one volt is induced in it by a current changing at the rate of one ampere per second'.

13. (i) Induced emf in a coil of N turns, E = N(Δφ/t)volts, where Δφ is the change in flux , in Webers , and t is the time taken for the flux to change , in seconds.

(ii) Induced emf in a coil of Inductance L henrys, E= L(ΔI/t)volts, where ΔI is the change in current , in amperes , and t is the time taken for the current to change, in seconds.

14. If a current changing from 0 to I amperes , produces a flux change from 0 to φ Webers. then $\Delta I = I$ and $\Delta\varphi = \varphi$ Then, induced emf

$E = \frac{N\Phi}{t} = \frac{LI}{t}$ from which, inductance of coil, $L = N\phi / I$ henrys.

15. Reluctance $S = \frac{mmf}{flux} = \frac{NI}{\Phi}$ from which $\Phi = \frac{NI}{S}$

Hence, inductance of coil $L = \frac{N\phi}{I} = \frac{N}{I} = \frac{NI}{S} = \frac{N^2}{S} i.e. L \alpha N^2$

16. **Mutually induced emf** in the second coil , $E_2 = M\frac{\Delta l_1}{t}$ volts where Miss the mutual inductance between two coils, in henrys, ΔI_1 is the change in current in the first coil, in amperes t is the time the current takes to change in the first coil, in seconds.

17. **A transformer** is a device which uses the phenomenon of mutual inductance to change the value of alternating voltages. A transformer is represented in Fig.12.74(a) and its circuit diagram symbol shown in Fig.12.74(b). When an alternating voltage is applied to the primary winding , an alternating current is produced in the winding . This current produces an alternating flux in the core which links with the secondary winding .The Flux induces an alternating emf in the secondary winding by mutual induction. Since E= $\frac{N\phi}{t}$

then $E_1 = \frac{N_1}{\Phi}$ *and* $E_2 = \frac{N_2}{\Phi}$ For an ideal transformer (i.e. no losses),

$\frac{E_1}{N_1} = \frac{E_2}{N_2} i.e. \frac{E_1}{E_2} = \frac{N_1}{N_2}$

$\frac{E_1}{E_2}$ is called the voltage ratio and $\frac{N_1}{N_2}$ the turns ratio.

If $N_{2\angle} N_1$ then $E_{2\angle} E_2$ and the device is termed a step down transformer.

If $N_{2|>\rangle} N_1$ then $E_2 > E_2$ and the device is termed a step up transformer.

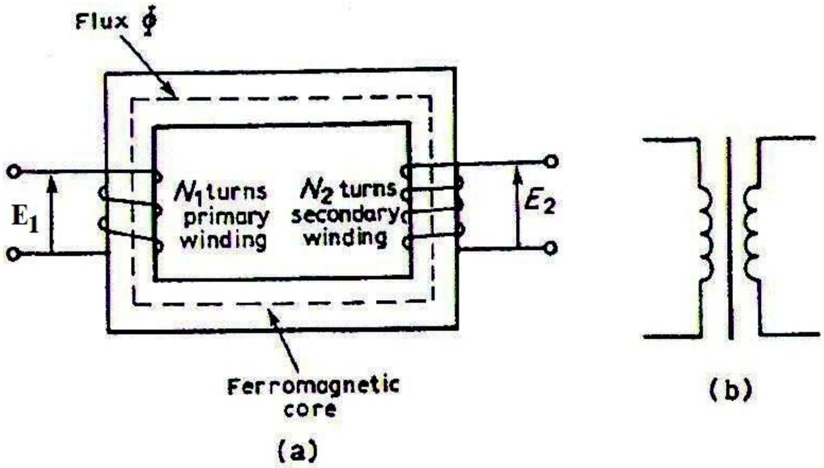


Fig.12.74

18. The energy W stored in the magnetic field of an inductor is given by:

$W = \frac{1}{2} LI^2$ joules

B. WORKED PROBLEMS ON ELECTROMAGNETIC INDUCTION

(a) DETERMINATION OF THE FORCE AND DIERECTION ON A CURRENT – CARRYING CONDUCTOR IN A MAGNETIC FIELD

Problem 1. A conductor carries a current of 20 A and is at right angles to a magnetic field having a flux density of 0.9 T. If the length of the conductor in the field is 30 cm , calculate the force acting on the conductor. Determine also the value of the force if the conductor is inclined at an angle 0f 30°to the direction of the field .

$B = 0.9$ T ; $I = 20$ A; $l = 30$ cm$= 0.30$m

Force F= BIl =(0.9)(20)(0.30) newtons when the conductor is at right angles to the field (as shown in Fig.12.75 (a)), I.e. F = 5.4 N When the conductor is inclided at 30° to the field (as shown in Fig.12.75 (b)) then Force F= BIl sinθ , i.e. F= =(0.9)(20)(0.30) sin 30° F= 2.7 N

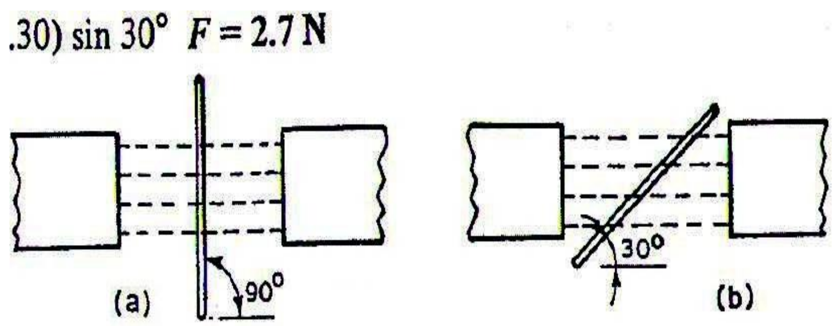


Fig.12.75

Problem 2. Determine the current required in a 400 mm length of conductor of an electric motor, when the conductor is situated at right angles to a magnetic field of flux density 1.2 T , if a force of 1.92 N is to be exerted on the conductor. If the conductor is vertical, the current flowing downwards and the direction of the magnetic field is from left to right , what is the direction of the force?

F= 1.92 N ; l = 400mm = 0.40 m; B= 1.2 T

Since $F = BI \; I = \dfrac{F}{Bl}$

Hence current $I = \dfrac{1.92}{(1.2)(0.4)} = 4A$

If the current flows downwards, the direction of its magnetic field due to the current alone will be clockwise when viewed from above. The lines of flux will reinforce (i.e. strengthen)the main magnetic field at the back of the conductor and will be in opposition in the front (i.e. weaken the field) .Hence the force on the conductor will be from back to front (i.e. towards the viewer). This direction may also have been deduced using Fleming's left-hand rule.

Problem 3. A conductor 350 mm long carries a current of 10 A and is at right angles to a magnetic field lying between two circular pole faces each of radii 60 mm. If the total flux between the pole faces is 0.5 m Wb , calculate the force exerted on the conductor.

$1 = 350mm = 0{,}35m, I = 10A$

$\Phi = 0.5mWb = 0.5 \times 10^{-3} Wb$

Hence $F = \dfrac{\phi}{A} Il = \dfrac{(0.5 \times 10^{-3})}{\pi (0.06)^2}(10)(0.35)N$

Area of pole face $A = \pi r^2 = \pi (0.06)^2 m$

Force $F = BIl$ and $B = \dfrac{\Phi}{A}$

i.e. force = 0.155 N

Problem 4. With reference to Fig.12.76 determine (a) the direction of the force on the conductor in Fig 8 (a) , (b) the direction of the force on the conductor in Fig.12.76 (b), (c) the direction of the current in Fig.12.76 (c) and (d) the polarity of the magnetic system in Fig 8 (d)

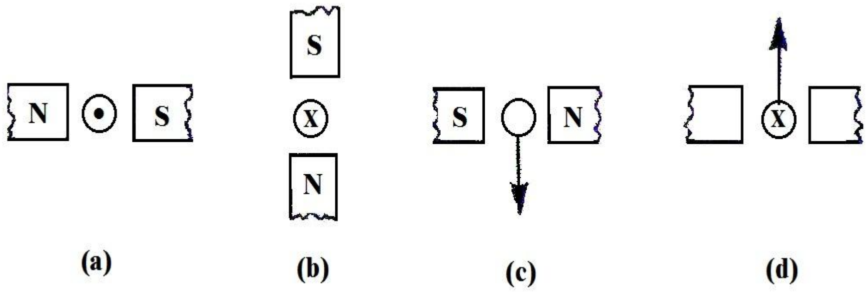


Fig.12.76

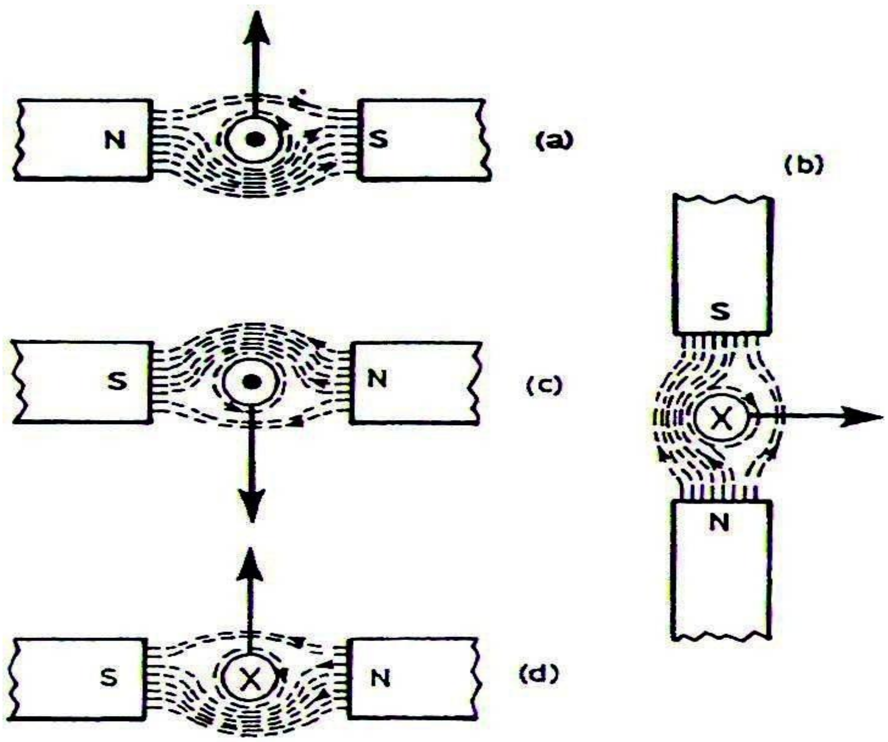


Fig.12.77

(a) The direction of the main magnetic field is from north to south, i.e. left to right. The current is flowing towards the viewer, and using the screw rule , the direction of the field is anticlockwise .Hence either by Fleming's left-hand rule , or by sketching the interacting magnetic field as shown in Fig.12.77(a), the direction of the force on the conductor is seen to be upwards.

(b) Using a similar method to part (a) it is seen that the force on the conductor is to the right (see Fig.12.77 (b)).

(c) Using Fleming's left-hand rule ,or by sketching as in Fig.12.77 (c) , it is seen that the current is towards the viewer , i.e. out of the paper.

(d) Similar to part(c) , the polarity of the magnetic system is as shown in Fig.12.77 (d)

MAGNITUDE AND DIRECTIONS OF INDUCED EMF'S

Problems 5. Determine the emf induced in a coil of 200 turns when there is a change of flux of 25 m Wb linking with it in 50 ms.

$$\text{Induced emf} \quad E = N\left(\frac{\Delta\Phi}{t}\right) = (200)\left(\frac{25\times10^{-3}}{50\times10^{-3}}\right) = 100V$$

Problem 6. A flux of 400μ Wb passing through a 150 turn coil is reversed in 40 ms. Find the average emf induced.

Since the flux reverses, the flux changes from + 400 μ Wb to − 400 μ Wb, i.e. a change of flux of 800 μ Wb.

$$\text{Induced emf } E = N\left(\frac{\Delta\Phi}{t}\right) = (150)\left(\frac{800\times10^{-6}}{40\times10^{-3}}\right) = \frac{800\times150\times10^{3}}{40\times10^{6}}$$

Hence , the average emf induced E= 3 V

Problem 7. Calculate the emf induced in a coil of inductance 12 H by a currenty changing at the rate of 4 A/s.

$$\text{Induced emf } E = L\left(\frac{\Delta I}{t}\right) = (12)(4) = 48V$$

Problem 8. An emf of 1.5 k V is induced in a coil when a current of 4 A collapses uniformly to zero in 8 ms.Determine the inductance of the coil.

Change in current $\Delta I = 4 - 0 = 4A; t = 8ms = 8\times10^{-3}s;$

$\frac{\Delta I}{t} = \frac{4}{8\times10^{-3}} = \frac{4000}{8} = 500A/s; E = 1.5kV = 1500V$

Since $E = L\left(\frac{\Delta I}{t}\right)$ *then* $L = \frac{E}{\left(\frac{M}{t}\right)} = \frac{1500}{500} = 3H$

Problem 9. An average emf 0f 40 V is induced in a coil of inductance 150 mH when a current of 6A is reversed. Calculate the time taken for the current to reverse.

$$E = 40V; L = 150mH = 0.15H$$

Change in current , $\Delta l = 6 - (-6) = 12A(\sin ce \quad the \quad current \ is \ reversed$

Since $E = L\left(\frac{\Delta I}{t}\right)$ *then time* $t = \frac{L\Delta l}{E} = \frac{(0.15)(12)}{40} = 0.045s$ or 45 ms

Problem 10. A conductor 300 mm long moves at a uniform speed of 4 m/s at right angles to a uniform magnetic field of flux density 1.25 T. Determine the current flowing in the conductor when (a) its ends are open-circuited , and (b) its ends are connected to a load of 20 Ω resistance.

When a conductor moves in a magnetic field it will have an emf induced in it but this emf can only produce a current if there is a closed circuit.

$$\text{Induced emf } E = Blv = (1.25)\left(\frac{300}{1000}\right)(4) = 1.5V$$

(a) If the ends of the conductor are open circuited no current will flow even though 1.5 V has been induced.

(b) From Ohm's law $I = \frac{E}{R} = \frac{1.5}{20} = 0.075A \quad or \quad 75mA$

Problem 11. At what velocity must a conductor 75 mm long cut a magnetic field of flux density 0.6 T if an emf of 9 V is to be induced in it? Assume the conductor , the field and the direction of motion are mutually perpendicular

Induced emf $E = Blv$

Hence velocity $v = \frac{E}{Bl}$

Hence $v = \frac{9}{(0.6)(75 \times 10^{-3})} = \frac{9 \times 10^3}{0.6 \times 75} = 200m/s$

Problem 12. A conductor moves with a velocity of 15 m/s at an angle of (a) 90^0, (b)60^0 and (c)30^0 to a magnetic field produced between two square-faced poles of side length 2cm. If the flux leaving a pole face is 5 μWb find the magnitude of the induced emf in each case.

V= 15m/s; length of conductor in magnetic field , l =2 cm = 0.02 m;

$A = 2 \times 2cm^2 = 4 \times 10^{-4} m^2; \Phi = 5 \times 10^{-6} Wb.$

$E_{90} = Blv \sin 90^0 = \frac{\Phi}{A} lv = \frac{(5 \times 10^{-6})}{(4 \times 10^{-4})}(0.02)(15) = 3.75mV$

$E_{60} = Blv$

$\sin 60^0 = E_{90} = \sin 60^0 = 3.75 \sin 60^0 = 3.25mV$

$30^0 = E_{90} = \sin 30^0 = 3.75 \sin 30^0 = 1.875.Mv$

Problem 13. The wing span of a metal aeroplane is 36 m. If the aeroplane is flying at 400 km/h, determine the emf induced between the wing tips. Assume the vertical component of the earth's magnetic field is 40μ T.

Induced emf across wing tips E= Blv

$B = 40\mu T 40 \times 10^{-6} T; l = 36m$

$v = 400\frac{km}{h} \times 1000\frac{km}{h} \times \frac{1h}{60 \times 60s} = \frac{(400)(1000)}{3600} = \frac{4000}{36} m \backslash s$

Hence $E = (40 \times 10^{-6})(36)\frac{4000}{36} = 0.16V$

Problem 14. The diagram shown in Fig.12.78 represents the generation of emf's Determine

(i) the direction in which the conductor has to be moved in Fig.12.78(a),

(ii)the direction of the induced emf in Fig.12.78(b), and

(iii)the polarity of the magnetic system in Fig.12.78(c)

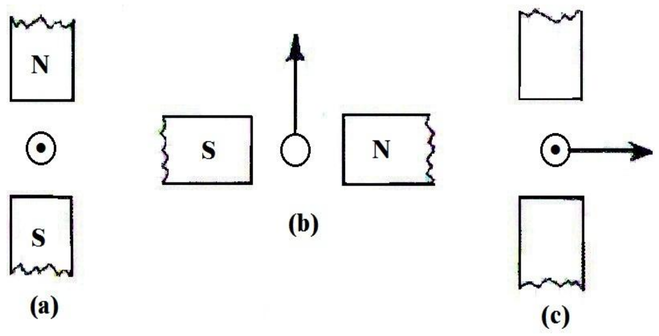


Fig.12.78

The direction of the emf ,and thus the current due to the emf , may be obtained either by Lenz's law or by Fleming 's Right-hand rule(i.e. GeneRator rule).

(h) Using Lenz's LAW : The field due to the magnetic and the field due to the current carrying conductor are shown in Fig.12.79(a) and are seen to reinforce to the left of the conductor .Hence the force on the conductor is to the right. However Lenz's law says that the direction of the induced emf is always such as to oppose the effect producing it. Thus the conductor will have to be moved to the left.

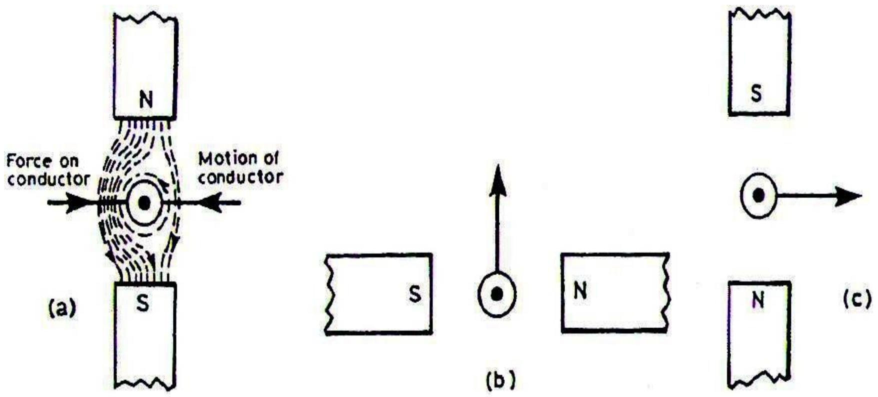


Fig.12.79

(ii) Using Fleming's right-hand rule:

First finger - Field, i.e. N→ S i.e. right to left
ThuMb - Motion ,i.e. upwards
Second finger - Emf, i.e. towards the viewer or out of the paper , as shown in Fig.12.79 (b).

(iii) The polarity of the magnetic system of fig.12.78 (c) is shown in Fig.12.79 (c) and is obtained using Fleming's right –hand rule.

(c) INDUCTANCE

Problem 15. Calculate the coil inductance when a current of 4A in a coil of 800 turns produces a flux of 5 mWb linking with the coil

For a coil , inductance $L = \frac{N\Phi}{I} = \frac{(800)(5\times10^{-3})}{4} 1H$

Problem 16. When a current of 1.5 A flows in a coil the flux linking with the coil is 90 μWb . If the coil inductance is 0.60 H calculate the number of turns of the coil.

For a coil $L = \frac{N\Phi}{I} .Thus N = \frac{LI}{\Phi} = \frac{(0.60)(1.5)}{90\times10^{-6}} = 100 turns$

Problem 17. When carrying a current of 3 A , a coil of 750 turns has a flux of 12 mWb linking with it.Calculate the coil inductance and the emf induced in the coil when the current collapses to zero in 18 ms.

Coil inductance $L = \frac{N\Phi}{I} = \frac{(750)(12\times10^{-3})}{3} = 3H$

Induced emf $E = \left(\frac{\Delta I}{t}\right) = 3\left(\frac{3-0}{18\times10^{-3}}\right) = 500V$

(Alternatively $E = N\left(\frac{\Delta\Phi}{t}\right) = (750)\left(\frac{12-10^{-3}}{18\times10^{-3}}\right) = 500V$

Problem 18. A coil has 200 turns and an inductance of 2 m H .How many y\turns would be needed to produce a 1.28 m H coil(assuming that the same core is used).

inductance $L \alpha N^2$

Hence $\frac{L_1}{L_2} = \frac{N_{1^2}}{N_{2^2}}$

i.e. $\frac{2\times10^{-3}}{1.28\times10^{-3}} = \frac{(200)^2}{N_{2^2}}$

$$N_2 = \sqrt{\left[(200)^2\left(\frac{1.28}{2}\right)\right]} = 160 turns$$

(a) MUTAL INDUCTANCE

Problem 19. Calculate the mutual inductance between two coils when a current changing at 200 A/s in one coil induces an emf of 1.5 V in the other.

Induced emf $E_2 = m\left(\frac{\Delta l_1}{t}\right) Hence 1,5 = M(200)$

Thus mutual inductance , $M = \frac{1.5}{200} = 0.0075 H or 7.5 mH$

Problem 20. The mutual inductance between two coils is 18 m H .Calculate the steady rate of change of current in one coil to induce an emf of 0.72 V in the other.

Induced $E_2 = M\left(\frac{\Delta l_1}{t}\right)$

Hence rate of change of current $\frac{\Delta l}{t} = \frac{F_2}{M} = \frac{0.72}{0.018} = 40 A/s$

Problem 21. Two coils have a mutual inductance of 0.2 H . If the current in one coil is changed from 10 A to 4A in 10 ms calculate (a) the, average induced emf in the second coil, and (b) the change of flux linked with the second coil if it is wound with 500 turns.

(a) Induced emf $E_2 = M\left(\frac{\Delta l_1}{t}\right) = (0.2)\left(\frac{10-4}{10\times10^{-3}}\right) = 120V$

(b) Induced emf $E = N\left(\frac{\Delta\Phi}{t}\right), Hence \Delta\Phi = \frac{Et}{N}$

Thus the change of flux $\Delta\Phi = \frac{120(10\times10^{-3})}{500} = 2.4 mWb$

(e) THE TRANSFORME

Problem 22. A transformer has 500 primary turns and 3000 secondary turns. If the primary voltage is 240 V determine the secondary voltage , assuming an ideal transformer.

For an ideal transformer, voltage ratio = turns ratio

i.e. $\frac{E_1}{E_2} = \frac{N_1}{N_2}$ Hence $\frac{240}{E_2} = \frac{500}{3000}$

Hence secondary voltage $E_2 = \frac{(3000)(240)}{(500)} = 1440 V or 1.44 kV$

Problem 23 An ideal transformer with a turns ratio of 2:7 is fed from a 240 V supply.Determine its output voltage

A turns ratio 2:7 means that the transformer has 2 tuns on the primary for every 7 turns on the secondary (i.e. a step-up transformer)

Thus $\dfrac{N_1}{N_2} = \dfrac{2}{7}$

For an ideal transformer $\dfrac{N_1}{N_2} = \dfrac{E_1}{E_2}$

Hence $\dfrac{2}{7} = \dfrac{240}{E_2}$

Thus the secondaryvoltage $E_2 = \dfrac{(240)(7)}{(2)} = 840V$

Further problems on transformers may be found in section C (c) ; Problems 31 to 34, page 77.

(f) ENERGY STORED IN AN INDUCTOR

Problem 24. An 8 H inductor has a current of 3 A flowing through it. How much energy is stored in the magnetic field of the inductor?

Energy stored , $W = \dfrac{1}{2}LI^2 = \dfrac{1}{2}(8)(3)^2 = 36J$

Problem 25. A flux of 25 m Wb links with a 1500 turn coil when a current of 3 A passes through the coil. Calculate (a) the inductance of the coil , (b) the energy stored in the magnetic field , and (c) the average emf induced if the current falls to zero in 150 ms.

(a) Inductance $L = \dfrac{N\Phi}{I} = \dfrac{(1500)(25\times10^{3})}{3} = 12.5H$

(b) Energy stored in field $W = \dfrac{1}{2}LI^2 = \dfrac{1}{2}(12.5)(3)^2 = 56.25J$

(c) Induced emf $E = N\left(\dfrac{\Delta\Phi}{t}\right) = (1500)\left(\dfrac{25\times10^{-3}}{150\times10^{-3}}\right) = 250$

C . FURTHER PROBLEMS ON ELECTROMAGNETIC INUCTION

(a) SHORT ANSWER PROBLEMS

1. What is electromagnetic induction?
2. State Faraday's Laws of electromagnetic induction.
3. State Lenz's Law.
4. Briefly explain the principle of the generator.
5. The direction of an induced emf in a generator may be determined using Fleming's ……….rule.

6. To calculate the force on a current –carrying conductor in magnetic field it is necessary to know the value of three quantities and their units.

7. The direction of the force on a conductor in a magnetic field may be predetermined using two methods .State each method.

8. Explain briefly the motor principle in terms of the interaction between two magnetic fields.

9. The direction of the magnetic field around a current-carrying conductor is given by the …… rule.

10. The emf E induced in a moving conductor may be calculated using the formula E= Blv . Name the quantities represented and their units.

11. What is self inductance ?State its symbol.

12. State and define the the unit of inductance.

13. When a circuit has an inductance L and the current changes at a rate of (ΔI/t) then the induced emf E is given by E = …… volts.

14. If a current of I A flowing in a coil of N turns produces a flux of ϕ Wb , the coil inductance L is given by L = ………henry's .

15. What is mutual inductance ?State its symbol.

16. The mutual inductance between two coils is M. The emf E_2 induced in one coil by a current changing at (ΔI_1/t) in it other is given by E_2 = ……… volts.

17. Briefly explain how a voltage is induced in the secondary winding of a transformer.

18. Draw the circuit diagram symbol for a transformer.

19. State the relationship between turns and voltage ratios for a transformer.

20. The energy W stored by an inductor is given by W = ……. joules.

(b) MULTI-CHOICE PROBLEMS

1. A current changing at a rate of 5 A/s in a coil of inductance 5 H induces and emf of

(a) 25 V in the same direction as the applied voltage;

(b) 1 V in the same direction as the applied voltage;

(c) 25 V in the opposite direction to the applied voltage;

(d) 1 V in the opposite direction to the applied voltage.

2. When a magnetic flux of 10 Wb links with a circuit of 20 turns in 2 s, the induced emf is

(a) 1 V; (b) 4 V; (c) 100 V ; (d) 400 V.

3. A current of 10 A in a coil of 1000 turns produces a flux of 10 m Wb linking with the coil. The coil inductance is

(a) 10^6 H; (b)1 H ; (c) 1μ H (d) 1 mH.

4. A conductor carries a current of 10 A at right angles to a magnetic field having a flux density of 500 mT . If the length of the conductor in the field is 20 cm the force on the conductor is

(a) 100 k N; (b) 1 k N; (c)100 N; (d) 1 N.

5. If a conductor is horizontal, the current flowing from left to right and the direction of the surrounding magnetic field is from above to below, the force exerted on the conductor is

(a) from left to right; (b) from below to above;

(c) away from the viewer; (d) towards the viewer.

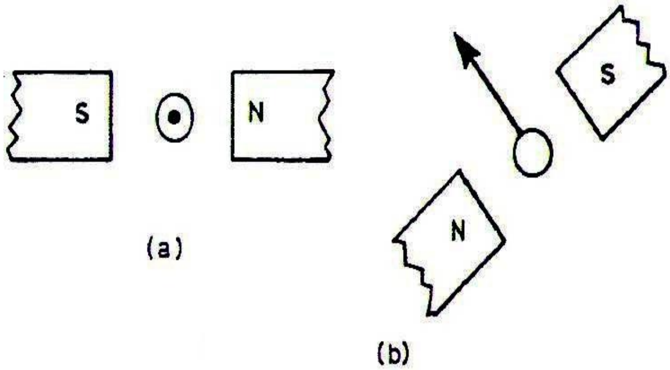


Fig. 12.80

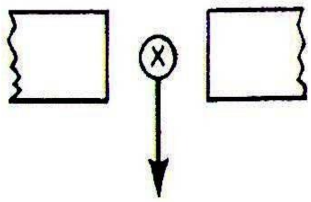


Fig.12.81

6. For the current –carrying conductor lyingin the magnetic field shown in Fig.12.80 (a) the direction of the force on the conductor is (a) to the left, (b) upwards, (c) to the right, (d) downwards.

7.For the current-carrying conductor lying in the magnetic field shown in Fig.12.80 (b) the direction of the current in the conductor is (a) towards the viewer, (b) away from the viewer.

8. An emf of 1V is induced in a conductor moving at 10 cm/s in a magnetic field of 0.5 T.The effective length of the conductor in the magnetic field is (a) 20 cm;(b) 5 m; (c) 20 m; (d) 50 m

9.Which of the following statements is false?

(a) Fleming's left-hand rule or Lenz's Law may be used to determine the direction of the induced emf.

(b) An induced emf is set up whenever the magnetic field linking that circuit changes.

(c) The direction of an induced emf is always such as to oppose the effect producing it.

(d) The induced emf in any circuit is proportional to the rate of change of the magnetic flux linking the circuit.

10. The mutual inductance between two coils , when a current changing at 20 A/s in one coil induces an emf of 10 mV in the other is

(a)0.5 H; (b) 200 mH; (c) 0.5 m H ; (d) 2 H.

11 A transformer has 800 primary turns and 100 secondary turns. To obtain 40 V from the secondary winding the voltage applied to the primary winding must be:

(a) 5 V ; (b) 320; (c) 2. 5 V; (d) 20 V.

12. An emf is induced into a conductor in the direction shown in Fig.12.81 when the conductor is moved at a uniform speed in the field between the two magnets. The polarity of the system is

(a) North pole on right, South pole on left;

(b) North pole on left , South pole on right.

(C) CONVENTIONAL PROBLEMS

Determination of the force and direction on a current – carrying conductor in a magnetic field.

1. A conductor carries a current of 70 A at right angles to a magnetic field having a flux density of 1.5T. If the length of the conductor in the field is 200 mm calculate the force acting on the conductor. What is the force when the conductor and field are at an angle of 45^0?

[21.0 N; 14,8N]

2. Calculate the current required in a 240 mm length of conductor of a dc motor when the conductor is situated at right angles to the magnetic field of flux density 1.25 T, if a force of 1.20 N is to be exerted on the conductor.

[4.0A]

3. A conductor 30 cm long is situated at right angles to a magnetic field . Calculate the strength of the magnetic field if a current of 15 A in the conductor produces a force on it of 3.6 N.

[0.80 T]

4. A conductor 300 mm long carries a current of 13 A and is at right angles to a magnetic field between two circular pole faces , each of diameter 80mm.If the total flux between the pole faces is 0.75 mWb calculate the force exerted on the conductor.

[0.582N]

5. (a) A 400 mm length of conductor carrying a current of 25 A is situated at right angles to a magnetic field between two poles of an electric motor . The poles have a circular cross-section. If the force exerted on the conductor is 80 N and the total flux between the pole faces is 1.27 mWb determine of a pole face.

(b) If the conductor in part (a) is vertical , the current flowing downwards and the direction of the magnetic field is from left to right, what is the direction of the 80 N force?

[(a) 14.2 mm; (b) towards the viewer]

Magnitude and directions of induced emf's

6. Find the emf induced in a coil of 200 turns when here is a change of flux of 30 mWb linking with it in 40 ms.

[150V]

7. An emf of 25 V is induced in a coil of 3000 turns when the flux linking with it changes by 12 mWb . Find the time , in ms, in which the flux makes the change.

[144ms]

8. An ignition coil having 10 000 turns has an emf of 8 k V induced in it. What rate of change of flux is required for this to happen?

[0.8 Wb/s]

9. A flux of 0.35 mWb passing through a 125 turn coil is reversed in 25 ms. Find the average emf induced.

[3.5 V]

10. Calculate the emf induced in a coil of inductance 6 H by a current changing at a rate of 15 A/s.

[90V]

11.An emf of 2 kV is induced in a coil when acurrent of 5A collapses uniformly to zero in ms. Determine the inductance of the coil.

[4 H]

12. An average emf of 50 V is induced in a coil of inductance 160 mH when a current of 7.5 A is reversed. Calculate the time taken for the current to reverse.

[48 ms]

13. A coil of 2500 turns has a flux of 10 mWb linking with it when carrying a current of 2A. Calculate the coil inductance and the emf induced in the coil when the current collapses to zer in 20 ms.

[12.5 H;1.25 kV]

14. A conductoer of length 15 cm is moved at 750 mm/s at right angles to uniform flux density of 1.2. T . Determine the emf induced in the conductor.

[0.135V]

15. Find the speed that a conductor of length 120 mm must be moved at rightangles to a magnetic field of flux density 0.6 T tu induce in it an emf of 1.8 V.

[25 m/s]

16. A 26 cm long conductor moves at a uniform speed of 8 m/s through a uniform magnetic field of flux density 1.2 T .Determine the current flowing in the conductor when (a) its ends are open-circuited , and (b) its ends are connected to A load of 15 Ω resistance.

[(a) 0; (b) 0.16 A]

17. A straight conductor 500 mm long is moved with constant velocity at right angles both to its length and to a uniform magnetic field. Given that the emf induced in the conductor is 2. 5 V and the velocity is 5 m/s , calculate the flux density of the magnetic field . If the

conductor forms part of a closed circuit of total resistance 5 ohms, calculate the force on the conductor.

[1T; 0,25 N]

18. A car is travelling at 80 km/h .Assuming the back axle of the car is 1.76 m in length and the vertical component of the earth's magnetic field is 40μT , find the emf generated in the axle due to motion.

[1.56 m V]

19. A conductor moves with a velocity of 20 m/s at an angle of (a) 90^0; (b) 45^0;(c) 30^0 to a magnetic field produced between two square faced poles of side length 2.5 cm. If the flux on the pole face is 60 m Wb find the magnitude of the induced emf in each case.

[(a) 48 V; (b) 33. 9 V ;(C) 24 V.]

Inductance

20. Calculate the coil inductance when a current of 5A in a coil of 1000 turns produces a flux of 8 mWb linking with the coil.

[1.6H]

21. A coil is wound with 600 turns and has a self inductance of 2.5 H. What current must flow to set up a flux of 20 m Wb?

[4.8A]

22. What a current of 2 A flows in a coil , the flux linking with the coil is 80μ Wb .If the coil inductance is 0.5 H calculate the number of turns of the coil .

[12 500]

23. A coil of 1200 turns has aflux of 15 mWb linking with it when carrying a current od 4A.Calculate the coil inductance and the emf induced in the coil when the current collapses to zero in 25 m/s.

[4.5 H; 720 V]

24. A coil has 300 turns and an inductance of 4.5 mH. How many turns would be needed to produce a 0.72 mH coil assuming the same core is used?

[120 turns]

25. A steady current of 5A when flowing in a coil of 1000 turns produces a magnetic flux of 500μ Wb . Calculate the inductance of the coil. The current of 5A is then reversed in 12. 5 ms. Calculate the emf induced in the coil.

[0.1 H; 80 V]

Mutual inductance

26. The mutual inductance between two coils is 150 mH .Find the emf induced in one coil when the current in the other is increasing at the rate of 30 A/s.

[4.5 V]

27. Determine the mutual inductance between two coils when a current changing at 50 A/s in one coil induces an emf of 80 mV in the other.

[1.6 mH]

28. Two coils have a mutual inductance of 0.75 H .Calculate the emf induced in one coil when a current of 2.5 A in the other coil is reversed in 15 ms.

[250 V]

29. The mutual inductance between two coils is 240 mH . If the current in one coil changes from 15 A to 6 A in 12 ms calculate (a) the average emf induced in the other coil and (b) the change of flux linked with the other coil if it is wound with 400 turns.

[(a) 180 V; (b) 5.4 m Wb]

30. A mutual inductance of 0.6 H exists between two coils. If a current 6 A in one coil is reversed in 0.8 s calculate (a) the average emf induced in the other coil and (b) the number of turns on the other coil if the flux change linking with the other coil is 5 mWb .

[(a) 0.9 V; (b) 144]

The transformer

31. A transformer has 800 primary turns and 2000 secondary turns. If the primary voltage is 160 V determine the secondary voltage assuming an ideal transformer.

[400 V]

32. An ideal transformer with a turns ratio of 3:8 is fed from a 240 V supply. Determine its output voltage

[640 V]

33. An ideal transformer with a turns ratio of 12:1 and is supply at 192 V. Calculate the secondary voltage.

[16 V]

34. A transformer primary winding connected across a 415 V supply has 750 turns. Determine how many turns must be wound on the secondary side if an output of 1.66 k V is required.

[3000turns]

Energy stored in an inductor

35. An inductor of 20 H has a current of 2.5 A flowing in it. Find the energy stored in the magnetic field of the inductor .

[62.5 J]

36. Calculate the value of the energy stored when a current of 30 mA is flowing in a coil of inductance 400 mH.

[0.18 m J]

37. The energy stored in the magnetic field of an inductor is 80 J when the current flowing in the inductor is 2A . Calculate the inductance of the coil.

[40 H]

38. A flux of 30 mWb links with a 1200 turn coil when a current o5 5A is passing through the coil. Calculate (a) the inductance of the coil, (b) the energy stored in the magnetic field, and (c) the average emf induced if the current is reduced to zero in θ. 20 s.

[(a) 7.2 H; (b) 90 J ; (c) 180 V]

Alternating voltages and current

A. FORMULAE AND DEFINITIONS ASSOCIATED WITH ALTERNATING VOLTAGES AND CURRENTS

1. Electricity is produced by generators at power stations and distributed by a vast network of transmission lines (called the National Grid system) to industry and for domestic use. It is easier and cheaper to generate alternating current (ac) than direct current (dc) and ac is more conveniently distributed than dc is needed in preference to ac, devices called rectifiers are used for conversion .

2. Let a single turn coil be free to rotate at constant angular velocity ω symmetrically between the poles of a magnetic system as shown in Fig.12.82. An emf is generated in coil(from Faraday's Law) which varies in magnitude and reversed its direction at regular intervals. The reason for this is shown in Fig.12.83. In positions (a), (e) and (i) the conductors of the loop are effectively moving along the magnetic field , no flux is cut and hence no emf is induced. In position (c) maximum flux is cut and hence maximum emf is induced .In position (g), maximum flux is cut and hence maximum emf is again induced.

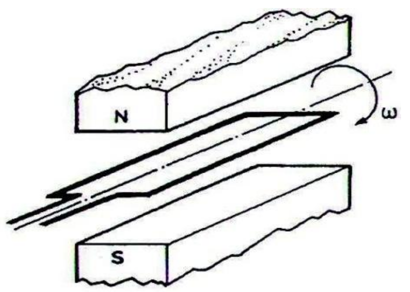


Fig. 12.82

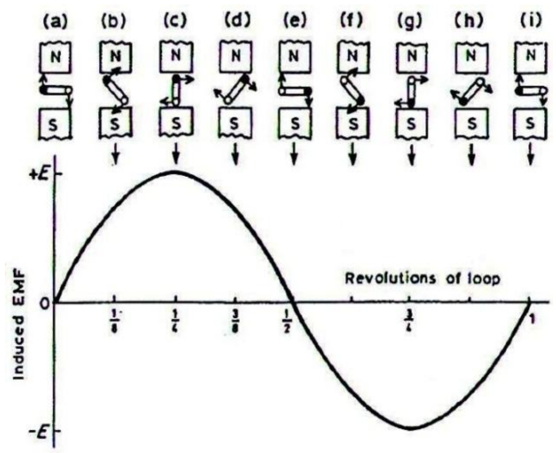


Fig.12.83

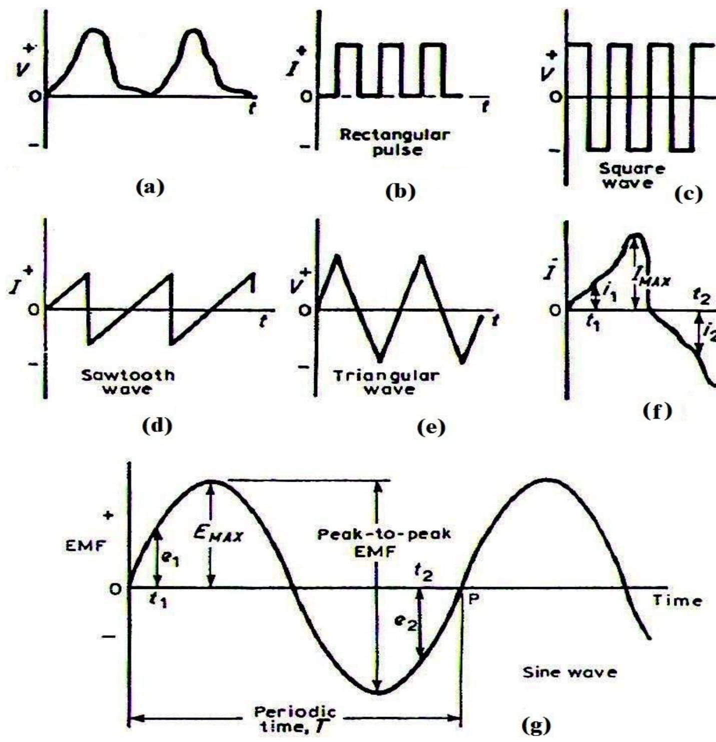


Fig.12.84

However , using Fleming's right –hand rule, the induced emf is in the opposite direction to that in position (c) and is thus shown as –E . In positions (b), (d) , (f) and (h) some flux is cut and hence some emf is induced. If all such positions of the coil are considered , in one revolution of the coil , one cycle of alternating emf is produced as shown. This is the principle of operation of the ac generator (i.e. the alternator).

3. If values of quantities which vary with time t are plotted to a base of time, the resulting graph is called a waveform .Some typical waveforms are shown in Fig.12.84 . Waveforms (a) and (b) are unidirectional waveforms, for, although, they vary considerably with time, they flow in one direction only (i.e. they do not cross the time axis and become negative). Waveforms (c) to (g) are called alternating waveforms ince their quantities are continually changing in direction (I.e. alternately positive and negative).

4. A waveform on the type shown in Fig12.84(g) is called a since wave. It is the shape of the waveform of emf produced by an alternator and thus the mains electricity supply is of sinusoidal' form.

5. One complete series of values is called a sycle (i.e. from O to P in Fig.12.84 (g)).

6. The time taken for an alternating quantity to complete one cycle is called the period or the periodic time , T , of the waveform.

7. The number of cycles completed in one second is called the frequency, f , of the supply and is measured in hertz , Hz . The standard frequency of the electricity supply in Great Britain is 50 H z.

$T = \frac{1}{f}\ f = \frac{1}{T}$

8. Instantaneous values are the values of the alternating quantities at any instant of time . They are represented by small letters. I,v , e etc ., (see Figs 12.84(f) and (g)).

9. The largest value reached in a half cycle is called the peak value or the maximum value or the crest value or the amplitude of the waveform .Such values are represented by V_{MAX} I_{MAX} etc.(see Figs 12.84 (f) and (g)). A peak –to – peak value of emf is shown in Fig. 12.84 (g) and is the difference between the maximum and minimum values in a cycle.

10. The average or mean value of a symmetrical alternating quantity, (such as a sine wave), is the average value measured over a half cycle , (since over a complete cycle the average value is zero).

Average or mean value = $\frac{area - under - the - curve}{length - of - base}$

The area under the curve is found by approximate methods such as the trapezoidal rule , the mid- ordinate rule or Simpson's rule. Average values are represented by, V_{AV} , I_{AV} etc.

For a sine wave, average value = 0.637 $_X$ maximum value (i.e. 2/π x maximum value).

11. The effective value of an alternating current is that current which will produce the same heating effect as an equivalent direct current. the effective value is called the root mean square (rms) value and whenever an alternating quantity is given , it is assumed to be the rms value. For example , the domestic mains supply in Great Britain is 240 V and is assumed to mean '240 Vrms'. The symbols used for rms values are I, V , E, etc. For a non – sinusoidal waveform as shown in Fig 4 the rms value is given by:

$$I = \sqrt{\left[\frac{i_1 + i_1^2 + ...i_n^2}{n}\right]}$$

where n is the number of intervals used

For a sine wave, rms value = 0.707 x maximum value

(i.e. $1\sqrt{2 \times \max imumvalue}$)

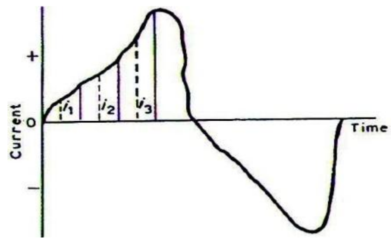


Fig.12.85

12. (a) Form factor = $\frac{rmsvalue}{averagevalue}$. For a sine wave , from factor = 1.11

(b) Peak factor = $\frac{\max imumvalue}{rmsvalue}$ For a sine wave , peak factor = 1.41

The values of form and peak factors give an inductance of the shape of waveforms.

13. In Fig.12.86, OA represents a vector that is free to rotate anticlockwise about 0 at angular velocity of ω rad/s . A rotating vector is known as a phasor. After time t seconds the vector OA has turned through an angle ωt. If the line BC is constructed perpendicular to OA as shown , then

sin $\omega t = \frac{BC}{OB}$ i.e. = $BC = OB \sin \omega t$

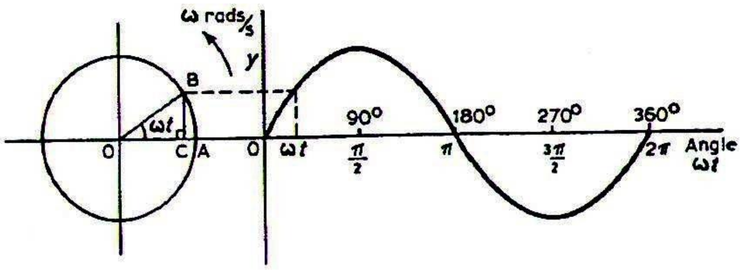


Fig.12.86

if all such vertical components are projected on to a graph of y against angle ωt (in radians), a sine curve results of maximum value OA. A ny quantity which varies sinusoidally can thus be represented as a phasor.

14. A sine curve may not always start at. o° To show this periodic function is represented by , $y = \sin(\omega t \pm \Phi)$ where Φ is a phase (or angle) difference compared

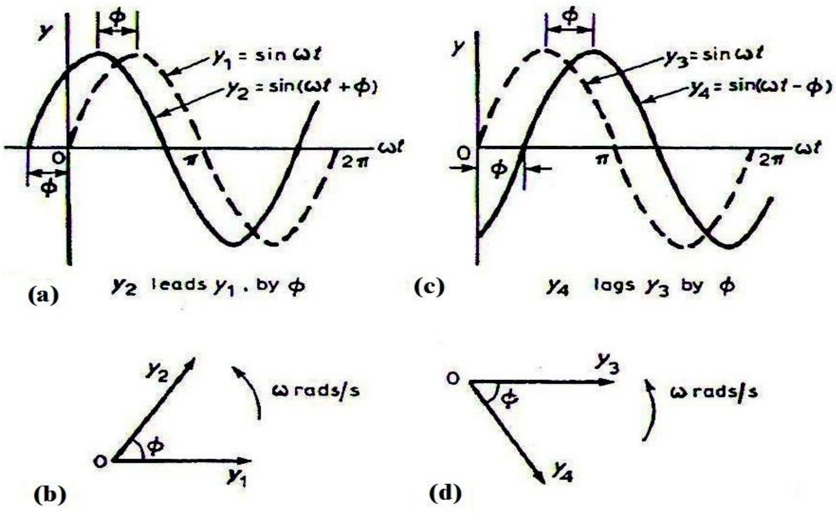


Fig.12.87

with $y = \sin \omega t$ In Fig.12.87(a) , $y_2 = \sin(\omega t + \Phi)$starts ϕ radians earlier than $y_1 = \sin \omega t +$ and is thus said to lead y_1 by ϕ radians . Phasors $y_1 and\ y_2$ are shown in Fig.12.87 (b) at the time when. $t = 0$ In Fig.12.87 (c) , $y_4 = \sin(\omega t - \Phi)$ start $\Phi\ radians$ later than $y_3 = \sin(\omega t.\Phi)$ and is thus said to lag y_3 by $\Phi\ radians$. Phasors $y_3 and\ y_4$ are shown in Fig.12.87 (d) at the time when $t = 0$

15. Given the general sinusoidal voltage, $v = V_{MAX}(\sin \pm \omega t)$

(i)Amplitude of maximum value = V_{MAX}

(ii) peak to peak value = $2V_{MAX}$

(i) Angular velocity = $\omega\ rads/s$

(ii) Periodic time,. $T = 2\pi/\omega$ seconds

(iii) Frequency , $f = \omega/2\pi Hz (Hence\ \omega = 2\pi f)$

(iv) ϕ = angle of lag or lead (compared with $V_{MAX} \sin \omega t$

16. The resultant of the addition (or subtraction) of two sinusoidal quantities may be determined either:

(a) by plotting the periodic functions graphically (see worked problems 13 and 16), or

(b) by resolution of phasors by drawing or calculation (see worked problems 14 and 15),

17. When a sinusoidal voltage is applied to a purely resistive circuit of resistance R, the voltage and current waveforms are in phase and $I = V/R$ (exactly as in a dc circuit). V and I are rms values.

18. For an ac resistive circuit , power $P = VI = I^2R = V^2/R$ watts (exactly as in a dc circuit). V and I are rms values.

19. The process of obtaining unidirectional currents and voltage s from alternating currents and voltages is called rectification. Automatic switching in circuits is carried out by devices called diodes.

20. Using a single diode ,as shown in Fig.12.88, half-wave rectification is obtained. When P is sufficiently positive with respect to Q diode D is switched on

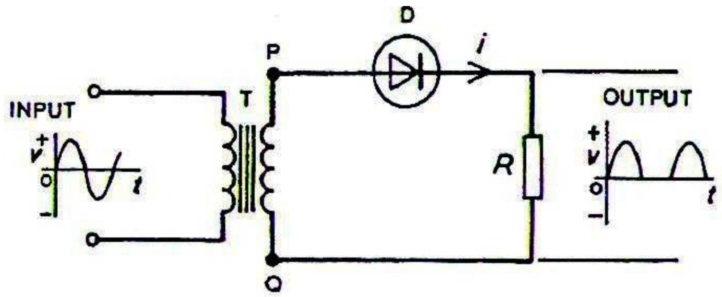


Fig.12.88

and current I flows. When P is negative with respect to, Q diode D is switched off. Transformer T isolates the equipment from direct connection with the mains supply and enables the mains voltage to be changed.

21. Two diodes may be used as shown in Fig.12.89 to obtain full wave rectification. *A* centre –tapped transformer T is used. When P is sufficiently positive with respect to Q, diode D_1 conducts and current flows (shown by the broken line in Fig.12.89).When S is positive with respects to Q , diode D_2 conducts and current flows (shown by continuous line in Fig.12.89).The current flowing in R is in the same direction for both half cycles of the input. The output waveform is thus as shown in Fig.12.89.

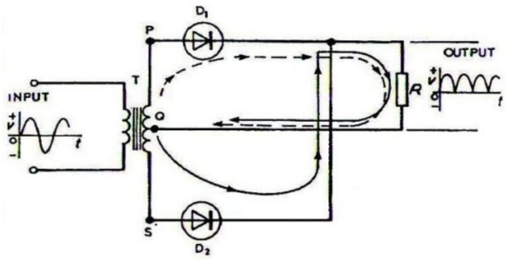


Fig. 12.89

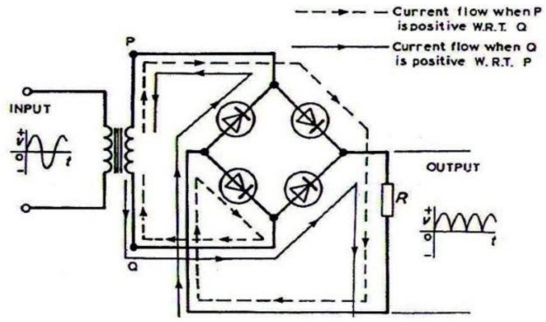


Fig.12.90

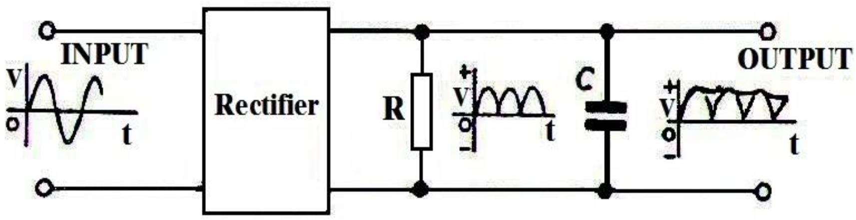


Fig.12.91

22. Four diodes may be used in a bridge rectifier , circuit ,as shown in Fig.12.90 to obtain full wave rectification . as for the rectifier shown in Fig.12.89, the current flowing in R is in the same direction for both half cycles of the input giving the output waveform shown.

23. To smooth the output of the rectifiers described above, capacitors having a large capacitance may be connected across the load resistors R. The effect of this is shown on them, output in Fig12.91.

B WORKED PROBLEMS ON ALTERNATING VOLTAGES AND CURRENTS

(a) FREQUENCY AND PERIODIC TIME

Problem 1. Determine the periodic time for frequencies of (a) 50 Hz and (b) 20 kHz

(a) Periodic Time $T = \frac{1}{f} = \frac{1}{50} = 0.02s \ or \ 20ms$

(b) Periodic Time $T = \frac{1}{f} = \frac{1}{20000} = 0.00005 \ s \ \ or \ 50ms$

Problem 2. Determine the frequencies for periodic times of (a) 4 ms, (b) 4 μs

(a) Frequency $f = \frac{1}{T} = \frac{1}{4 \times 10^{-3}} = Frequency \ f = \frac{1}{T} = \frac{1}{4 \times 10^{-3}} = \frac{1000}{4} = 250Hz$

(b) $= 250Hz$

(c) $Frequency \ f = \frac{1}{T} = \frac{1}{4 \times 10^{-6}} = \frac{100000}{4} = 250000Hz \ \ or$

$= 250KHz \ or \ \ 0.25MHz$

Problem 3. An alternating current completes 5 cycles in 8 ms. What is its frequency?

Time for 1 cycle $\frac{8}{5} ms = 1.6ms = periodic \ time \ T$

Frequency $f = \frac{1}{T} = \frac{1}{1.6 \times 10^{-3}} = \frac{1000}{1.6} = \frac{10000}{1.6} = 625Hz$

(b) AC VALUES OF NON- SINUSOIDAL WAVEFORMS

Problem 4. For the periodic waveforms shown in Fig.12.92 determine for each: (i) frequency; (ii) average value half a cycle ; (iii) rms value; (iv) form factor; and (v) peak factor.

(a) Triangular waveform (Fig.12.92(a))

(i) Time for 1 complete cycle = 20 ms = periodic time, T.

Hence frequency $f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = \frac{1000}{20} = 50Hz$

(ii) Area under the triangular waveform for a half cycle

$\frac{1}{2} \times base \times heigt = \frac{1}{2} \times (10 \times 10^{-3}) \times 200 = 1volt\sec ond$

Average value of waveform

$= \frac{area \ under \ curve}{length \ of \ base} = \frac{1 \ volt \ second}{10 \times 10^{-3} \ second} = \frac{1000}{10} = 100 \text{ V}$

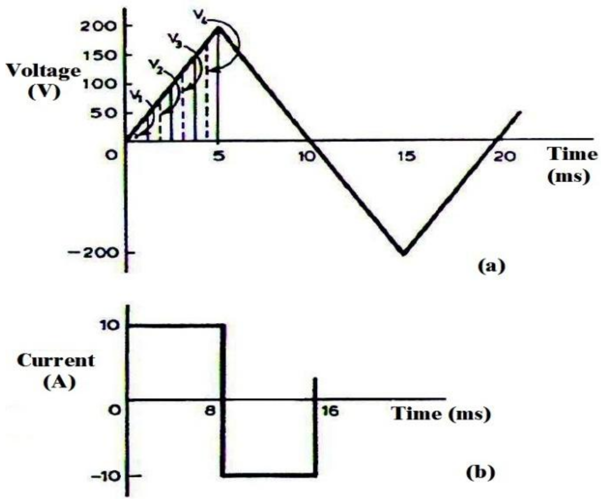


Fig.12.92

(iii) In Fig 12.92,(a), the first1/4 cycle is divided into 4 intervals.
Thus rms value

$$= \sqrt{\left[\frac{i_1^2 + i_2^2 + i_3^2 + i_4^2 +}{4}\right]} = \sqrt{\left[\frac{25^2 + 75^2 + 125^2 + 175^2}{4}\right]} = 114.6V$$

(Note that the greater the number of intervals chosen, the greater the accuracy of the result . For example, if twice the number of ordinates as that chosen above is used, the rms value is found to be 115. 6 V)

(iii) From factor $= \frac{rms\ value}{average\ value} = \frac{114.6}{100} = 1.15$

(iv) Peak factor $= \frac{maximum\ value}{rms\ value} = \frac{200}{114.6} = 1.75$

(b) Rectangular waveform (Fig.12.92 (b))

(i) Time for 1 complete cycle = 16 ms = periodic time T.

Hennce frequency , $f = \frac{1}{T} = \frac{1}{16 \times 10^{-3}} = \frac{1000}{16} = 62.5Hz$

(ii) Average value of over half a cycle = $\frac{area\ under\ curve}{length\ of\ base}$

$$\frac{10\times\left(8\times10^{-3}\right)}{8\times10^{-3}}=10A$$

(iii) The rms value $=\sqrt{\left[\frac{i_1^2+i_2^2+......i_n^2}{n}\right]}=10A$

however many intervals are chosen , since the waveform is rectangular.

(iv)Form factor $=\frac{rms\ value}{average\ value}=\frac{10}{10}=1$

(v) Peak factor $=\frac{maximum\ \ value}{armsvalue}=\frac{10}{10}=1$

Problem 5. The following table gives the corresponding values of current and time for a half cycle of alternating current.

time t (ms)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4. 0	4 .5	5.0
current I (A)	0	7	1.4	23	40	56	68	76	60	5	0

Assuming the negative half cycle is identical in shape to the positive half cycle , plot the waveform and find (a) the frequency of the supply , (b) the instantaneous values of current after 1.25 ms and 3.8 ms, (c) the peak or maximum value, (d) the mean or average value , and (e) the rms value of the waveform.

The half cycle of alternating current is shown plotted in Fig.12.93.

(a) Time for a half cycle = 5 ms . Hence the time for 1 cycle , i.e. the periodic time, T = 10 ms or 0.01 s.

frequency , $f=\frac{1}{T}=\frac{1}{0.01}=100Hz$

(b) Instantaneous value of current after 1.25 ms is 19 A, from Fig.12.93

Instantaneous value of current after 3.8mis is 70 A, from Fig.12.93

(c) Peak or maximum value = 76 A

(d) Mean or average value = $\frac{area\ under\ curve}{length\ of\ base}$

Using the mid –ordinate rule with 10 intervals , each of width 0.5 ms gives:

Area under curve (see Fig.12.93)

$$\left(0.5\times10^{-3}\right)[3+10+19+30+49+63+73+72+30+2]=\left(0.5\times10^{-3}\right)(351)$$

Hence mean or average value $=\frac{\left(0.5\times10^{-3}\right)(351)}{0.5\times10^{-3}}=35.1$

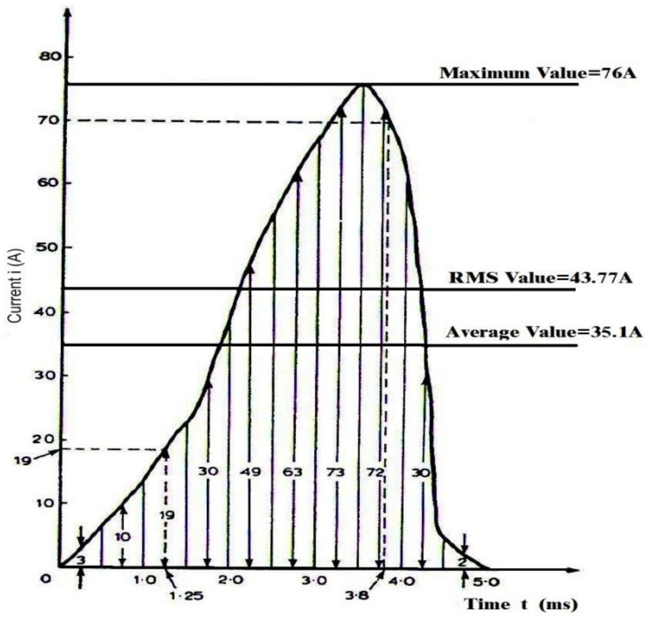


Fig.12.93

(e) rms value = $\sqrt{\left[\dfrac{3^2+10^2+19^2+30^2+49^2+73^2+72^2+30^2+2^2}{10}\right]}$

$$=\sqrt{\left[\frac{19157}{10}\right]}=43.8A$$

(c) AC VALUES OF SINUSOIDAL WAVEFORMS

Problems 6. Calculate the rms value of a sinusoidal current of maximum value 20 A

For a sine vawe,

$$rms\ \ value = 0.707\times \max imum\ value$$
$$= 0.707\times 20 = 14.14A$$

Problems 7. Determine the park and mean values for a 240 V mains supply.

For a sine wave , rms value of voltage $V = 0.707\times V_{MAX}$

A 240 V mains supply means that 240 V is the rms value.

Hence $V_{MAX} = \dfrac{7}{0.707} = \dfrac{240}{0.707} = 339.5V = peak$ value

Mean value $V_{AV} = 0.637\times V_{MAX} = 0.637\times 339.5 = 216.3V$

Problems 8. A supply voltage has a mean value of 150 V. Determine its maximum value and its rms value.

For a sine wave ,mean falue = $0.637\times \max imum\ value$

Hence maximum value = $\dfrac{meanvalue}{0.637} = \dfrac{150}{0.637} = 166.5V$

Further problems on ac values of sinusoidal waveforms may be found in section C (c), problems 8 to 12 , page 98.

(d) $v = V_{MAX}\sin(\omega t \pm \Phi)$

Problem 9. An alternating voltage is given by $v = 282.8\sin 314t volts$ .Find (a) the rms voltage, (b) the frequency and (c) the instantaneous value of voltage when. $t = 4ms$

(a) The general expression for an alternating voltage is $v = V_{MAX}\sin(\omega t \pm \Phi)$

Comparing $v = 282.8\sin 314t$ with this general expression gives the peak voltage as 282.8 V

Hence the rms voltage $= 0.707 \times \max imum\ value = 0.707 \times 282.8 = 200V$

(b) Angular velocity , $\omega = 314 rads/s\ i.e. 2\pi f = 314$

Hence frequency, $f = \dfrac{314}{2\pi} = 50Hz$

(c) When $t = 4ms, v = 282.8\sin(314 \times 4 \times 10^{-3})$

$= 282.8\sin(1.256)$

$1.256 radians = \left(1.256 \times \dfrac{180}{\pi}\right)^{\circ} = 71.96^{\circ} = 71^{\circ}58$

$Hence\ v = 282.8\sin 71^{\circ}58$

$= 268.9V$

Problem 10 .An alternating voltage is given by $v = 75\sin(200\pi t - 0.25)$ volts. Find (a) the amplitude , (b) the peak – to – peak value , (c) the rms value, (d) the periodic time , (e) the frequency; and (f) the phase angle (in degrees and minutes) relative to 75 sin 200 πt.

Comparding $v = 75\sin(200\pi t - 0.25)$with the general expression $v = V_{MAX}\sin(\omega t \pm \Phi)$ gives:

(a) Amplitude , or peak value = 75 V

(b) peak – to – peak value = $2 \times 75 = 150V$

(c) the rms value = 0.707 x maximum value = $0.707 \times 75 = 53V$

(d) Angular velocity , $\omega = 200\pi rads/s$

Hence periodic time , $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{200\pi} = \dfrac{1}{100} = 0.01s\ or\ 10ms$

(e) Frequency , $f = \dfrac{1}{T} = \dfrac{1}{0.01} = 100Hz$

(f) Phase angle, ϕ = = 0.25 radians lagging 75 sin 200 πt

$0{,}25 rads = \left(0.25 \times \dfrac{180}{\pi}\right)^{\circ} = 14.32^{\circ} = 14^{\circ}19$

Hence phase angle = $14^{\circ}19$

(lagging)

Problem 11. An alternating voltage v, has a periodic time of 0.01 s and a peak value of 40 V. When time t is zero, $v = -20V$ .Express the instantaneous voltage in the form l.

$v = V_{MAX}\sin(\omega t \pm \Phi)$

Amplitude , $V_{MAX} = 40V$

Periodic time $T = \frac{2\pi}{\omega}$ Hence angular velocity , $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi$ rads/s

$v = V_{MAX}\sin(\omega \pm \Phi)$ thus becomes $v = 40\sin(200\pi t + \varphi)V$.

when time $t = 0, v = -20V$

i.e. $-20V = 40\sin\varphi$

sin $\varphi = \frac{-20}{40} = -5$

Hence $\varphi = \arcsin -0.5 = -30° = \left(-30 \times \frac{\pi}{180}\right) rads = \frac{\pi}{6}$ rads

thus $v = 40\sin\left(200\pi t \frac{\pi}{6}\right)V$

Problem 12 .The current in an ac circuit at any time t seconds is given by: $120\sin(100\pi t + 36) ampers$ Find

(a) the peak value , the periodic time, the frequency and phase angle relative $to 120\sin 100\pi t;$

(b) the value of the current when $t = 0$

(c) the value of the current when $t = 8ms$

(d) the value of the current first reaches 60 A, and the time when the current is first a maximum

(a) Peak value = 120 A

Periodic time $T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi}(\sin ce\, \omega = 100\pi)$

$= \frac{1}{50} = 0.02s \quad or 20ms$

Frequency $f = \frac{1}{T} = \frac{1}{0.02} = 50Hz$

Phase angle $= 0.36 rads = \left(0.36 \times \frac{180}{\pi}\right)° = 20°38' leading$

(a) When $t = 0, I = 120\sin(0 + 0.36) = 120\sin 20°38' = 42.29A$

(b) When $t = 8ms, I = 120\sin\left[100\pi\left(\frac{8}{10^3}\right)\right] = 120\sin 2.8733$

$= 120\sin 164°38' = 31.80A$

(d) When $I = 60A.60 = 120\sin(100\pi t + 0.36)\frac{60}{120} = \sin(100\pi t + 0.36)$

$$(100\pi t + 0.36) = \arcsin 0.5 = 30° = \frac{\pi}{6} rads = 0.5236 rads$$

Hence time, $t = \dfrac{0.5236 - 0.36}{100\pi} = 0.5208 ms$

(c) When the current is a maximum , $I = 120A$

Thus $120 = 120\sin(100\pi t + 0.36)$

$1 = \sin(100\pi t + 0.36)$

$$(100\pi t + 0.36) = \arcsin 1 = 90° = \frac{\pi}{2} rads = 1.5708 rads$$

Hence time, $t = \dfrac{1.5708 - 0.36}{100\pi} = 3.854 ms$

(a) COMBINATION OF PERIODIC FUNCTIONS

Problem 13. The instantaneous values of two alternating currents are given by $i_1 = 20\sin\omega t$ amperes and $i_2 = 10\sin(\omega t + \pi/3)$ amperes. by plotting $i_1 + i_2$ on the same axes, using the same scale, over on cycle ,and adding ordinates at intervals , obtain a sinusoidal expression for $i_1 + i_2 + i_1 = 20\sin\omega t$ and $i_2 = 10\sin\left(\omega t + \dfrac{\pi}{3}\right)$ are shown plotted in Fig.12.94.

Oedinates of i_1 and i_2 are added at, , say, $15°$ intervals (a pair of dividers are useful for this). For example

at $30°$, $i_1 + i_2 = 10 + 10 = 20A$

at $60°$, $i_1 + i_2 = 8{,}7 + 17{,}3 = 26A$

at $150°$

$i_1 + i_2 = 10 + (-5) = 5A$ and so on

ωt (degrees)	0	30	60	90
$\sin\omega t$	0	0.5	0.866	1
$i_1 = 20\sin\omega t$	0	10	17.32	20
$(\omega t + 60)$	60	90	120	150
$\sin(\omega t + \frac{\pi}{3})$	0.866	1	0.866	0.5
$i_2 = 10\sin(\omega t + \frac{\pi}{3})$	8.66	10	8.66	5

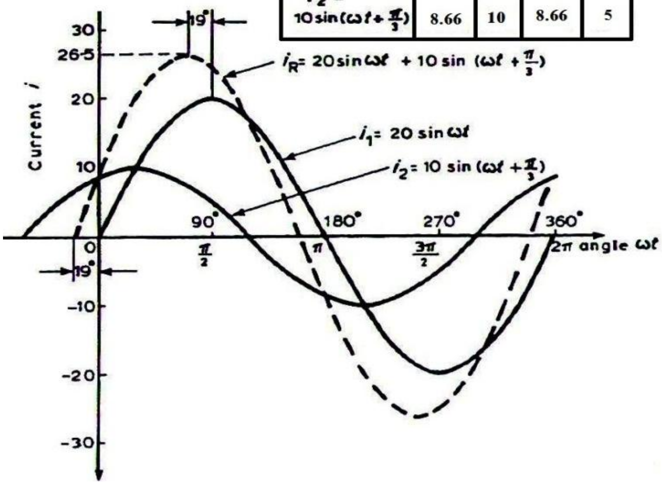


Fig.12.94

The resultant waveform for $i_1 + i_2$ is shown by the broken line in Fig.12.94. It has the same period , and hence frequency , as i_1 and i_2 . The amplitude or peak value is 26.5 A. The resultant waveform leads the curve $i_1 = 20\sin\omega t$ by 19 ^{0}

i.e. $\left(19 \times \frac{\pi}{180}\right)$ rads = 0. 332 rads

Hence the sinusoidal expression for the resultant $i_1 + i_2$ is given by:

$i_R = i_1 + i_2 = 26.5\sin(\omega t + 0.332)A$

Problem 14. Two alternating voltages are represented by v_1 = 50 sin ωt volts and v_2 = 100 sin (ωt – π/6) V. Draw the phasor diagram and find , by calculation, a sinusoidal expression to represent. $v_1 + v_2$

Phasors are usually drawn at the instant when the time t = 0. Thus v_1 is drawn horizontally 50 units long and v_2 is drawn 100 units long lagging $i_1 by \pi/6$ rads ,i.e. 30^0 . This is shown in Fig.12.95 (a) where 0 is the point of rotation of the phasors . Procedure to draw phasor diagram to represent: $v_1 + v_2$

(i) Draw v_1 horizontal 50 units long, i.e. oa of Fig.12.95(b)
(ii) Join v_2 to the end of v_1 at the appropriate angle, i.e. ab of Fig (12.95b).
(iii) The resultant $v_R = v_1 + v_2$ is given by the length ob and its phase angle may be measured with respect to v_1

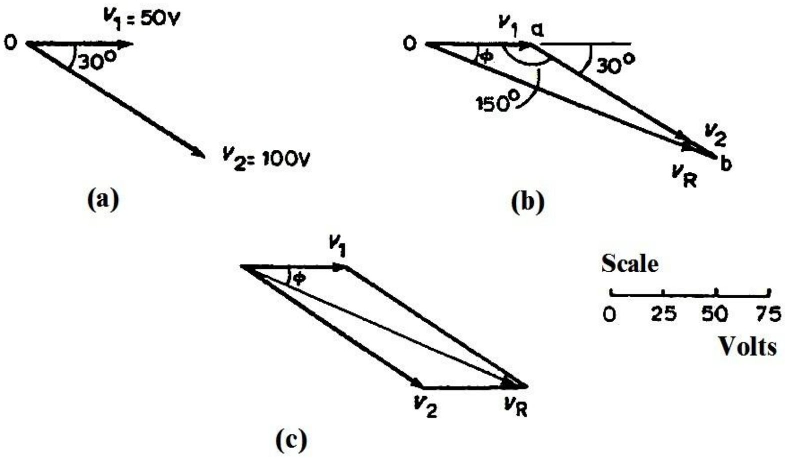


Fig.12.95

Alternatively , when two phasors are being added the resultant is always the diagonal of the parallelogram , as shown in Fig.12.95(c).

From the drawing , by measurement , v_R = 145 V and angle φ = 20^0 laggig . v_1 A more accurate solution is obtained by calculation , using the cosine and sine rules. Using the cosine rule on triangle oab of Fig.12.95(b) gives:

$v_R{}^2 = v_1{}^2 + v_2{}^2 - 2v_1v_2\cos 150°$

$50^2 + 100^2 - 2(50)(100)\cos 150°$

$= 2500 + 10\,000 - (-8660)$

$v_R = \sqrt{(21\,160)} = 145.5\,V$

Using the sine rule, $\frac{100\sin 150^{\circ}}{\sin\Phi}$

$\sin\ \Phi = \frac{100\sin 150^{\circ}}{145.5} = 0.3436$

$\phi = \arcsin 0.3436 = 20°6' = 0.35 radians, and lags\ v_1$

Hence $v_R{}^2 = v_1{}^2 + v_2{}^2 = 145.5\sin(\omega t - 0.35)\mathrm{V}$

Problem 15. Find a sinusoidal expression for ($v_1 + v_2$) of Problem 13 (a) by drawing phasors , (b) by calculation.

(a) The relative positions of v_1 and v_2 at time t = 0 are shown as phasors in Fig.12.96 (a) . The phasor diagram in Fig 15 (b) shows the resultant i_R and i_R is measured as 26 A and angle ϕ as 19^0(i.e.0.33 rads) (i.e. 0.33 rads)leading i_1 Hence , by drawing, $v_R = 26\sin(\omega t + 0.33)A$

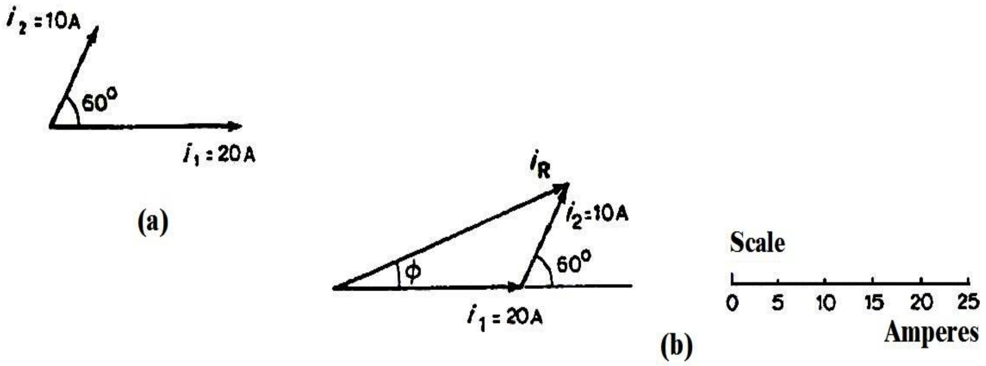


Fig.12.96

(b) From Fig.12.96 (b) , by the cosine rule:

$i_R{}^2 = 20^2 + 10^2 - 2(20)(10)\cos 120°$

from which i_R = 26.46 A

by the sine rule $\frac{10}{\sin\phi} = \frac{26.46}{\sin 120°}$

from which $\phi = 19°10'(i.e.0.333 rads)$

Hene , by calculation $i_R = 26.46\sin(\omega t + 0.333)A$

Problem 16. Ttwo alternating voltages are given by $v_1 = 120\sin\omega t$ volts and $i_2 = 200\sin(\omega t - \pi/4) volts$ Obtain sinusoidal expressions for $v_1 - v_2$ (a) by plotting waveforms , and (b) resolution of phasors.

(a) $v_1 = 120\sin\omega t$ volts and $v_2 = 200\sin(\omega t - \pi/4)$ are shown plotted in Fig.12.97. Care must be taken when subtracting values of ordinates especially when at least one of the ordinates is negative. For example

at $30°.v_1 - v_2 = 60 - (-52) = 112V$

at $60^2.v_1 - v_2 = 104 - 53 = 52V$

at $150°.v_1 - v_2 = 60 - 193 = -133V$ and so on

The resultant waveform, $v_R = v_1 - v_2$ is shown by the broken line in Fig.12.97.

The maximum value of v_R is 143 V and the waveform is seen to lead v1 by 99^0 (i.e. 1.73 radians).

Hence , by drawing $v_R = v_1 - v_2$ = 143 sin (ωt +1.73) volts

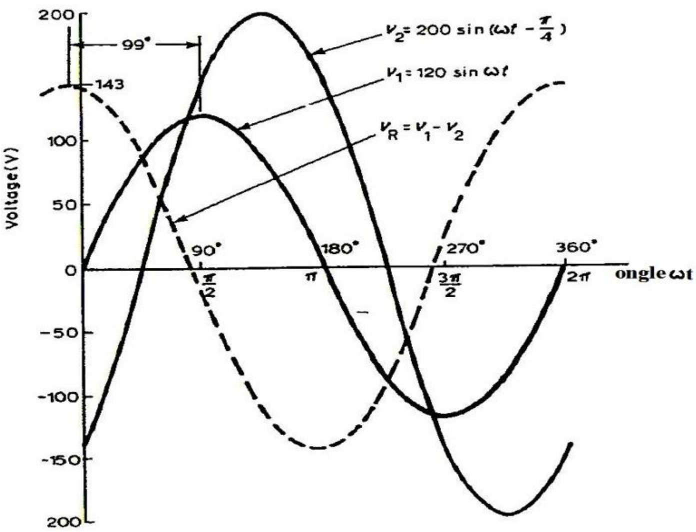


Fig.12.97

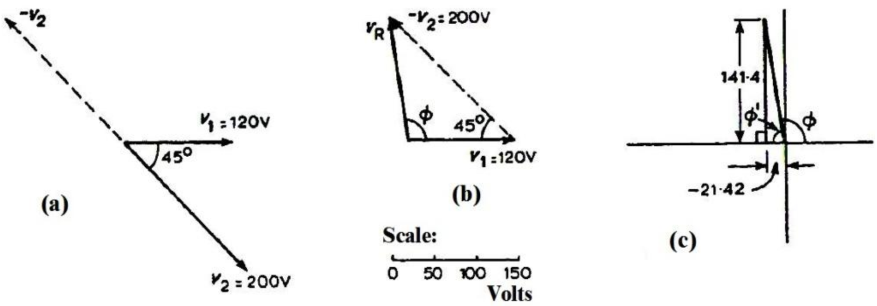


Fig.12.98

(b) The relative positions of v_1 and v_2 are shown at time $t = 0$ as phasors in Fig.12.98 (a) . Since the resultant of $v_1 - v_2$ is required, - v_2 is drawn in the opposite direction to + v_2 (shown by the broken line in Fig.12.98 (a)). The phasor diagram with the resultant is shown in Fig.12.98 (b) where - v_2 is added phasorially to v_1

By resolution :

Sum of horizontal components of . v_1 and v_2 = 120 cos 0^0- 200 cos 45^0 = -21.42 Sum of vertical components of . v_1 and v_2 = 120 cos 0^0+ 200 sin45^0 = 141.4 From Fig.12.98 (c) ,

resultant $v_R = \sqrt{[(-21.42)]^2 + (141.4^2)]} = 143.0$

and tan $\phi = \dfrac{141,4}{21,42}$ = tan 6. 6013,from which

$\phi = \arctan 6.6013 = 81°23$ 'and

$\phi = 93°37 or 1.721 radians$

Hence , by resolution of phasors , $v_R = v_1 - v_2$ = 143.0 sin (ωt +1.721)volts.

Problem 17. Determine the current flowing in a 20 Ω resistor when a 240 V, 50 Hz supply voltage is applied across the resistor.Find also the power disspated by the resistor.

PURELY RESITIVE AC CIRCUITS

For a purely resistive ac circuit

current $I = \dfrac{V}{R} = \dfrac{240}{20} = 12A$

power $P = VI = 240 \times 12 = 2880W$
$= 2.88kW$

Problem 18. A sinusoidal voltage of maximum value 50 V causes a current of maximum value 4 A to flow through a resistance .Find the value of the resistance and the power developed. What is the energy dissipated in 2 minutes?

rms value of voltage , $V = 0.707 \times \max imum\ value = 0.707 \times 50 = 35.35V$

rms value of current , $I = 0.707 \times \max imum\quad value = 0.707 \times 4 = 2.828A$

For a purely resistive ac circuit, resistance $R = \dfrac{V}{I} = \dfrac{35.35}{2.828} = 12.5\Omega$

Power developed . $P = VI = 35.35 \times 2.828 = 100W$

Energy dissipated = $\dfrac{Power \times time = 100 \times (2.60)Ws}{12000J = 12kJ}$

C FURTHER PROBLEMS ON ALTERNATING VOLTAGES AND CURRENTS

(a) SHORT ANSWER PROBLEMS

1. Briefly explain the principle of the simple alternator
2. What is the difference between an alternating and a unidirectional waveform.
3. What is meant by (a) waveform ; (b) cycle.
4. The time to complete one cycle of a waveform is called the . . . .
5. What is frequency?Name its unit.
6. The mains supply voltage hasa special shape of waveform called a . . . .
7. Define peak value.
8. What is meant by the rms value
9. The domestic mains electricity supply voltage in Great Britaiin is . . . . .
10. What is the mean value of a sinusoidal alternating emf which has a maximum value of 100 V.
11. The effective value of a sinusoidal waveforms is . . . . X maximum value.
12. What is a phasor quantity?
13. Complete the statement : Form factor = ... , and for a sine wave , form factor = ...
14. Complete the statement : Peak factor = ... , and for a sine wave , : Peak factor = ...
15. A sinusoidal current is given by $I = I_{MAX}\sin(\omega t \pm \alpha)$What do the symbols, I_{max} ω and α *represent*
16. A sinusoidal voltage of 250 V is applied across a pure resistance of 5Ω. What is (a) the current flowing , and (b) the power developed across the resistance?
17. How is switching obtained when converting ac to dc?
18. Draw an appropriate circuit diagram suitable for half –wave rectification.

P2.1 Determine the voltages V_1 and V_2 in the network in Fig.P2.1 using voltage division.

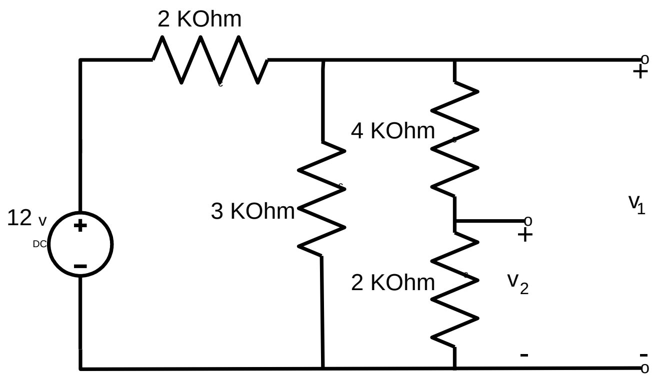


Fig.P2.1

P2.2 Find the currents I_1 and I_0 in the circuit in Fig.P2.2 using current division.

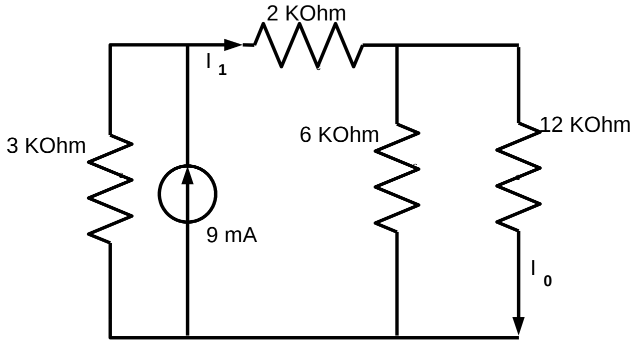


Fig.P2.2

P2.3 Find the resistance of the network in Fig.P2.3 at the terminals A-B.

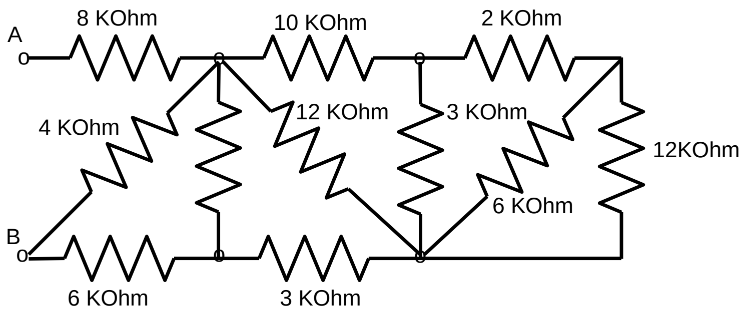


Fig.P2.3

P2.4 Find the resistance of the network shown in Fig.P2.4 at the terminals A-B.

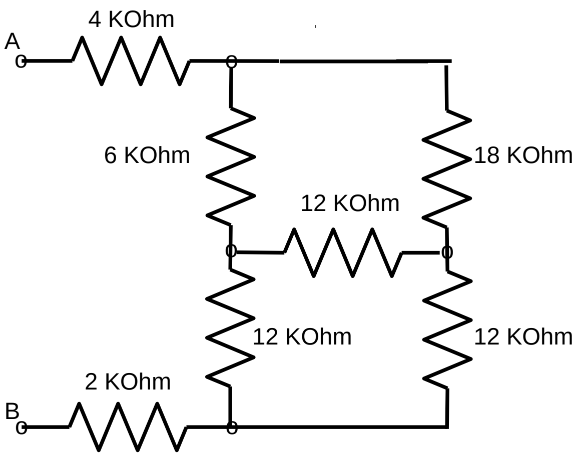


Fig.P2.4

P2.5 Find all the currents and voltages in the network in Fig.P2.5.

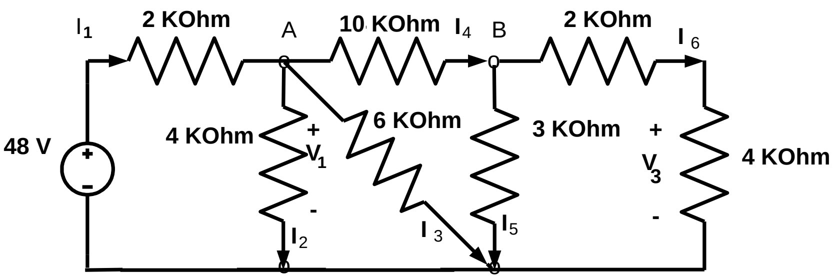


Fig.P2.5

P2.6 In the network in Fig.P.2.6, the current in the 4 KOhm resistor is $I_5 = 3mA$. Find the input voltage. V_5

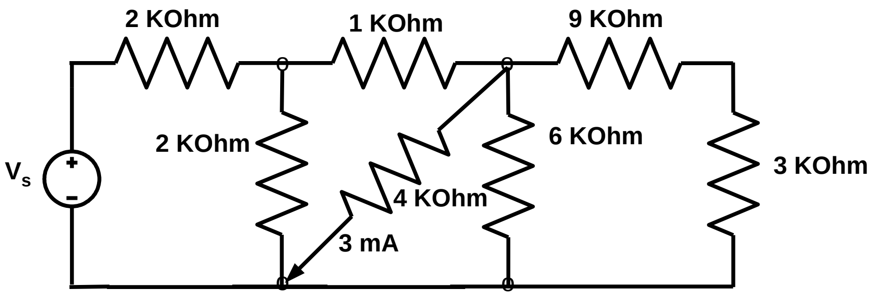


Fig.P2.6

SOLUTIONS

S2.1. We recall that if the circuit is of the form

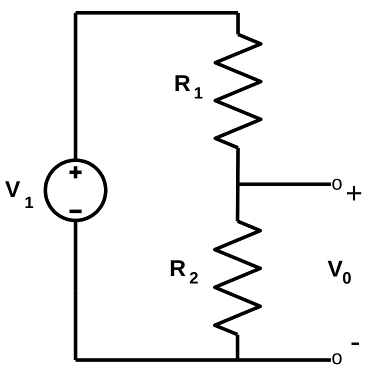


Fig.S2.1(a)

Then using voltage division $V_0 = \left(\dfrac{R_2}{R_1 + R_2} \right) \times V_1$ that is the voltage V_1 divides between the two resistors in direct proportion to their resistances. With this in mind, we can draw the original network in the form

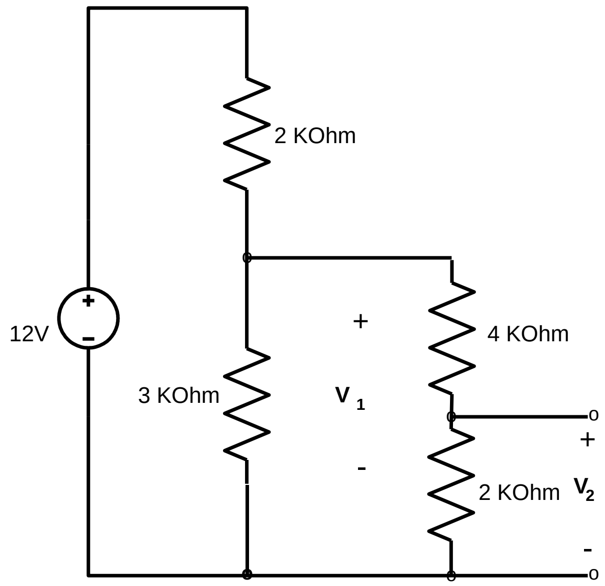


Fig.S2.1(b)

Now voltage division can be sequentially applied. From Fi.S2.1 (c).

$$V_1 = \left(\frac{2K}{2K + 2K} \right) \times 12 = 6V$$

Then from the network in Fig.S2.1(b)

$$V_2 = \left(\frac{2K}{2K + 4K} \right) \times V_1 = 2V$$

S2.2 If we combine the 6K and 12K Ohm resistors, the network is reduced to that shown in Fig.S2.2(a).

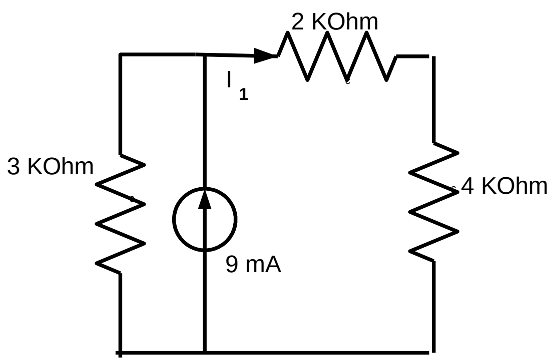


Fig.S2.2(a).

The current emanating from the source will split between the two parallel paths, one which is the 3KOhm resistor and the other is the series combination of the 2K and 4KOhm resistors. Applying current division

$$I_1 = \frac{9}{K}\left(\frac{3K}{3K+\left(2K+4K\right)}\right) = 3mA$$

Using KCL or current division we can also show that the current in the 3KOhm resistor is 6mA. The original circuit in Fig.S2.2(b) indicates that I_1 will now be split between the two parallel paths defined by the 6K and 12KOhm resistors.

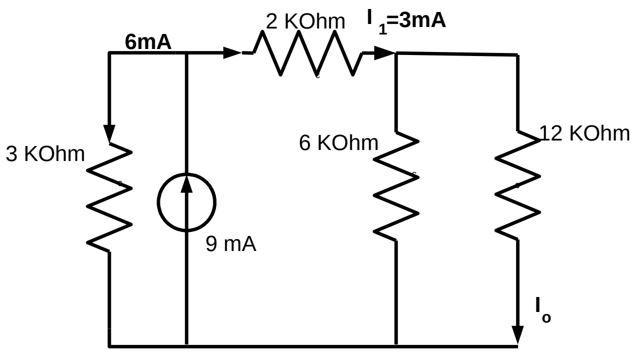


Fig.S2.2(b)

Applying current division again

$$I_0 = I_1\left(\frac{6K}{6K+12K}\right)$$

$$I_0 = \frac{3}{K}\left(\frac{6K}{18K}\right) = 1mA$$

Likewise the current in the 6KOhm resistor can be found by KCL or current division to be 2mA. Note that KCL is satisfied at every node.

S2.3 To provide some reference points, the circuit is labeled as shown in Fig. S2.3(a).

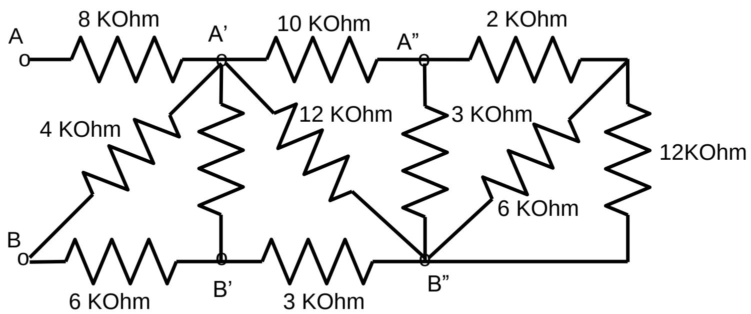


Fig. S2.3(a)

Starting at the opposite end of the network from the terminals A-B, we begin looking for resistors that can be combined, e.g. resistors that are in series or parallel. Note that none of the resistors in the middle of the network can be combined in anyway. However, at the right-hand edge of the network, we see that the 6K and 12K ohm resistors are in parallel and their combination is in series with the 2Kohm resistor. This combination of 6K (parallel 12K+2K) is in parallel with the 3KOhm resistor reducing the network to that shown in Fig.S2.3(b).

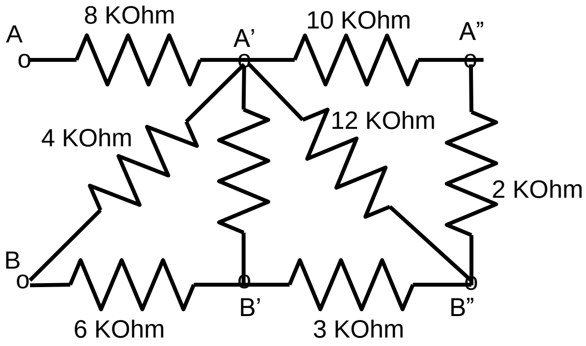


Fig.S2.3(b)

Repeating the process, we see that the 2KOhm resistor is in series with the 10KOhm resistor and that combination is in parallel with the 12KOhm resistor. This equivalent ((2K+10K) in parallel with 12K) =6KOhm) resistor is in series with 3KOhm resistor and that combination is in parallel with the A'B'=18 KOhm resistor. So equivalent resisto will be 6KOhm, and thus the network is reduced to that shown in Fig.S2.3(c).

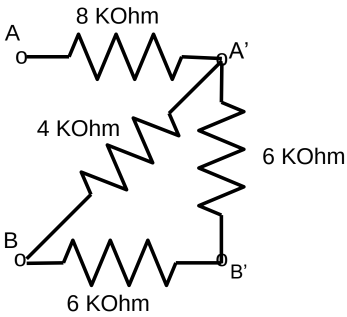


Fig.S2.3(c)

At this point we see that two 6K resistors are in series and their combination in parallel with 4KOhm resistor. And the last one is in series with 8KOhm resistor and yielding a total resistance $R_{AB} = 3K + 8K = 11KO$hm.

S2.4 An examination of the network indicates that there are no series or parallel combination of resistors in the network. However, if redraw the network in the form shown in Fig.S4(a), we find that the networks have two deltas back to back.

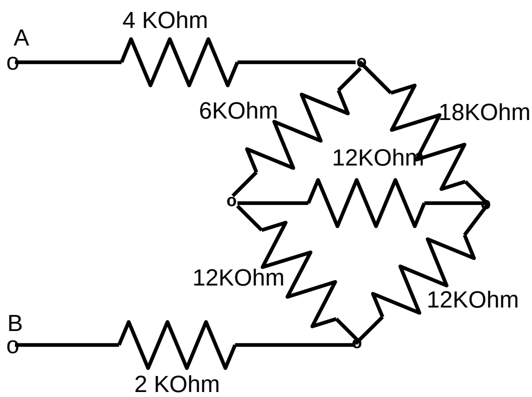


Fig.S4(a)

If we apply the $\Delta \rightarrow Y$ transformation to either delta, the network can be reduced to a circuit in which the various resistors are either in series or parallel. Employing the $\Delta \rightarrow Y$ transformation to the upper delta, we find the new elements using the following equations as illustrated in Fig.S2.4(b).

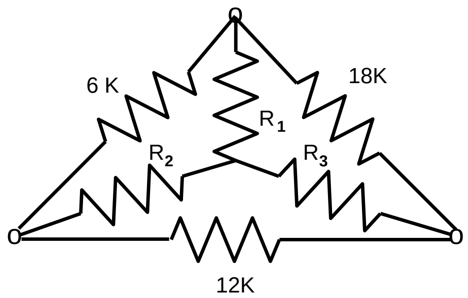


Fig.S2.4(b)

$$R_1 = \frac{(6K)(18K)}{6K + 12K + 18K} = 3KOhm$$
$$R_2 = \frac{(6K)(12K)}{6K + 12K + 18K} = 2KOhm$$
$$R_3 = \frac{(12K)(18K)}{6K + 12K + 18K} = 6KOhm$$

The network is now reduced to that shown in Fig.S2.4(c)

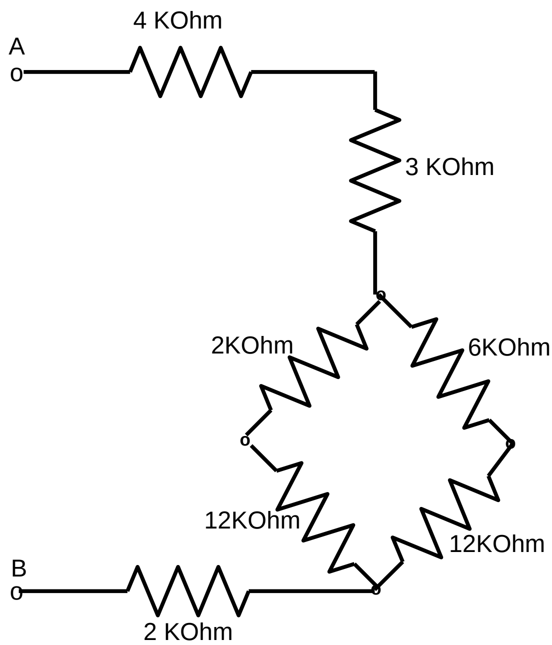


Fig.S2.4(c)

Now the total resistance, R_{AB} is equal to the parallel combination of (2K+12K) and (6K+12K) in series with the remaining resistors i.e.

$$R_{AB} = 4K + 3K = \frac{14K \times 18K}{(14K + 18K)} + 3K = 16.875 \text{ KOhm}$$

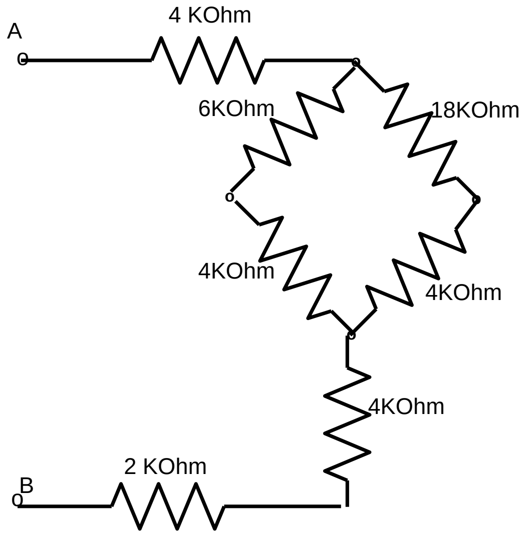


Fig.2.4(d)

In this case total resistance R_{AB} is

$$R_{AB} = 4K + \frac{(6K + 4K)(18K + 4K)}{(6K + 4K + 18K + 4K)} + 4K + 2K = 16.875 KOhm$$

Which is of course, the same as earlier result.

S2.5 Ourapproach to this problem will be to first find the total resistance seen by the source, use it to find I_1 and then apply Ohm's law, KCL, KVL, current division and voltage division to determine the remaining unknown quantities. Starting at the opposite end of the network from the source, the 2K and 4KOhm resistors are in series and that combination is in parallel with the 3KOhm resistor yielding the network in Fig.S2.5(a).

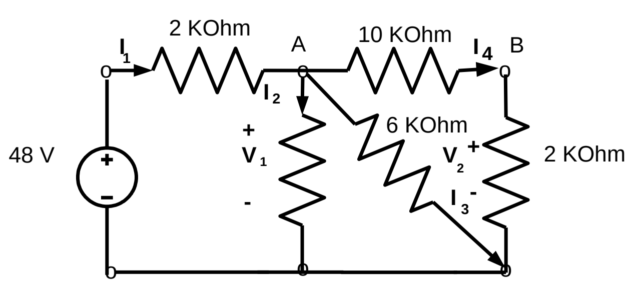


Fig.S2.5(a)

Proceeding, the 2K and 10KOhm resistors are in series and their combination is in parallel with both the 4K and 6KOhm resistors. The combination (10K+2K) and 6K gives 2KOhm. Therefore, this further reduction of the network is as shown in Fig.S2.5(b) .

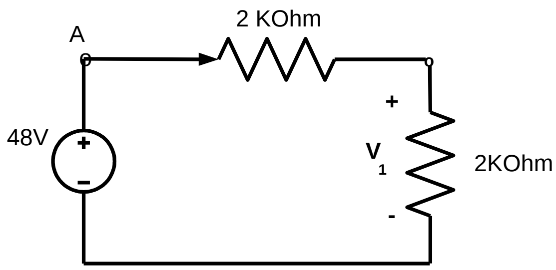


Fig.S2.5(b)

Now I_1 and V_1 can be easily obtained.

$$I_1 = \frac{48}{2K+2K} = 12mA$$

And by Ohm's law . $V_1 = 2KI_1 = 24V$

Or using voltage division

$$V_1 = 48\left(\frac{2K}{2K+2K}\right) = 24V$$

Once V_1 is known, I_2 can be obtained using Ohm's law

$$I_2 = \frac{V_1}{4K} = \frac{24}{4K} = 6mA$$

$$I_3 = \frac{V_1}{6K} = \frac{24}{6K} = 4mA$$

I_4 can be obtained using KCL at node A. As shown from the circuit diagram.

$$I_1 = I_2 + I_3 + I_4 \ (\frac{12}{K} = \frac{6}{K} + \frac{4}{K} + I_4)$$

So $I_4 = 2mA$

The voltage V_2 is then $V_2 = V_1 - 10KI_4 = 4V$

Or using voltage division $V_2 = V_1\ (\ \frac{2K}{10K+2K}\ ) = 4V$

Knowing, V_2 I_5 can be defined using Ohm's law

$$I_5 = \frac{V_2}{3K} = \frac{4}{3}mA$$

And also

$$I_6 = \frac{V_2}{2K+4K} = \frac{2}{3}mA$$

Current division can also be used to find I_5 and . I_6

$$I_5 = I_4\left(\frac{2K+4K}{2K+4K+3K}\right) = \frac{4}{3}mA \quad \text{and} \quad I_6 = I_4\left(\frac{3K}{3K+2K+4K}\right) = \frac{2}{3}mA$$

Finally V_3 can be obtained using KVL or voltage division

$$V_3 = V_2 - 2KI_6 = 8/3V \quad \text{and} \quad V_3 = V_2\left(\frac{4K}{2K+4K}\right) = \frac{8}{3}V$$

The network is labeled with all currents and voltages in Fig.S2.6.

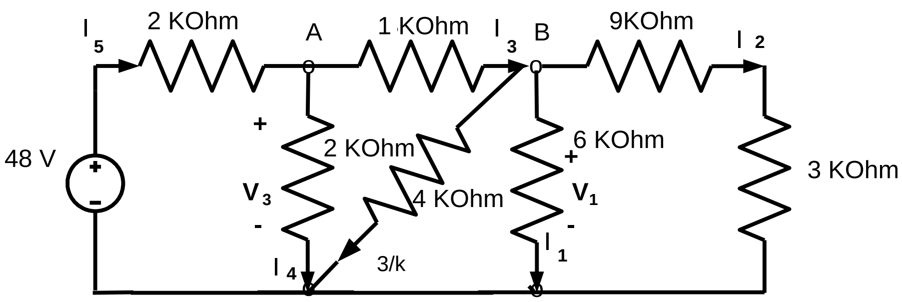


Fig.S2.6

Given 3 mA current in the 4KOhm resistor, the voltage. $V_1 = (3/K)(4K) = 12V$
$I_1 = V_1/6K = 2mA$ and . $I_2 = V_1/(9K + 3K) = 1mA$ Applying KCL at node B, . $I_3 = 3/K + I_1 + I_2 = 6mA$
Then using Ohm's law

$$V_2 = I_3(1K) = 6V$$

KVL can then be used to obtain V_3 i.e. $V_3 = V_2 + V_1 = 18V$. Then

$$I_4 = V_3 / 2K = 9mA \quad \text{and} \quad V_5 = V_3 + V_4 = 15mA$$

Using Ohm's law $V_4 = (2K)I_5 = 30V$ and finally . $V_5 = V_4 + V_3 = 48V$

APPANDIX A

Transformer Tests Using Simulink

Objectives:

The main objective is to obtain transformer equivalent circuit parameters by simulating three transformer tests.

This part of the lab consists of three different simulations:

Open Circuit Test

Short Circuit Test

Load Test

Simulations are designed to follow the actual hardware experiments as closely as possible. That will give you a chance to compare the simulation results to those of the actual experiment. For the tests, we will use the Matlab Power System Blockset that provides models of the main elements of power systems such loads, transformers, etc.

Simulink diagrams for each test will be provided during the experiment. The single-phase transformer used in these simulations has the following nameplate information and the equivalent circuit is given in Fig. A1.

Nameplate Information:

Rated Power: 2500 VA

Rated Voltage: 220/220 V

Rated Frequency: 50 Hz

Ideal transformer

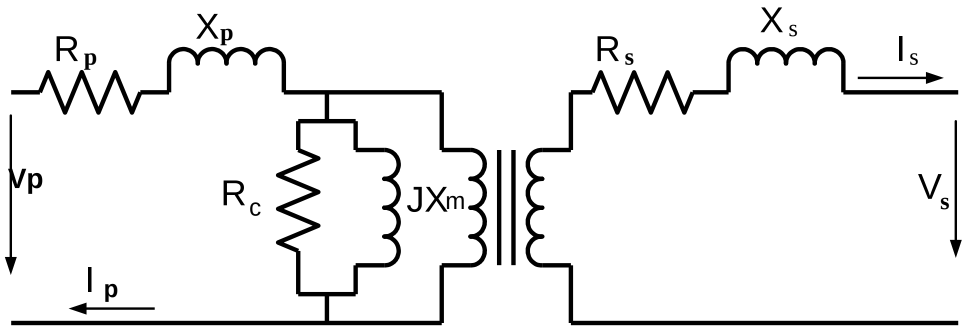


Figure A1: Equivalent circuit of a transformer

No Load Test:

This test will provide you with core losses of the transformer. Using simulation results you will able to determine the magnetization resistance and reactance of the transformer (Rc and Xm). The Simulink diagram is depicted in **Figure A 2.** Please open the Simulink diagram called xformer_noload and spend some time to understand it. As shown in Figure A2, we have a single-phase transformer that is connected to an RLC load. An AC source is

connected to the primary side of the transformer. We use several measurement blocks of the Power System Blockset to measure real and reactive power of the primary, voltages and currents (including phase angles) of the primary and

secondary. The simulations parameters are already set up for you. In order to simulate the no load situation under various voltage level, a very small amount of real and reactive power loads are chosen (double-click on the series

RLC load to see the values). Please follow the steps below:

Set the AC source peak voltage value to 0 V. To do that, double-click the AC source and set the peak amplitude (V) to 0. Do not change anything else and save the changes.

Run the simulation (Simulation/Start).

Double-click on the powergui (blue box on the left) to see the RMS values of voltages and currents: click Steady-State Voltages and Currents, then choose RMS values. in the dialog box for Units, record measurements in Table A1: primary voltage (Primary Voltage) , scondary voltage (Secondary Voltage), primary current(Current Measurement), secondary current (Current Measurement 1).

Type real_reactive_power (length(real_reactive_power),:) in the MATLAB workspace (and hit “Enter”) to obtain input real and reactive powers. It will give you the values of P and Q. The first component of the row is the input real power (P in watts) and the second component is the input reactive power (Q in VARS).

Record your data in a table format shown in Table A1.

Repeat the steps from 1 to 5 for the voltage values: 30, 60, 90, 120, 150, and 170 V.

Record your data in the same table.

Note that the data at 170 V corresponds to the rated rms voltage (120 V). Why is this so? The rms values of the primary voltage and current, and the input real power will be used to compute the magnetization resistance and reactance of the transformer (Rc and Xm).

For your lab report

Compute and record values for Rc and Xm

Provide the table containing all the data

Plot current vs. voltage for both the primary and secondary side

Plot the input real power vs. input voltage (primary voltage)

Compare your results with those obtained from the actual hardware experiments.

• **Information on powergui:** Please refer to the list to obtain the measurements in Table A1 from the list in

powergui:

1. Vp =Primary Voltage
2. Ip = Current Measurement
3. Vs = Secondary Voltage
4. Is = Current Measurement1

Table A1: Example Table for No Load Test

Vac peac Value (V)	RMS Primary Voltage, Vp(V	RMS Primary Current, Ip(A)	Ip Phase Angle (degrees)	RMS Secondary Voltage Vs(V)	RMS Secondary Current, Is(A)	Ip Phase Angle (degrees)	Input Real Power P (W)	Input Reactive Power Q (VAR)	RMS Primary Current, Ip(A)
0									
60									
120									
180									
240									
300									
380									

Table A1: Example Table for No Load Test

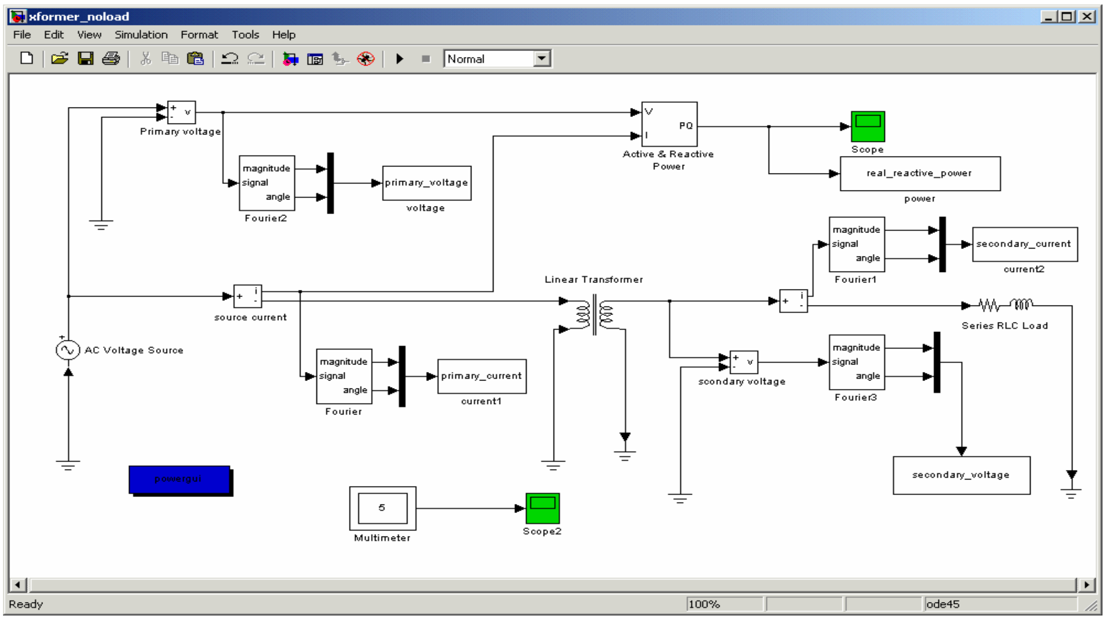


Figure A2: Simulink diagram for no load test

Short Circuit Test:

The short circuit test of a transformer provides the copper losses of the transformer windings when it supports the rated load. Voltage, current, and the input real power measurements enable us to compute equivalent resistance and
reactance of windings referred to the primary side when the secondary side of the transformer is short-circuited.

Note that the rated primary current is 2500 (VA)/120 (V) = 20.83 A. We will run the short circuit test for several voltage values of the AC source connected to the primary side of the transformer to obtain measurement of several
variables over a range of the primary current. Open simulink file: xformer_short, you will get the simulation diagram as in Figure A3. Take a look at it to understand its components. For simulations, please follow the steps below:
Set the AC source peak voltage value to 0 V. To do that, double-clicking the AC source and set the peak amplitude (V) to 0. Do not change anything else and save the changes.
Run the simulation (Simulation/Start).

Double-click on the powergui (blue box on the left) to see the RMS values of voltages and currents: click Steady-State Voltages and Currents, in the dialog box, then choose RMS values. as Units, record measurements in Table A1: primary voltage (Primary voltage) , secondary voltage (Secondary voltage), primary current(Current Measurement), secondary current (Current Measurement 1). Type real_reactive_power(length(real_reactive_power),:) in the MATLAB workspace (and hit "Enter") to obtain input real and reactive powers. It will give you the values of P and Q. The first component of the row is the input real power (P in watts) and the second component is the input
reactive power (Q in VARS).
Record your date in a table format shown in Table A2.
Repeat the steps from 1 to 5 for the voltage values: 5, 10, 15, 17, 20 V.
Record your data in the table below.
Note that the data at 20 V approximately corresponds to the rated rms current (20.83A V). The rms values of the primary voltage and current, and the input real power will be used to compute the equivalent winding resistance and reactance of windings, which are referred to the primary side (Req and Zeq).

For your lab report

Compute and record Req and Zeq.
Provide the table containing all the data
Plot current vs. voltage for both primary and secondary
Plot the input real power vs. input voltage (primary voltage)
Compare you results with those of the actual hardware experiment.

Table A2: Example table for Short Circuit Test

Vac peac Value (V)	RMS Primary Voltage, Vp(V)	RMS Primary Current, Ip(A)	Ip Phase Angle (degrees)	RMS Secondary Voltage Vs(V)	RMS Secondary Current, Is(A)	Ip Phase Angle (degrees)	Input Real Powr P (W)	Input Reactive Power Q (VAR)	Vac peac Value (V)
0									
5									
10									
15									
17									
20									

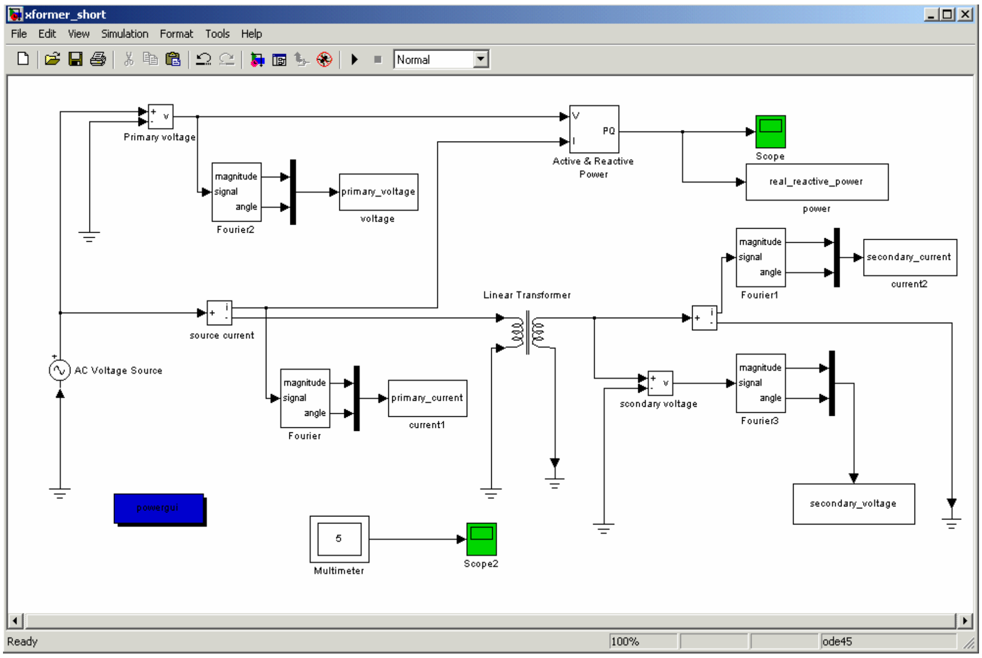


Figure A3: The Simulink diagram for short circuit test

Load Test:

The load test is designed to study the effects of the three different types of loads on the voltage regulation and efficiency of the transformer. Fig.A4 shows the Simulink diagram for the load test. Observe that a series RLC load is connected to the secondary side. Double-click in the series RLC load in the Simulink diagram. A dialog box shown in Fig.A5 will pop up. This dialog box allows you to specify the real, inductive reactive and capacitive reactive power components of the load. We will simulate the following load types:

1.Load 1-resistive load: Nominal voltage = 220 V, Nominal frequency = 50 Hz, Active power P (W) =2500, Inductive reactive power QL = 0 and Capacitive reactive power Qc = 0.

2.Load 2-Inductive load: Nominal voltage = 220 V, Nominal frequency = 50 Hz, Active power P (W)=2500 Inductive reactive power QL = 1090 Var and Capacitive reactive power Qc = 0.

3.Load 3-Capacitive load: Nominal voltage = 220 V, Nominal frequency = 50 Hz, Active power P (W) =2500, Inductive reactive power QL = 0 and Capacitive reactive power Qc = 1500 Var.

In order to simulate each load, follow the steps below:

Set the AC source voltage peak amplitude to 170 V, phase to 0 and frequency to 60 Hz.

Run the simulation for each load type (load 1, load 2 and load 3)
Record all the data shown in Table A3.
For your lab report

- Provide the table containing all the data
- Compute the voltage regulation in percent for each load type
- Calculate the efficiency of the transformer for each load type

Table A3: Example Table for Short Circuit Test

Vac peac Value (V)	RMS Primary Voltage, Vp(V)	RMS Primary Current, Ip(A)	Ip Phase Angle (degrees)	RMS Secondary Voltage Vs(V)	RMS Secondary Current, Is(A)	Ip Phase Angle (degrees)	Input Real Power P (W)	Input Reactive Power Q (VAR)
Load1								
Load2								
Load3								

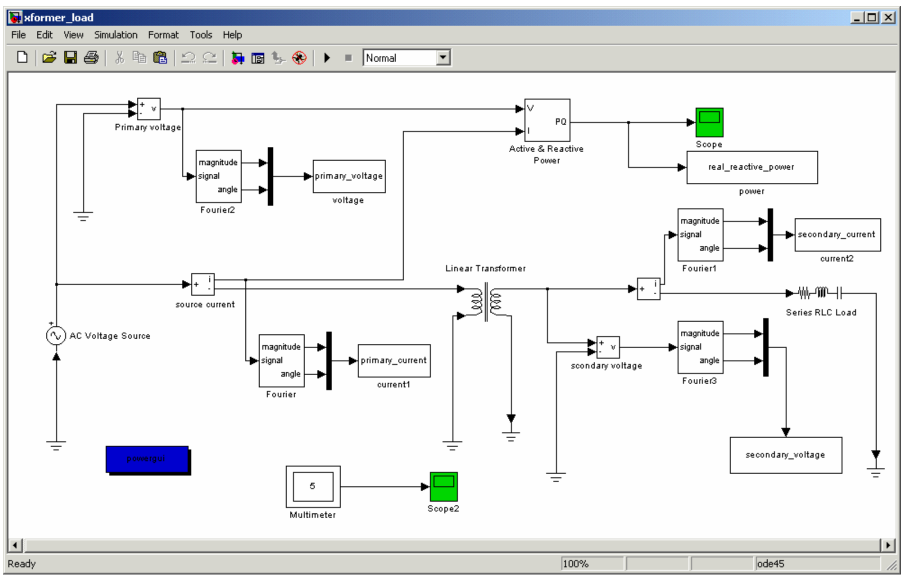


Figure A4: The Simulink diagram for the load test

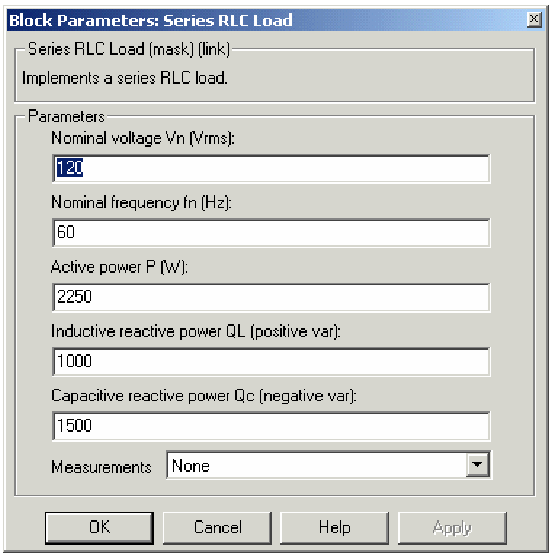


Figure A5: The RLC load dialog box

Appendix B

Laboratory Work №1
DC Series Circuit Analysis

Objectives

The main objectives of the work are to

- verify Ohm's law for DC circuit;
- plot the potential diagram;
- compare the measured and calculated values of currents and voltages.

Basic Concepts

The circuit diagram of DC unbrunched circuit is shown in Figure 1.1. According to Ohm's law for the closed circuit current flowing through any closed loop is directly proportional to the algebraic sum of electromotive forces and inversely proportional to the arithmetical sum of circuit resistances:

$$I = \frac{\sum\limits_k E_k}{\sum\limits_k R_k} = \frac{E_1 - E_2 + E_3}{R_1 + R_{01} + R_2 + R_{02} + R_3 + R_{03} + R_4} \qquad 1.1$$

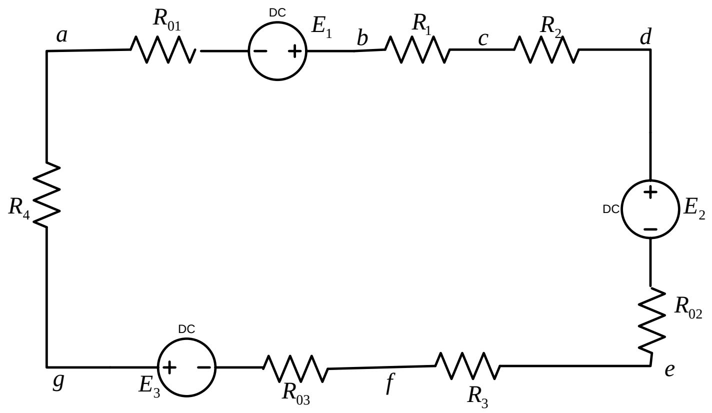


Fig. 1.1

Ohm's law for the active abcde branch is written as follows:

$$I = \frac{\varphi_a - \varphi_e + \sum E_{ae}}{\sum R_{ae}} = \frac{V_{ae} + E_1 - E_2}{R_1 + R_{01} + R_2 + R_{02}} \qquad 1.2$$

Ohm's law for the passive bcd branch is

$$I = \frac{\varphi_b - \varphi_d}{\sum R_{bd}} = \frac{V_{bd}}{R_1 + R_2} \qquad 1.3$$

The plot of distribution of potentials around a loop is called as potential diagram. In order to plot the potential diagram, it is necessary to lay off the values of resistances on the abscissa axis; corresponding potentials should be laid off on the ordinate axis.

Experimental Procedures

1. Select the necessary elements (two DC voltage sources, switcher, and three resistors) from the panel of elements (Fig1.3) and assemble an electric circuit on the workshop according to the circuit diagram shown in Figure 1.2.

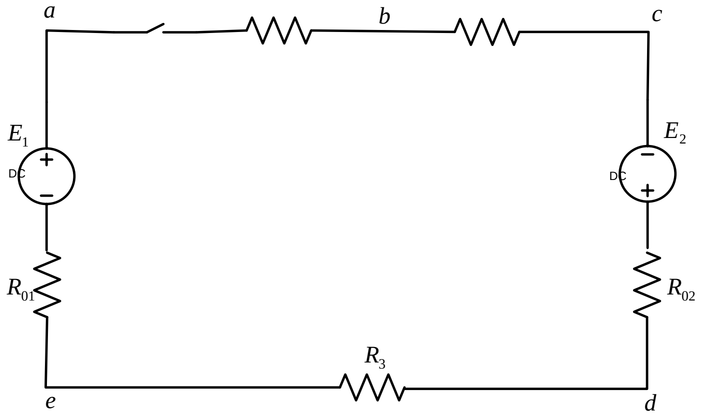


Fig. 1.2

By double clicking on each element, the "*detail parameters*" window will be displayed. Assign certain (e.g. R_1 =100Ohm, R_{01}=50Ohm, R_2=200Ohm, R_{02}=50Ohm, R_3=300Ohm, E_1=15V, E_2 =10V) parameters to each element. Display the digital multimeter from the control panel and measure the current flowing through the circuit.

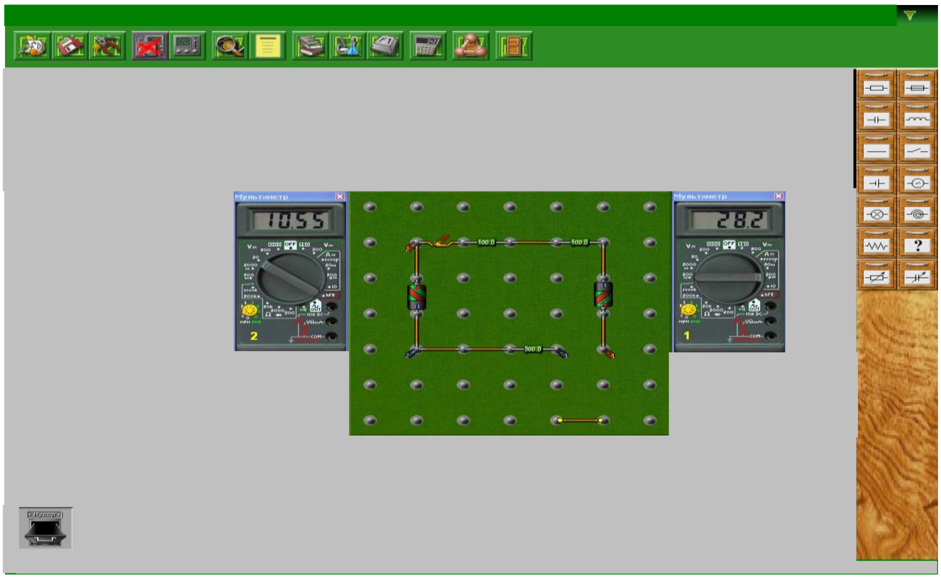


Fig.1.3

Enter the selected and measured values in Table 1.1.

Table1.1

E_1	E_2	R_1	R_{01}	R_2	R_{02}	R_3	I
V	V	Ohm	Ohm	Ohm	Ohm	Ohm	mA

Calculate the current by the expression:

$$I = \frac{E_1 + E_2}{R_1 + R_{01} + R_2 + R_{02} + R_3} \quad 1.4$$

Enter the calculated current in the Table 1.2. The obtained current represents the theoretical value of current based on the Ohm's law for the closed circuit.

2. In order to verify Ohm's law for the passive (For example abc) branch, display another multimeter from the control panel and measure the potentials of the points a and c relative to the point e (potential of which is assumed to be zero). Calculate current by the formula:

$$I = \frac{\varphi_a - \varphi_c}{R_1 + R_2} \quad 1.5$$

Enter the results in Table 1.2 and compare the calculated value with the measured value given in the Table 1.1.

Table 1.2

Verification of Ohm's Law						
for Closed Circuit	for Passive Branch			for Active Branch		
I	φ_a	φ_c	I	φ_b	φ_d	I
mA	V	V	mA	V	V	mA

3. To verify Ohm's law for the active (for example bcd) branch, measure the potentials of the points b and d relative to point e. calculate the current by the following formula

$$I = \frac{\varphi_b - \varphi_d + E_2}{R_2 + R_{02}} \quad 1.6$$

Enter the result in Table 1.2 and compare it with the value of current given in the Table 1.1.

4. Calculate theoretically the potentials of the points a, b, c, d relative to the point e by means of the following formulas:

$\varphi_a = E_1 - IR_{01}$, $\varphi_b = \varphi_a - IR_1$, $\varphi_c = \varphi_b - IR_2$, $\varphi_d = \varphi_c - IR_{02} + E_2$, $\varphi_e = \varphi_c - IR_3 = 0$

Insert the results in Table 1.3

Table1.3

Calculated				
φ_a	φ_b	φ_c	φ_d	I
V	V	V	V	mA

Compare the theoretical and measured values of potentials.

5. Plot the potential diagram according to obtained results. Presumable shape of potential diagram is shown in Figure1.4.

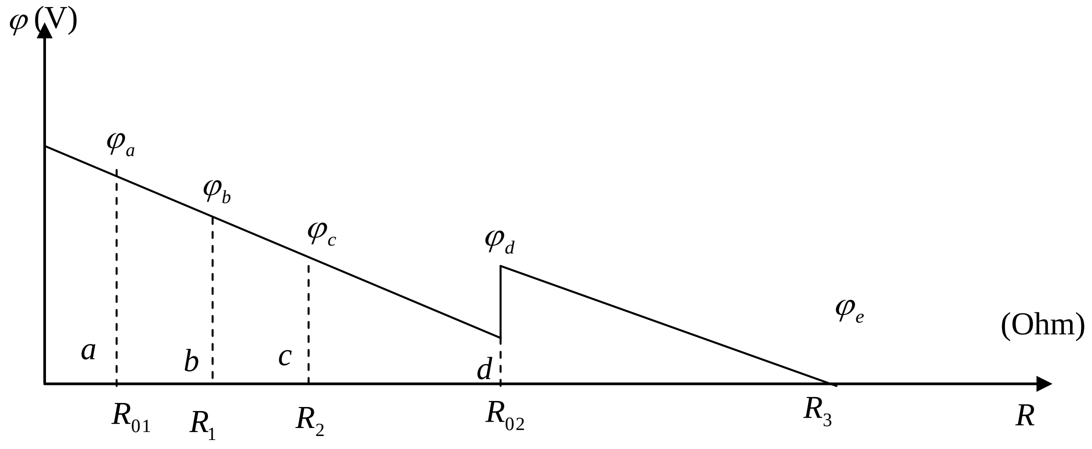


Fig1.4

6. Determine the current in diagram form. Insert the result in Table1.3.

Answer the Questions

1. Formulate Ohm's law. Write its expressions for the active and passive branches.
2. What is a potential diagram?
3. Is it possible to determine the voltage between two points with the help of the potential diagram? Give an example
4. Is it possible to determine the current from the potential diagram? Give an example.

Laboratory Work 2

AC Series R-L Circuit Analysis

Objectives

The main objectives of this work are to

- analyze a sinusoidal steady-state circuit with a resistor and an inductive coil (inductor) connected in series;
- plot the curves of frequency characteristics;
- compare the experimental and theoretical data.

Basic Concepts

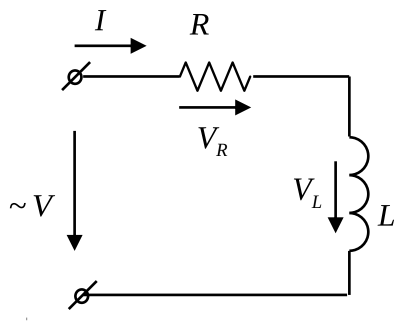


Fig. 2.1

Assume, the sinusoidal current with angular frequency $\omega = 2\pi f$ and initial phase Ψ_i flows through the circuit shown in Figure 2.1

$$i = I_m \sin(\omega t + \Psi_i) = \sqrt{2} I \sin(\omega t + \Psi_i) \qquad 2.1$$

Here I_m and I are the amplitude and root mean square (rms) values of current respectively. Instantaneous values of voltages across the resistor and inductor will be written as:

$$v_R = iR = I_m R \sin(\omega\, t + \Psi_i) = \sqrt{2} V_R \sin(\omega t + \Psi_i) \qquad 2.2$$

$$v_L = L\frac{di}{dt} = I_m \omega L \sin(\omega t + \Psi_i + \frac{\pi}{2}) = \sqrt{2} I X_L \sin(\omega t + \Psi_i + \frac{\pi}{2}) = \sqrt{2} V_L \sin(\omega t + \Psi_i + \frac{\pi}{2})$$

2.3

where $V_R = IR$ and $V_L = IX_L$ are the rms values of voltage drops across the resistor and inductor respectively. The quantity

$$X_L = \omega L$$

2.4

represents the inductive reactance of the coil.

According to KVL, the driving voltage

$$v = v_R + v_L = I_m Z \sin(\omega t + \Psi_i + \varphi) = V_m \sin(\omega t + \Psi_i + \varphi),$$

where Z is the circuit impedance

$$Z = \sqrt{R^2 + X_L^2} \qquad 2.5$$

and φ is the phase difference between the input voltage and current:

$$\varphi = \Psi_u - \Psi_i = \tan^{-1}\frac{X_L}{R} \qquad 2.6$$

The previous expression shows that the phase difference is positive; it means that the current lags the voltage by the angle φ.

The amplitude and rms values of current are determined by the following expressions:

$$I_m = \frac{V_m}{Z} \qquad I = \frac{V}{Z} \qquad 2.7$$

Expected shapes of frequency characteristics curves are shown in Figure 2.2

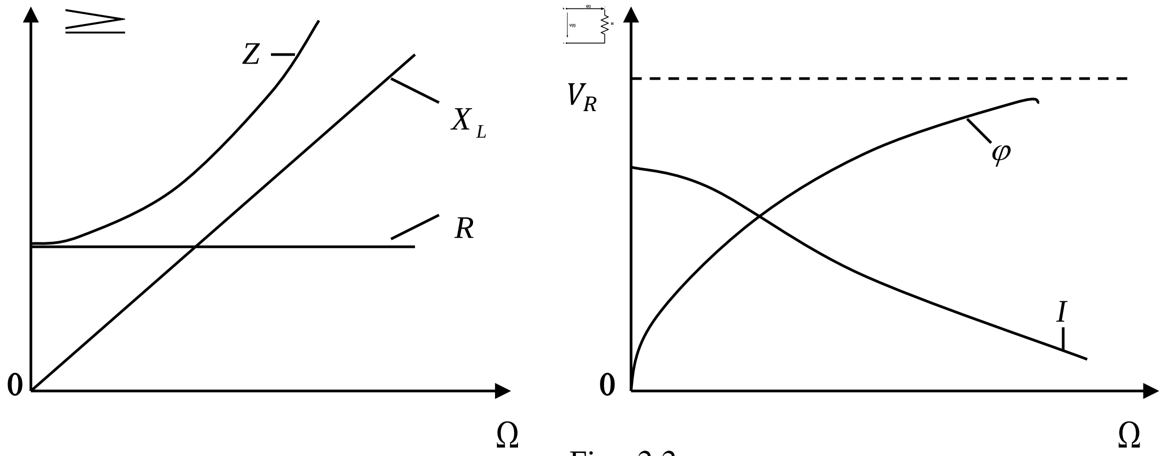


Fig. 2.2

Experimental Procedure

1. Display the AC generator (alternator), resistor, inductor and switcher from the panel of elements and construct the circuit on the workshop according to the circuit diagram shown in Figure 2.3.

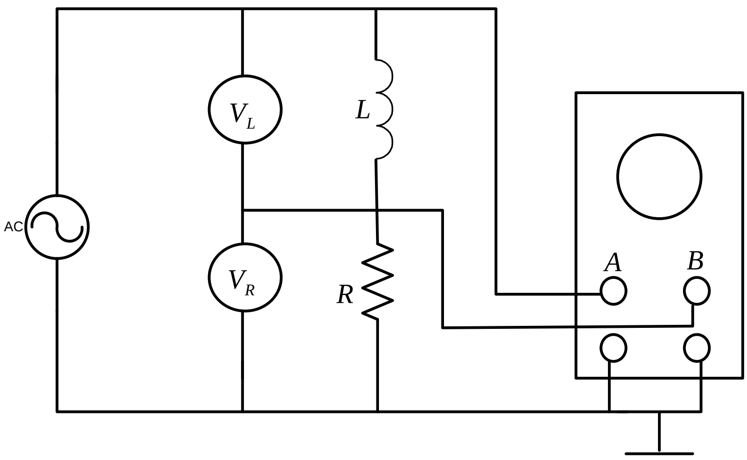


Fig. 2.3

2. Assign the certain parameters to each element (e.g.$V = 10V,\ R = 100Ohm,\ L = 0.1H$), display two digital multimeters from the control panel and measure the voltage drops V_R and V_L on the following frequencies:

$$f = 100,\ 200,\ 300,\ 500,\ 800,\ 1000,\ 1500,\ 2000,\ 3000\ Hz.$$

Enter the results of measurements in Table 2.1.

Fig.2.4

3. To measure the phase difference, display the dual-beam oscillograph from the control panel (Fig.2.4). Connect the terminals of channel A to the outputs of generator, and channel B - to the ends of resistor. Two sinusoids will display on the screen, one of which corresponds to the applied voltage, another one – to the input current. Count the number of divisions n between the peak values of sinusoids. Calculate the corresponding period of time by the formula:

$$\Delta t = S_t n \qquad 1.8.$$

where S_t is the time scale of the oscillograph. Then calculate the phase difference as

$$\varphi = f \Delta t 360^0 \qquad 1.9$$

Enter the results of measurements in Table2.1.

Table 2.1

No	Measure				Calculate						
	f	V_R	V_c	Δt	by Experimental Data			by Formulae			
								3.4	3.6	3.7	3.8
					φ	I	Z	X_L	Z	φ	I
	Hz	V	V	sec	Deg	mA	Ohm	Ohm	Ohm	Deg	mA
1											
2											
3											
4											
5											
6											
7											
8											
9											

4. Calculate current and impedance using experimental data:

$$I = \frac{V_R}{R}, \qquad Z = \frac{V}{I}.$$

1.10

5. For each frequency calculate the inductive reactance, circuit impedance, phase displacement and current through the circuit by the formulae 2.4, 2.5, 2.6, 2.7.

Enter the results in Table 2.1.

6. Plot in scale the frequency characteristics

$$X_L = F(f),\ Z = F(f),\ I = F(f),\ \varphi = F(f)$$

Compare the experimental and theoretical data.

Answer the Questions

1. Write Ohm's law for the rms and amplitude values of current and voltage.
2. How is the phase displacement calculated?
3. Draw the phasor diagram for series R-L circuit.
4. How does the inductive reactance depend on frequency?

Laboratory Work 3
AC Series R-C Circuit Analysis

Objectives

The main objectives of this work are to

- analyze a sinusoidal current circuit with a series connected resistor and capacitor;
- build the curves of frequency characteristics of the circuit;
- compare the experimental and theoretical data.

Basic Concepts

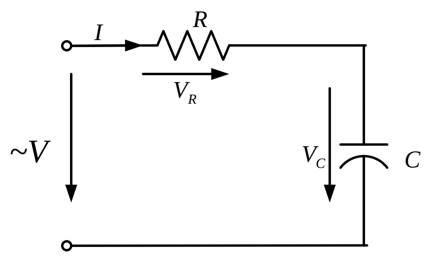


Fig. 3.1

Assume a sinusoidal current with the angular frequency and initial phase Ψ_i flows through the circuit shown in Figure 3.1:

$$i = I_m \sin(\omega t + \psi_i) = \sqrt{2} I \sin(\omega t + \psi_i),$$

3.1

where I_m is the amplitude and I -the rms value of current.

The instantaneous values of voltage drops across the circuit elements will be written as

$$v_R = iR = I_m R \sin(\omega t + \psi_i) = \sqrt{2} V_R \sin(\omega t + \psi_i)$$

3.2

$$v_c = I_m X_c \sin(\omega t + \psi_i - \frac{\pi}{2}) = \sqrt{2} I X_c \sin(\omega t + \psi_i - \frac{\pi}{2}) = \sqrt{2} V_c \sin(\omega t + \psi_i - \frac{\pi}{2}) ,$$

3.3

where $V_R = IR$ and $V_c = IX_c = I\dfrac{1}{\omega C}$ are the rms values of voltage drops across the resistor and capacitor respectively.

Capacitive reactance is defined as

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi fC} .$$

3.4

According to KVL, the instantaneous value of applied voltage will be written as follows:

$$v = v_R + v_c = I_m Z \sin(\omega t + \psi_i + \varphi) = V_m \sin(\omega t + \psi_i + \varphi) , \qquad 3.5$$

where Z is a circuit impedance:

$$Z = \sqrt{R^2 + X_c^2} .$$

3.6

Phase difference between the applied voltage and input current

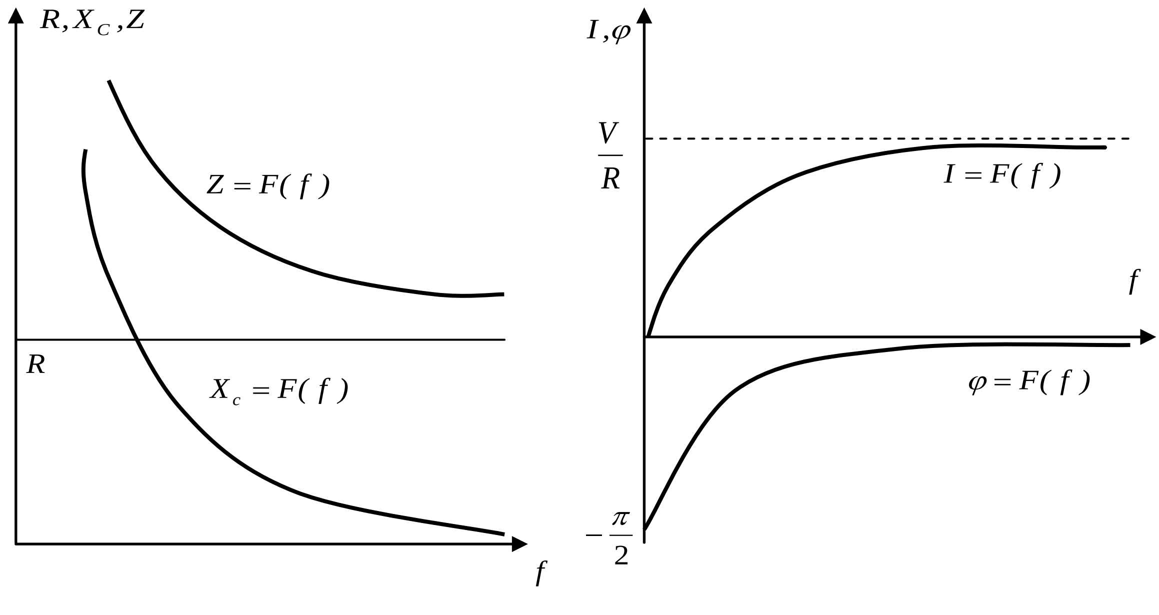
$$\varphi = -arctg \frac{X_c}{R} . \qquad 3.7$$

The previous expression shows that the phase difference is negative; it means that the input current leads the applied voltage by the angle φ.

The amplitude and rms values of current are determined as follows:

$$I_m = \frac{V_m}{Z} , \qquad I = \frac{V}{Z} . \qquad 3.8$$

Expected curves of frequency characteristics are shown in Figure 3.2



Experimental Procedure

1. Display the AC generator (alternator), resistor, capacitor, and switcher from the panel of elements and construct the circuit on the workshop according to the circuit diagram shown in Figure 3.3(Fig.3.4).

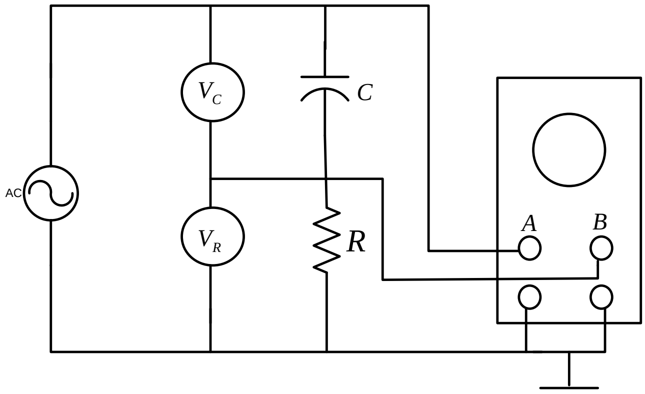


Fig. 3.3

Assign the certain parameters to each circuit element (e.g. $V = 10V$, $R = 1000\,Ohm$; $C = 1\,\mu F$).

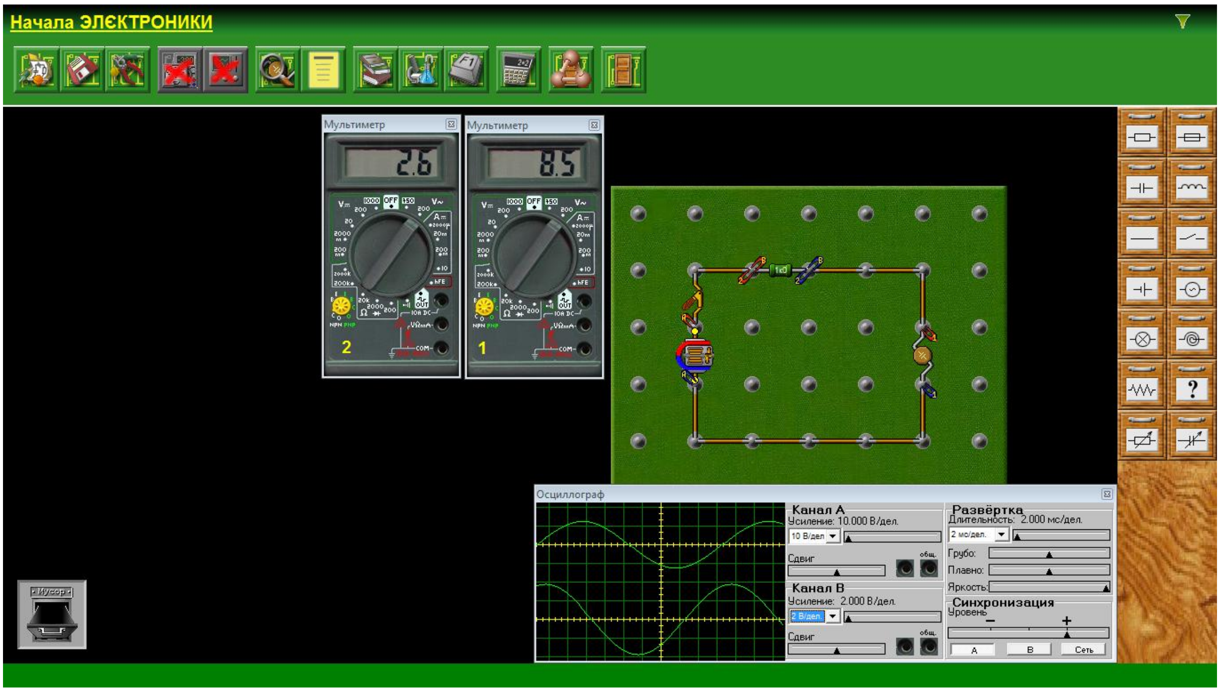


Fig.3.4

2. *Display two digital multimeters from* the control panel. Take the readings of voltmeters V_R and V_c for the following frequencies: $f = 200,\ 500,\ 800,\ 1000,\ 1500,\ 3000,\ 6000,\ 10000\ (\mathrm{Hz})$.

3. To measure the phase difference, display a dual-beam oscillograph from the control panel. Connect the terminals of channel A to the outputs of generator, and channel B - to the ends of resistor. Two sinusoids will display on a screen, one of which corresponds to the applied voltage, another one – to the input current. Count the number of divisions n between the peak values of sinusoids. Calculate the corresponding time interval by the formula:

$$\Delta t = S_t n$$

where S_t is the time scale of the oscillograph. Then calculate the phase difference as

$$\varphi = f \Delta t 360^0$$

Enter the results in Table3.1.

4. Calculate current through the circuit and circuit impedance by experimental data:

$$I = \frac{V_R}{R} \qquad 3.9$$

$$Z = \frac{V}{I}. \qquad 3.10$$

5. Calculate the capacitive reactance, circuit impedance, phase displacement, and current through the circuit by the formulae 3.4, 3.6, 3.7, 3.8. Enter the results in Table 3.1.

6. Plot in scale the frequency characteristics:

$$X_c = F(f), \quad Z = F(f), \ \psi = F(f), \quad I = F(f)$$

Table 3.1

No	Measure				Calculate						
	f	V_R	V_c	Δt	by Experimental Data			by Formulae			
								3.4	3.6	3.7	3.8
					φ	I	Z	X_C	Z	φ	I
	Hz	V	V	sec	deg	mA	Ohm	Ohm	Ohm	deg	mA
1											
2											
3											
4											
5											
6											
7											
8											
9											

Compare the measured and calculated data.

Answer the Questions

1. Write Ohm's law for the rms values of current and voltage.
2. How is the phase displacement calculated?
3. Draw the phasor diagram for series R-C circuit.
4. How does the capacitive reactance depend on frequency?

Laboratory Work №4
Analysis of a Star Connected Three-Phase Circuit Using the *Simulink* Software Package

1. Objective

Objective of the work is to analyze a star connected three-phase circuit in case of balanced or unbalanced load, in the presence and absence of a neutral wire.

2. Basic Concepts

A three-phase circuit is a combination of single-phase circuits with sinusoidal emf-s having the same frequencies but phase-displaced from one another by the angle 2π/3 and supplied by a single source of energy.

Instantaneous values of the balanced three-phase emf-s are given as follows:

$$\begin{aligned} e_A &= E_m \sin \omega t, \\ e_B &= E_m \sin\left(\omega t - 120^0\right) \\ e_C &= E_m \sin\left(\omega t + 120^0\right) \end{aligned} \qquad 4.1$$

If the three common ends of each phase are connected together at a common terminal (neutral), and the other three ends are connected to the 3-phase line, the system is star or wye (Y) connected. Figure 4.1 illustrates a star-to-star (Y-Y) connection scheme of a three-phase generator and a load.

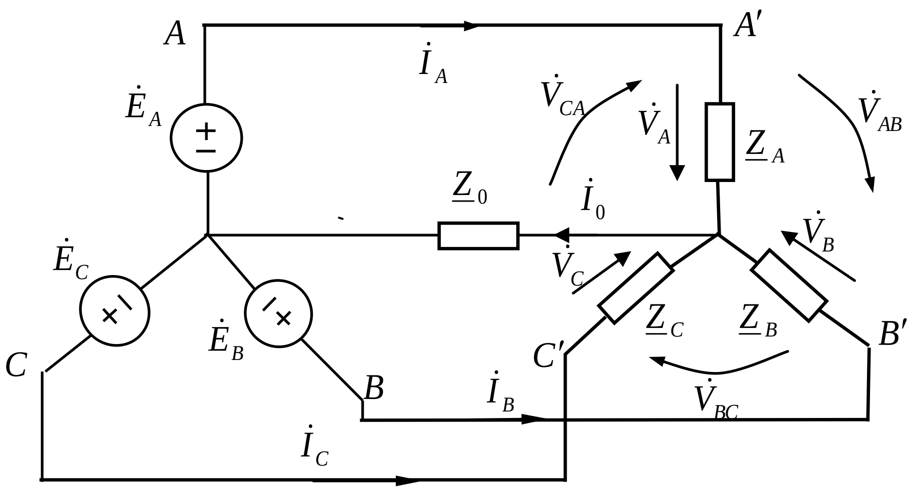


Fig.4.1

Currents I_A, I_B, I_C flowing through the line wires are called the line currents. Current I_0 flowing through the neutral wire is called the neutral current. Voltages between initial and last points of the phases (V_A, V_B, V_C) are called the phase voltages, and voltages between initial points of the phases (between lines) – line voltages (V_{AB}, V_{BC}, V_{CA}).

Complex impedances of the load phases are:

$$\underline{Z}_A = R_A + jX_A, \qquad \underline{Z}_B = R_B + jX_B, \qquad \underline{Z}_C = R_C + jX_C, \qquad 4.2$$

The complex impedance of neutral wire

$$\underline{Z}_0 = R_0 + jX_0.$$

Line and phase voltages of the load are related with each other as follows:

$$\dot{V}_{AB} = \dot{V}_A - \dot{V}_B, \quad \dot{V}_{BC} = \dot{V}_B - \dot{V}_C, \quad \dot{V}_{CA} = \dot{V}_C - \dot{V}_A.$$

Voltage across the neutral wire

$$\dot{V}_0 = \frac{\dot{E}_A \underline{Y}_A + \dot{E}_B \underline{Y}_B + \dot{E}_C \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C + Y_0}, \qquad 4.3$$

where $\underline{Y}_A = \frac{1}{\underline{Z}_A}, \underline{Y}_B = \frac{1}{\underline{Z}_B}, \; \underline{Y}_C = \frac{1}{\underline{Z}_C}, \; \underline{Y}_0 = \frac{1}{\underline{Z}_0}$ are the corresponding complex admittances.

When the internal resistance of generator is negligible, phase voltages are calculated as follows:

$$\dot{V}_A = \dot{E}_A - \dot{V}_0, \; \dot{V}_B = \dot{E}_B - \dot{V}_0, \; \dot{V}_C = \dot{E}_C - \dot{V}_0. \qquad 4.4$$

But phase currents and current in neutral wire

$$\dot{I}_A = \dot{V}_A \underline{Y}_A, \; \dot{I}_B = \dot{V}_B \underline{Y}_B, \; \dot{I}_C = \dot{V}_C \underline{Y}_C, \; \dot{I}_0 = \dot{V}_0 \underline{Y}_0.$$

In the presence of neutral wire $Y_0 = \infty, V_0 = 0$ and

$$\dot{I}_A + \dot{I}_B + \dot{I}_C = I_0.$$

In the absence of neutral wire, we'll have: $Y_0 = 0, I_0 = 0$.

If the load is unbalanced, the active, reactive and apparent powers should be found separately for each phase. As an example for phase A

$$P_A = V_A I_A cos\varphi_A, \quad Q_A = V_B I_B \sin\varphi_A \quad S_A = V_A I_A.$$

The total active, reactive and apparent powers are

$$P = P_A + P_B + P_C, \; Q = Q_A + Q_B + Q_C, \quad S = S_A + S_B + S_C.$$

In the case of balanced three phase system

$$P = 3V_{ph} I_{ph} cos\varphi, \quad Q = 3V_{ph} I_{ph} sin\varphi, \quad S = 3V_{ph} I_{ph};$$

In terms of line quantities, we have:

$$P = \sqrt{3} V_L I_L cos\varphi, \quad Q = \sqrt{3} V_L I_L sin\varphi, \quad S = \sqrt{3} V_L I_L.$$

Consider the different modes of operation of three-phase circuit:

1. Balanced Load

When three-phase loading is balanced (symmetrical), phase admittances are the same $\underline{Y}_A = \underline{Y}_B = \underline{Y}_C$ and respectively, voltage across the neutral wire equals zero both in presence and in absence of neutral wire. Phase voltages of the load equal the phase emf-s of generator: $\dot{V}_A = \dot{E}_A$, $\dot{V}_B = \dot{E}_B$, $\dot{V}_C = \dot{E}_C$.

Phasor diagram in case of balanced load is shown in Figure 4.12. The relationships between the line and phase quantities are the following: $V_L = \sqrt{3} V_{Ph}$, $I_L = I_{Ph}$.

In case of balanced loading when there is no neutral wire, phasor diagram is the same.

2. Unbalanced Load

In the presence of neutral wire $\underline{Y}_A \neq \underline{Y}_B \neq \underline{Y}_C$, and the neutral current $\dot{I}_0 = \dot{I}_A + \dot{I}_B + \dot{I}_C$. The phase voltages of load are equal to the phase voltages of generator. Corresponding phasor diagram is shown in Figure 4.2 b.

If there is no neutral wire, $Y_0 = 0$ and the voltage across the neutral wire is determined as follows:

$$\dot{V}_0 = \frac{\dot{E}_A \underline{Y}_A + \dot{E}_B \underline{Y}_B + \dot{E}_C \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C}.$$

In this case, the load phase voltages are not equal to the generator emf-s and are determined by the expression 4.4. The corresponding phasor diagram is shown in Figure 4.2c.

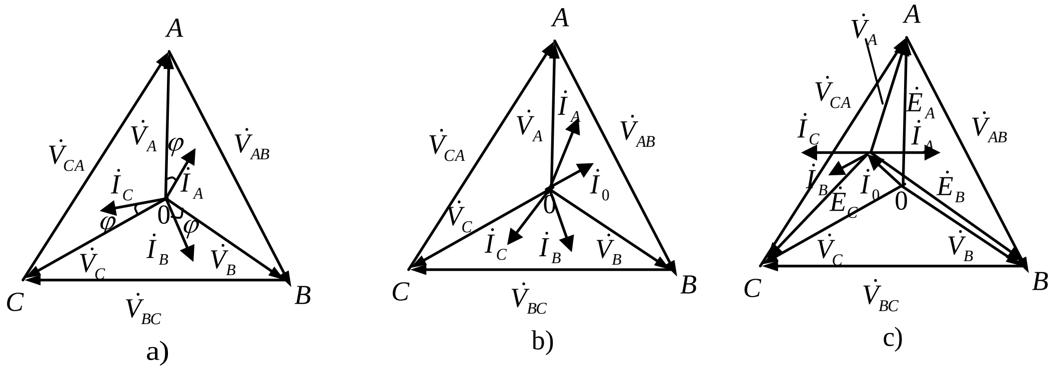


Fig.4.2

3. Used Devices and Equipment

Necessary devises and equipment are shown in Figure 4.3, which illustrates the *Simulink* blocks to be used during experimental procedure.

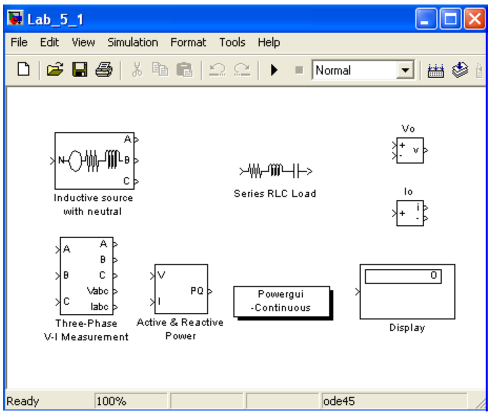


Fig.4.3

Figure 4.4 illustrates the dialog box of block parameters of 3-phase inductive source. This dialog box contains the fields for the voltage amplitude, initial phase of the phase voltage V_A, frequency and the internal parameters of the source (resistance and inductance). These parameters shall be selected with the help of the head and cannot be changed until the end of the laboratory work.

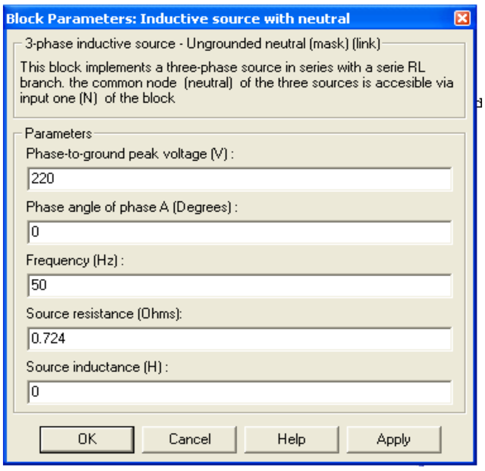


Fig. 4.4

The dialog box of block parameters of series RLC load is shown in Figure4.5. It contains the fields for the voltage and frequency of load, which should be equal to the supply voltage and frequency. Here the values of active and reactive powers shall be recorded too.

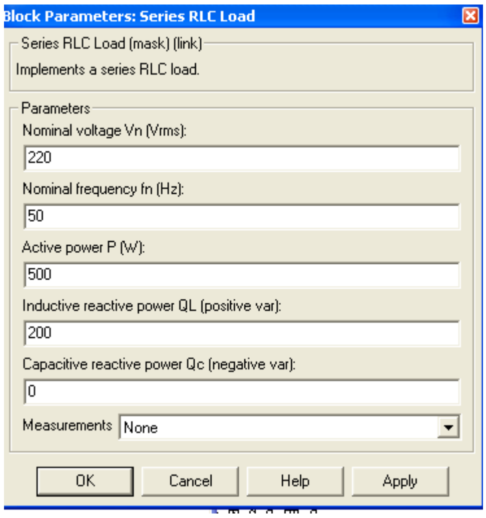


Fig.4.5

4. Experimental Procedure

The *Simulink* model of the system is represented in Figure 4.6.

After simulating, the values of active and reactive powers for A, B, C phases will appear in the *Display* block.

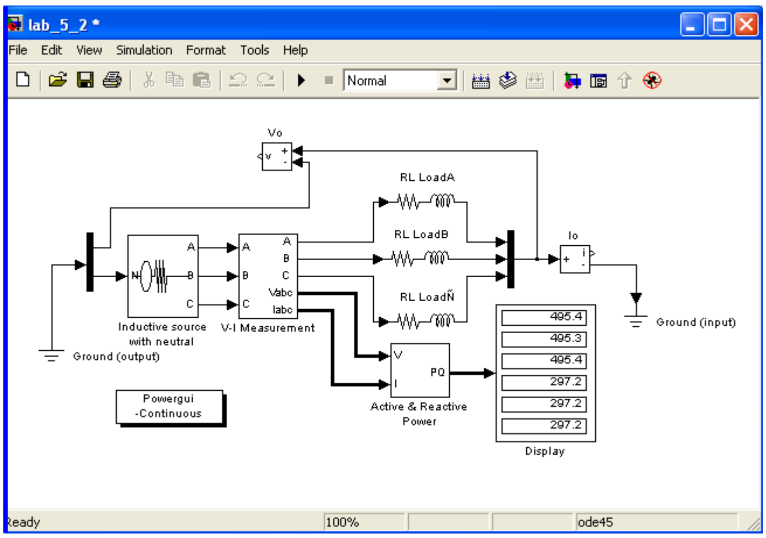


Fig.4.6

The measured values by *Powergui-Continuous* block are shown in Figure 4.7. Except for the phase voltages and currents the neutral voltage V_0 and current I_0 are measured too.

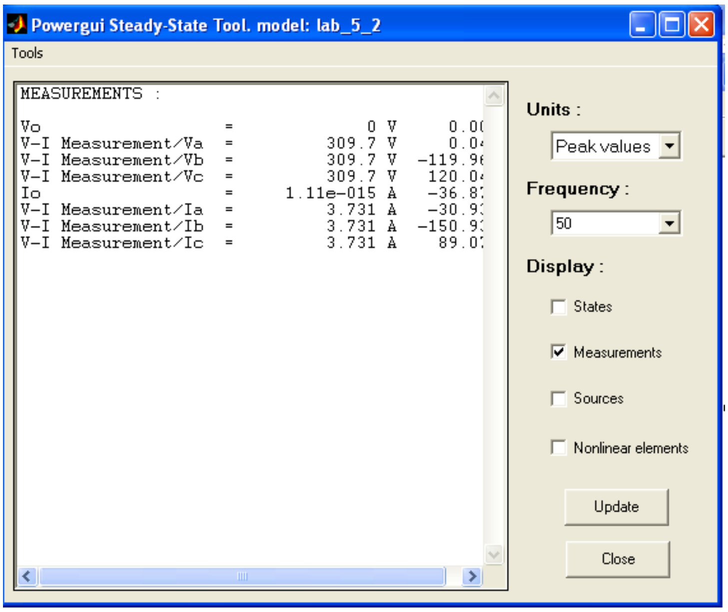


Fig.4.7

The results of measuring should be entered in Table 4.1

Table 4.1

Operation Modes	Measured													
	V_A	V_B	V_C	V_0	I_A	I_B	I_C	I_0	P_A	P_B	P_C	Q_A	Q_B	Q_C
1.Balanced Load with Neutral														
2.Balanced Load without Neutral														
3.Unbalanced Load with Neutral														
4. Unbalanced Load without Neutral														

To perform a simulation without neutral wire, it is necessary to disconnect N neutral point of power supply from "*Ground.*"

To draw the phasor diagram, the initial phases measured by *Powergui -Continuous* block should be written in Table 4.2.

Table 4.2

Operation Modes	Measured							
	φ_{V0}	φ_{I0}	φ_{V_A}	φ_{V_B}	φ_{V_C}	φ_{I_A}	φ_{I_B}	φ_{I_C}
1.Balanced Load with Neutral								
2.Balanced Load without Neutral								
3.Unbalanced Load with Neutral								
4.Unbalanced Load without Neutral								

5. Answer the Questions

1. What is a three-phase electric circuit?
2. What is a star connection of three-phase circuit?
3. What is the purpose of a neutral wire?
4. Write relationships between the line and phase quantities in case of star connection;
5. How the active, reactive, and apparent powers can be calculated in case of balanced load?

Laboratory Work 5

Analysis of a Delta Connected Three Phase Circuit Using the *Simulink* Software Package

1. Objective

Objective of the work is to analyze a delta connected three-phase circuit in case of balanced or unbalanced load.

2. Basic Concepts

If the three phases are connected in series to form a closed loop, the system is *delta* connected. Figure 5.1 illustrates a star-delta (Y-Δ) connection of a generator and a load.

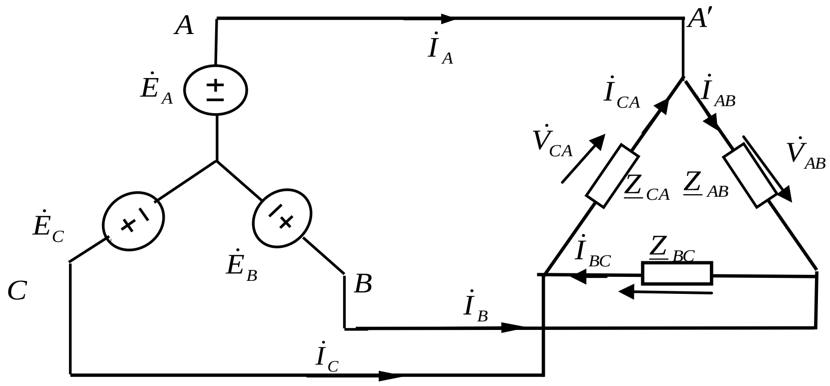


Fig.5.1

In case of delta connection, the line and phase voltages are the same, but the line and phase currents are related as follows:

$$\dot{I}_A = \dot{I}_{AB} - \dot{I}_{CA}, \quad \dot{I}_B = \dot{I}_{BC} - \dot{I}_{AB}, \quad \dot{I}_C = \dot{I}_{CA} - \dot{I}_{BC}. \tag{5.1}$$

Phase currents are determined as

$$\dot{I}_{AB} = \dot{U}_{AB} \cdot \underline{Y}_{AB}, \quad \dot{I}_{BC} = \dot{U}_{BC} \cdot \underline{Y}_{BC}, \quad \dot{I}_{CA} = \dot{U}_{CA} \cdot \underline{Y}_{CA}. \tag{5.2}$$

In case of the balanced load $\underline{Z}_{AB} = \underline{Z}_{BC} = \underline{Z}_{CA}$ and the rms values of phase currents are identical. Line and phase currents are related with the expression $I_{\mathrm{L}} = \sqrt{3} I_{\mathrm{Ph}}$ (see the phasor diagram in Figure 5.2a).

If the load is unbalanced, $\underline{Z}_{AB} \neq \underline{Z}_{BC} \neq \underline{Z}_{CA}$ and the rms values of phase currents are proportional to their admittances. Corresponding phasor diagram is shown in Figure 5.2b.

If one phase of the load is disconnected, current flowing through it becomes zero but the rest of two currents do not change, as well as their phase voltages remain constant. For example, when disconnecting CA phase, we'll get:

$$I_{CA} = 0, \ \dot{I}_A = \dot{I}_{AB}, \ \dot{I}_B = \dot{I}_{BC} - \dot{I}_{AB}, \ \dot{I}_C = -\dot{I}_{BC}$$

Phasor diagram is shown in Figure 5.2c.

When disconnecting two phases, currents through them become zero. Current through the third phase does not change. For example, in case of disconnection of AB an CA phases, we can write:

$$I_{AB} = \dot{I}_{AC} = 0, \quad \dot{I}_A = 0, \quad \dot{I}_B = \dot{I}_{BC}, \quad \dot{I}_C = -\dot{I}_{BC} \quad \text{(Fig.5.2d).}$$

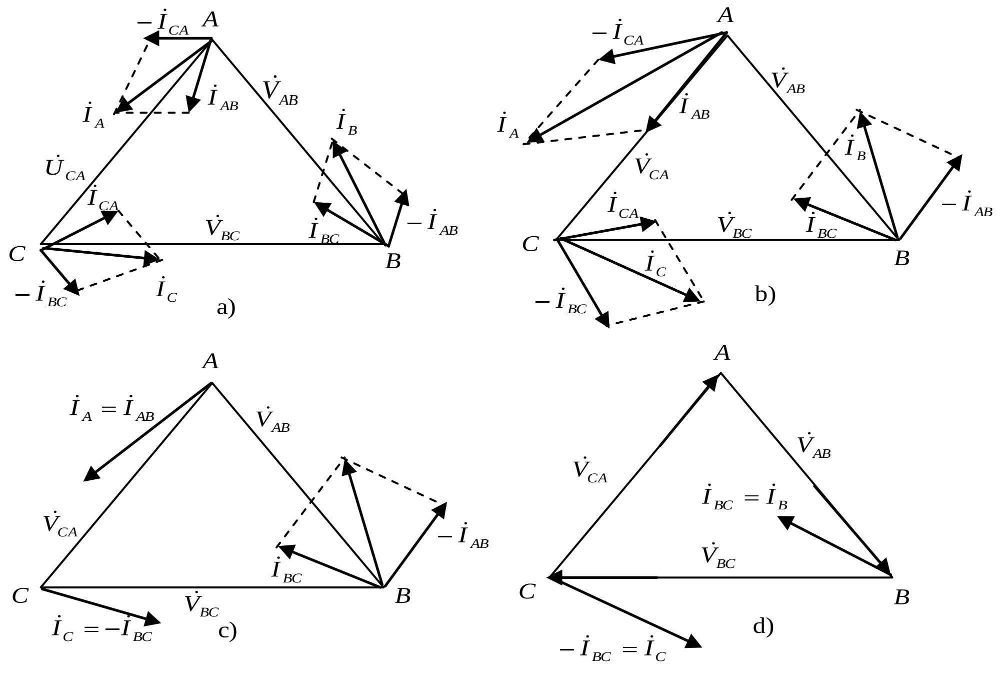


Fig.5.2

The active, reactive and apparent powers in the case of balanced loading are determined as

$$P = 3V_{ph}I_{ph}cos\varphi = \sqrt{3}V_L I_L cos\varphi,$$

$$Q = 3V_{ph}I_{ph}sin\varphi = \sqrt{3}V_L I_L sin\varphi,$$

$$S = 3V_{ph}I_{ph} = \sqrt{3}V_L I_L.$$

3. Used Devices and Equipment

Devices and equipment used in this work are identical with the devices and equipment used in the work No4.

4. Experimental Procedure

Figure 5.3 illustrates the *Simulink* model of simulation of wye-delta connected three-phase circuit.

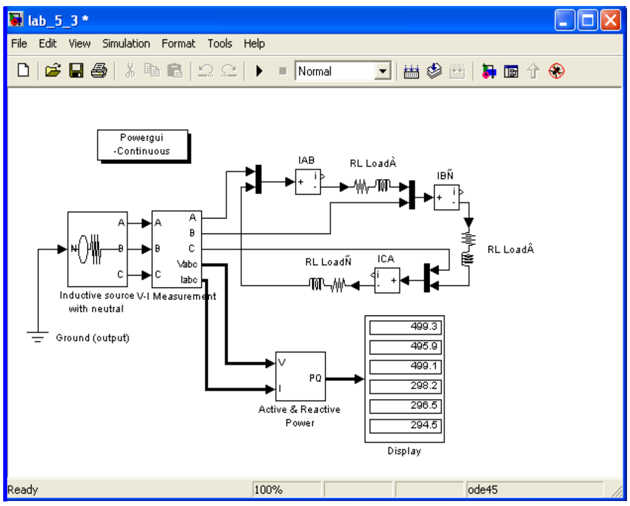


Fig.5.3

Dialog box of block parameters of series RLC load is shown in Figure5.4. It contains the fields for the voltage and frequency of load, which should be equal to the supply voltage and frequency.

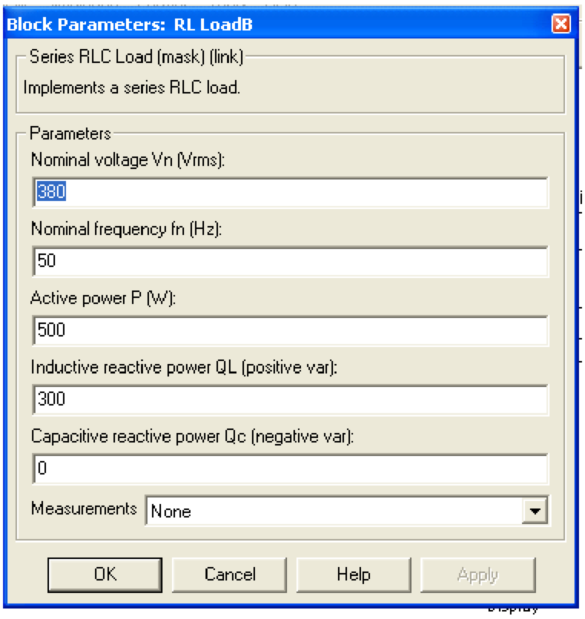


Fig.5.4

After simulating, the values of active and reactive powers will appear on the *Display* block. Line and phase currents, as well as line (phase) voltages are measured by means of *Powergui-Continuous* block (Fig.5.5).

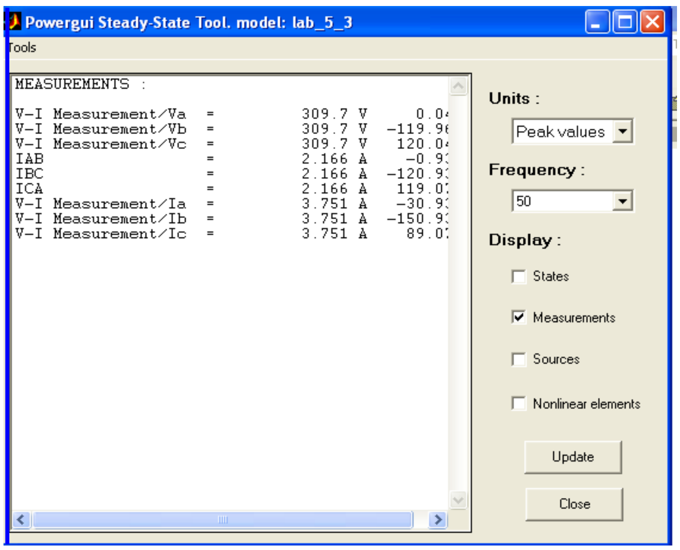


Fig.5.5

Results of simulation are entered in Table 5.1. The source and load parameters shall be chosen with the help of the head and cannot be changed until the end of experimental procedures.

Table 5.1

Operation Modes	Calculated														
	V_{AB}	V_{BC}	V_{CA}	I_A	I_B	I_C	I_{AB}	I_{BC}	I_{CA}	P_A	P_B	P_C	Q_A	Q_B	Q_C
1.Balanced Load															
2. Unbalanced Load															
3. Disconnection of One Phase															
4.Disconnection of Two Phases															

To plot the phasor diagrams, the initial phases measured by *powergui* block shall be recorded in the separate Table 5.2.

Table 5.2

Operation Modes	Measured								
	$\varphi_{V_{AB}}$	$\varphi_{V_{BC}}$	$\varphi_{V_{CA}}$	$\varphi_{I_{AB}}$	$\varphi_{I_{BC}}$	$\varphi_{I_{CA}}$	φ_{I_A}	φ_{I_B}	φ_{I_C}
1..Balanced Load									
2. Unbalanced Load									
3. Disconnection of One Phase									
4. Disconnection of Two Phases									

5. Answer the questions:

1. What is a delta connected three-phase circuit?
2. What is the relationship between the line and phase quantities in case of delta connection?
3. Write expressions for the active, reactive, and apparent powers in case of symmetrical load.
4. What does the disconnection of one or two phases cause in delta connected three-phase circuit?

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