FRACTAL REPRESENTATION OF FLUID FLOW INTO POROUS MEDIA

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Summary

In this paper fluid flow into the porous media is discussed. With conjunction of Diffusion equation, according to Darcy's law and conservation of mass equation, and Pore-Solid Fractal model is created new model that explains fractal look on fluid flow in porous media. The new – fractalization coefficient is proposed. This approach is inverse perspective of fluid flow into ground, where new property of homogenous liquid is got from characteristics of ground.

Keywords: Darcy's Law, Diffusion Equation, Pore-Solid fractal, Fractalization Coefficient.

1. Diffusion Equation for Fluid flow in Porous Media

Transient flow of a fluid through a porous medium is governed by a certain type of partial differential equation known as a diffusion equation. In order to derive this equation, we combine Darcy's law, the conservation of mass equation, and an equation that describes the manner in which fluid is stored inside a porous rock. Let's step by step lead ourselves to diffusion equation.

1.1. Darcy's Law

The basic law governing the flow of fluids through porous media is Darcy's law, which was formulated by the French civil engineer Henry Darcy in 1856 on the basis of his experiments on vertical water filtration through sand beds. Darcy found that his data could be described by

$$Q = \frac{\mathrm{CA}\Delta(\mathrm{P}-\rho \mathrm{gz})}{\mathrm{L}} \tag{1}$$

where: P = pressure [Pa], ρ = density [kg/m^3], g = gravitational acceleration [m/s^2], z = vertical coordinate (measured downwards) [m], L = length of sample [m], Q = volumetric flowrate [m^3/s], C = constant of proportionality [m^2/Pa s], A = cross-sectional area of sample [m^2].

Subsequent to Darcy's initial discovery, it has been found that, all other factors being equal, Q is inversely proportional to the fluid viscosity, μ [Pa s]. It is therefore convenient to factor out μ, and put $C = k/\mu$, where k is known as the permeability, with dimensions [m^2].

It is also more convenient to work with the volumetric flow per unit area, $q = Q/A$. Darcy's law is therefore usually written as

$$q = \frac{Q}{A} = \frac{k}{\mu}\frac{\Delta(\mathrm{P}-\rho \mathrm{gz})}{\mathrm{L}} \tag{2}$$

where the flux q has dimensions of [m/s]. It is perhaps easier to think of these units as [m^3/m^{2}s].

For transient processes in which the flux varies from point- to-point, we need a differential form of Darcy's law. In the vertical direction, this equation would take the form the minus sign is included because the fluid flows in the direction from higher to lower potential. The differential form of Darcy's law for one-dimensional, horizontal flow is

$$q_h = \frac{Q}{A} = \frac{-k}{\mu}\frac{\mathrm{d}(\mathrm{P}-\rho g z)}{\mathrm{dz}} = \frac{-\mathrm{k}}{\mu}\frac{\mathrm{dP}}{\mathrm{dz}} \tag{3}$$

The permeability is a function of rock type, and also varies with stress, temperature, etc., but does not depend on the fluid; the effect of the fluid on the flow rate is accounted for by the viscosity term in eq. (4) or (5).

Permeability has units of m^2, but in petroleum engineering it is conventional to use „Darcy" units, defined by $1\text{Darcy}=0.987\text{x}10^{-12}\text{m}^2\approx10^{-12}\text{m}^2$

The Darcy unit is defined such that a rock having a permeability of 1 Darcy would transmit 1 cc of water (with viscosity 1 cP) per second, through a region of 1 sq. cm. cross-sectional area, if the pressure drop along the direction of flow were 1 atm per cm.

The numerical value of k for a given rock depends on the diameter of the pores in the rock, d, as well as on the degree of interconnectivity of the void space. Very roughly speaking, $k = d^2/1000k$. Typical values for unfractured rock are given in the following table:

Rock Type	k (Darcies)	k (m^2)
coarse gravel	$10^3 – 10^4$	10^{-9} - 10^{-8}
sands, gravels	$10^0 – 10^3$	10^{-12} - 10^{-9}
fine sand, silt	10^{-4} - 10^0	10^{-16} - 10^{-12}
clay, shales	10^{-9} - 10^{-6}	10^{-21} - 10^{-18}
limestones	10^0 - 10^2	10^{-12} - 10^{-10}
sandstones	10^{-5} - 10^1	10^{-17} - 10^{-11}
weathered chalk	10^0 - 10^2	10^{-12} - 10^{-10}
unweathered chalk	10^{-9} - 10^{-1}	10^{-21} - 10^{-13}
granite, gneiss	10^{-8} - 10^{-4}	10^{-20} - 10^{-16}

Darcy's law is a macroscopic law that is intended to be meaningful over regions that are much larger than the size of a single pore. In other words, when we talk about the permeability at a point "(x,y,z)" in the reservoir, we cannot be referring to the permeability at a mathematically infinitesimal "point", because a given point may, for example, lie in a sand grain, not in the pore space The property of permeability is in fact only defined for a porous medium, not for an individual pore. Hence, the permeability is a property that is in some sense “averaged out” over a certain region of space surrounded the mathematical point (x,y,z). This region must be large enough to encompass a statistically significant number of pores.

1.2. Conservation of mass equation

Darcy’s law in itself does not contain sufficient information to allow us to solve transient (i.e., time-dependent) problems involving subsurface flow. In order to develop a complete governing equation that applies to transient problems, we must first derive a mathematical expression of the principle of conservation of mass.

Consider flow through a one-dimensional tube of cross-sectional area A; In particular, let’s focus on the region between two locations x and $x+\Delta x$:

The main idea behind the application of the principle of conservation of mass is
Flux in - Flux out = Increase in amount stored.

Consider the period of time between time t and time $t+\Delta t$. The amount of fluid mass stored in the region is denoted by m, V is the pore volume of the rock contained in the slab between x and $+\Delta x$. We have the formula $\mathrm{m} = \rho\phi\mathrm{V} = \rho\phi\mathrm{A}\Delta\mathrm{x}$. Where ϕ- is porosity.
From this the conservation of mass equation is derived:

$$-\mathrm{A}\left[\rho\mathrm{q}(\mathrm{x}+\Delta\mathrm{x}) - \rho\mathrm{q}(\mathrm{x})\right] = \frac{d(\rho\phi)}{dt} A\Delta x\ . \tag{4}$$

Here we temporarily treat ρq as a single entity.

For one-dimensional flow, such as through a cylindrical core A is constant. So divide both sides by AΔx, and let $\Delta\mathrm{x} \to 0$. We will get the basic equation of conservation of mass for 1-D linear flow in a porous medium. It is exact, and applies to gases, liquids, high or low flowrates, etc.

$$-\frac{d(\rho q)}{dx} = \frac{d(\rho\phi)}{dt}\ . \tag{5}$$

1.3 Diffusion Equation

Now by Combining Darcy’s law to mass conservation equation and then using chain rule of differentiation we can get the following:

$$\frac{d(\rho\phi)}{dt} = \rho\frac{d\phi}{dt} + \phi\frac{d\rho}{dt} = \rho\phi\left[\left(\frac{1}{\phi}\frac{\mathrm{d}\phi}{\mathrm{dP}}\right) + \left(\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dP}}\right)\right]\frac{\mathrm{dP}}{\mathrm{dt}} = \rho\phi(\mathrm{C}_\phi + \mathrm{C}_\mathrm{f})\frac{\mathrm{dP}}{\mathrm{dt}}\ , \tag{6}$$

where C_f is the compressibility of the fluid,

C_ϕ is the compressibility of the rock formation.

Now look at the left-hand side of eq. (5). The flux q is given by Darcy’s law eq. (3):

$$-\frac{d(\rho q)}{dz} = \frac{\rho k}{\mu}\left[\frac{d^2P}{dz^2} + C_f\left(\frac{dP}{dz}\right)^2\right]. \tag{7}$$

Now equate eqs (6) and (7):

$$\frac{d^2P}{dz^2} + C_f\left(\frac{dP}{dz}\right)^2 = \frac{\rho\phi(C_\phi + C_f)}{k}\frac{dP}{dt}. \tag{8}$$

Practice shows that, for liquids, the nonlinear term $C_f\left(\frac{dP}{dz}\right)^2$ in eq. (8) is small. In practice, it is usually neglected. So we have the one-dimensional, linear form of the diffusion equation:

$$\frac{dP}{dt} = \frac{k}{\mu\phi C_t}\frac{d^2P}{dz^2}, \tag{9}$$

where C_t is total compressibility - $C_t = C_\phi + C_f$.

2. Pore-Solid Fractal Model

The Pore-Solid Fractal model originates from two studies. Neimark developed the 'self-similar multiscale percolation system', a representation of a disordered, disperse medium that exhibits a fractal interface between solid and pore phases. Perrier Independently proposed a multiscale model of soil structure which combines a fractal pore number–size distribution and a fractal solid number–size distribution. Although these two models have been developed in different contexts, using slightly different definitions, and presenting different local geometrical shapes, they are nevertheless equivalent.

This homogeneous material can be identified either with the solid phase of the porous medium (shown in black in Fig.1) ('pore mass fractal'), or the pore phase (shown in white in Fig.1) ('solid mass fractal').

D is fractal dimension, d – Euclid dimension, i – number of iterations.

Two main options have been considered in previous studies: 1. Iterations are carried out ad infinitum, and the fractal set of (Nz). i subregions vanishes. The model represents only solid in the so-called pore mass fractal or only pores in the solid mass fractal. 2. A lower cutoff of scale is sumed, considering a finite number of recursive iterations m. The (Nz). m subregions created at the last iteration step $i = s$ m will undergo no further division and the fractal set is assumed to model the complementary phase: in a pore mass fractal it is associated with the pore phase (shown in very light gray) and in a solid mass fractal it is associated with the solid phase (shown in black).

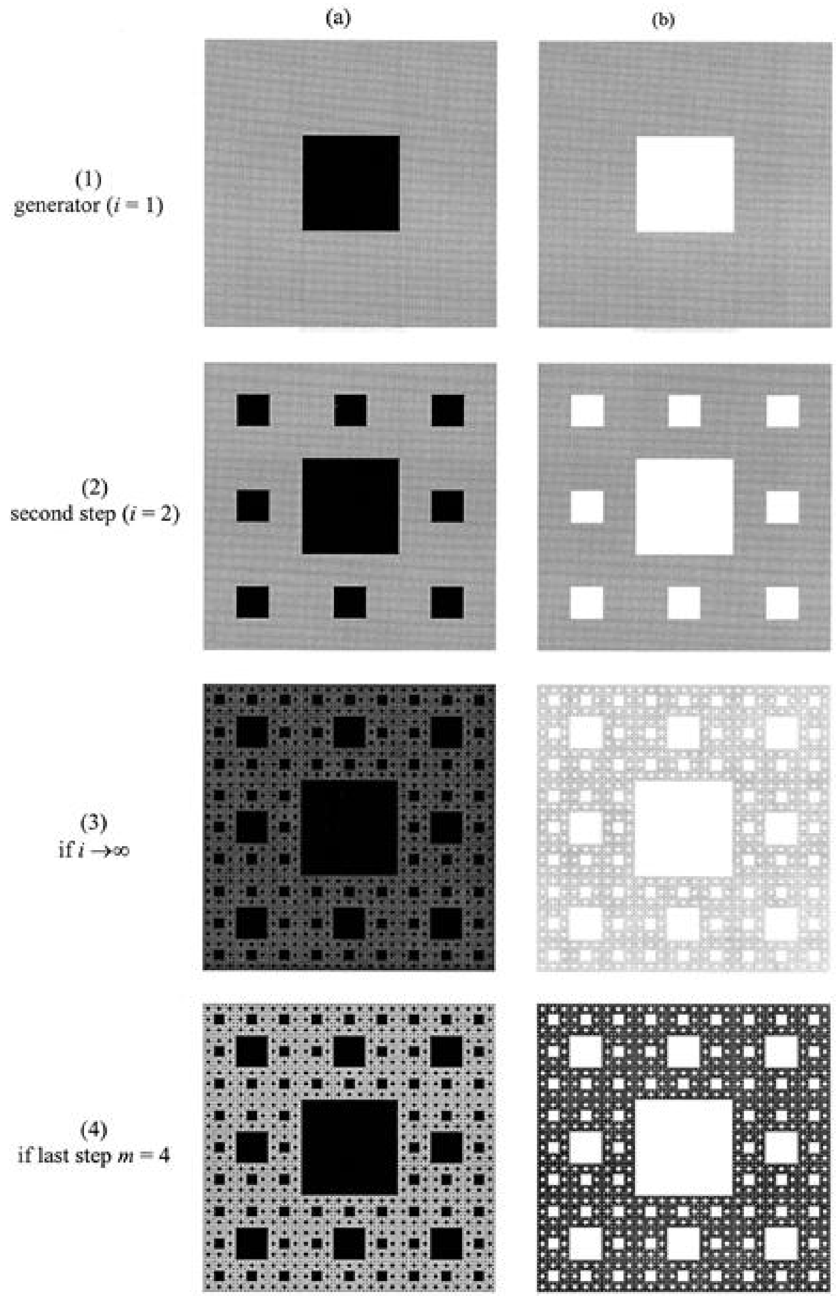

Fig.1. $d = 2$, $n = 3$, $z = 8/9$, $D = 1.893$

Following the approach of Neimark, which combines pores and solids in the model in an interesting symmetrical setting, we define the $(1 - z)$. proportion of the generator as a mixture of pore and solid defined as follows:

$$(1 - z) = (x + y) ,$$

where x denotes the proportion of pore phase, y the proportion of solid phase and z represents the proportion of the generator where the whole shape is replicated at each step. Solids and pores generated at each step are kept whereas the fractal set is transformed.

Derived from mentioned fractal dimension is :

$$D = d + \frac{\log(1-x-y)}{logn} \tag{10}$$

shows that for a given Euclidean dimension d, the value of the fractal dimension D of a PSF model depends only on the value of parameters n, x and y.

The $N(1-z)$ subregions are divided into $Nx = 4$ pore subregions (white) and $Ny = 3$ solid subregions (black). The fractal set (light gray). Corresponds to $Nz = 2$ subregions where the whole shape is replicated at next iteration step.

Parameters x, y and z can be considered as probabilities $x + y + z = 1$ and mathematical calculations can be done in a probabilistic way. However, for sake of simplicity, we will consider here that x, y and z are proportions and Nx, Ny, Nz refer to the number of subregions of each type, to get simple proofs based only on counting.

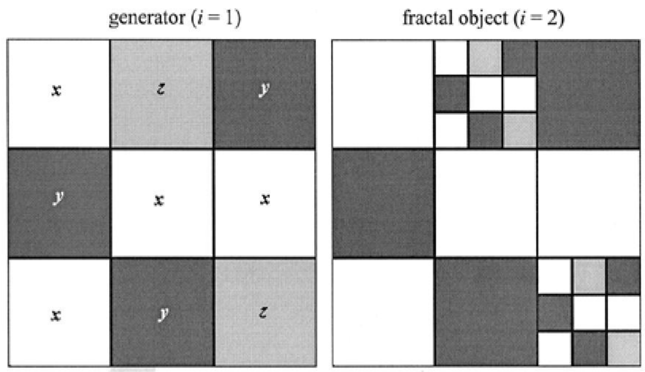

Fig.2. $d = 2$, $n = 3$, $z = 2/9$, $x = 4/9$, $y = 3/9$, $D = 2 + \log(1\text{-}7/9)/\log 3 = 0.631$

Since x represents the proportion of pores kept at step 1 by the generator, zx is the proportion of pores added in the replicates generated at step 2, and so on. Thus the porosity ϕ_i at step i is the following sum:

$$\phi = x + zx + z^2x + \cdots + z^{i-1}x = x\sum_{j=0}^{i-1} z^j = x(\frac{z^i-1}{z-1}) \tag{11}$$

From where we can get formula of porosity:

$$\phi = \frac{x}{x+y}(1 - z^i) \tag{12}$$

The number of iterations i increases to infinity, $z^i \to 0$. Eq (12) shows that a PSF model exhibits a finite value of the total porosity.

3. Fractal Representation of Permeability

The main purpose of this paper is an attempt to represent fluid permeability in fractal terms. Let's conjoin fluid diffusion equation eq. (9) and total porosity equation eq (12) by equaling porosities of both sides:

$$\frac{x}{x+y}(1 - z^i) = \frac{k}{\mu C_t \frac{dP}{dt}} \frac{d^2P}{dz^2} \tag{13}$$

In terms of experiment where fluid flux is small enough we can neglect pressure, P≈0; Consequently we can represent permeability as follows $\phi = \frac{k}{\mu C_f}$.

So we have:

$$\frac{x}{x+y}(1 - z^i) = \frac{k}{\mu C_f} \tag{14}$$

Left hand side of equation above represents fractal model of total porosity of porous media, while right hand side is porosity formula represented by fluid characteristics. This approach helps us measure fluid permeability with fractal terms e.i. experimentally if we will picture fractal representation of certain media and then we pour certain fluid on it so that flux is small enough ($q \to 0$) and therefore P≈0. Then after selecting maximal permeability level (via microscopic camera) of that fluid, we will be able to link fractal measures of the media at that level at $i = m$ iteration, where liquid will stop leaking into pores, to liquid characterizations and get some coefficient that we will call liquid fractalization coefficient for that certain liquid.

Measuring this coefficient for other one phase transportations of liquids will give us systemized set of coefficients that in the future can be used as additional characteristic of liquids.

This approach is inverse perspective of fluid flow into ground or rocks. Unlike traditional models where scientist first measure liquid characteristics, like viscosity or density, and then from this basis calculate permeability of fluid in media, by knowing this new fractalization coefficient we will be able to measure permeability of certain fluid into soil, clay, silt, or etc. only by knowing local porous media environment.

We hope that this model, only in practically refined form, will find its ground and be useful for water industry, oil industry or for other fields hydrogeology.

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ფორიან გამტარში სითხის ფილტრაციის ფრაქტალური მოდელირება

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რეზიუმე

განხილულია ფორიან გამტარში ერთგვაროვანი სითხის გადინების მოდელის შექმნის საკითხი. შედგენილია დიფუზიის განტოლება ერთფაზიანი გადინებისთვის დარსის კანონზე დაყრდნობით. ასევე განხილულია მყარ-ფორიანი ფრაქტალური მოდელი. ამ ორი მოდელის შეჯერებით მიღებულია ფორიან გამტარში სითხის გადინების ფრაქტალური მოდელი. შემოთავაზებულია სითხის ახალი მახასიათებელი ე.წ. დაფრაქტალების კოეფიციენტი. ეს მიდგომა აღწერს ფორიან გამტარში სითხის გადინების შებრუნებულ ვარიანტს, როცა გამტარის თვისებიდან გამომდინარე ვადგენთ ერთგვაროვანი სითხის მახასიათებელს.

ФРАКТАЛЬНОЕ МОДЕЛИРОВАНИЕ ФИЛЬТРАЦИИ ЖИДКОСТЕЙ В ПОРИСТЫХ СРЕДАХ

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Резюме

Рассматривается вопрос создания модели движения однородных житкостей в пористых средах. Построено уравнение диффузии для однофазового движения на основе закона Дарси. Рассмотрена также пористо-твердая фрактальная модель. На основе согласования этих моделей создана новая фрактальная модель, которая объясняет фильтрацию жидкостей в пористых средах. Предложена новая характеристика жидкости, т.н. коэффициент дефрактализации. Этот подход описывает обратный вариант движения жидкости в пористых средах, когда исходя из свойства среды строятся характеристики однородной жидкости.