MODELING OF CONTROL SYSTEM OF AIRPLANE FLIGHT CONDITIONS

Mchedlishvili Nino, Mosashvili Ia, Kipshidze David
Georgian Technical University

Summary

Navigation of an airplane is generally difficult aerodynamic problem. Achievement of object, effective and safety flight is unavailable without high-technology automatic control systems. In the thesis free rotation in flying of airplane around three axes is learnt, problem of control system synthesis of airplane pitch is arisen, which is solved by software Matlab by usage of modern synergetic methods, namely, catastrophe theory.

Keywords: Modeling. Stability. Robust system. Catastrophe theory. Elliptic umbilic. Hyperbolic umbilic. Structural modeling. Pitch. List. Heading.

1. Introduction

Main issue of control system projection and study is in guarantee of its stability. Lots of scientific work is dedicated to research of control system stability of airplane by classic methods and new technologies, when system parameters are unchangeable and well-known in advance. More difficult problem regarding to creation of robust control system is actual, or control when there is uncertainty of parameter.

There are many methods for projection of robust control system. Almost in all of them boundaries of parameters are calculated in which system will functions with desirable features, firstly it will be stable. Presently there are lots of research in which reduction of influence over stability of indefinite changes in small boundaries of parameters have been successfully achieved. But the methods guaranteeing stability of projected control system are rare in case of quite big boundaries of indefinitely changeable parameters.

In the process of study of this problem, we have searched an approach which we have considered as an original and we have been interested in it. Approach has been proposed by Victor Ten and it refers to projection of robust control system [1]. The method is based on results of catastrophe theory, usage of structurally stable functions, which gives an opportunity to make projected nonlinear system stable in quite big boundaries of variable parameters of dynamic objects in the favor of bifurcation of balance points.

It is known that catastrophe theory uses several functions characterized with stable structure. For today there is much kind of such structures including seven nonlinearities given by Rene Thom which are named “Thom's 7 elementary catastrophes”. They are: fold, cusp, swallowtail, butterfly, hyperbolic umbilic, elliptic umbilic, parabolic umbilic [2].

2. Research of second-round system with catastrophe theory

Let`s consider general second-round system in which we choose such meanings of parameters as it will be unstable.

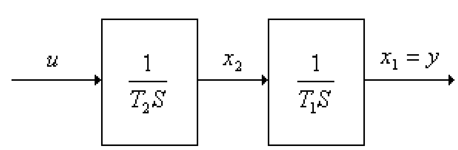

Fig.1. General second-round system

Select feedback control law in a form of elliptic umbilic [1]:

$$u = -x_2^3 + 3x_2x_1^2 - k_1\left(x_1^2 + x_2^2\right) + k_2x_2 + k_3x_1 \qquad (1)$$

System offered by controller may be represented in such a way:

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{1}{T_1}x_2 \\ \dfrac{dx_2}{dt} = \dfrac{1}{T_2}\left(-x_2^3 + 3x_2x_1^2 - k_1\left(x_1^2 + x_2^2\right) + k_2x_2 + k_3x_1\right) \end{cases} \qquad (2)$$

$$y = x_1 .$$

System has the following balance points:

$$x_{1s}^1 = 0,\ x_{2s}^1 = 0; \qquad (3)$$

$$x_{1s}^2 = \frac{k_3}{k_1},\ x_{2s}^2 = 0 \qquad (4)$$

And appropriate stability conditions:

$$\begin{cases} -\dfrac{k_2}{T_2} > 0, \\ \dfrac{k_3}{T_1T_2} < 0, \end{cases} \qquad (5)$$

$$\begin{cases} -\dfrac{3k_3^2 + k_2k_1^2}{k_1^2T_2} > 0, \\ \dfrac{k_3}{T_1T_2} > 0. \end{cases} \qquad (6)$$

(3) and (4) points are alternatives.

Let us assume that T_1 parameter may be changed but will be remained positively. If both k_2 and k_3 is negative and $|k_2| < 3\dfrac{k_3^2}{k_1^2}$, then meaning of T_2 parameter is unessential. It can obtain any meaning, both positive and negative. In the moment of changing balance point (3) becomes unstable (disappeared). The same is the condition when balance point (3) is stable, but (4) becomes unstable (disappeared).

Results of modeling in Matlab computer system are presented accordingly on image 2. We can see that phase paths strive for (0,0) and $\left(\dfrac{k_3}{k_1},0\right)$ balance points.

On image 2 phase portrait is constructed for the following meanings of parameters: $k_1 = 1;\ k_2 = -5;\ k_3 = -2;\ T_1 = 100$ and for variable parameters $T_2 = \mathrm{var}$ (from -4500 to 4500 pitch 1000) with initial condition $x = (-1,0)$.

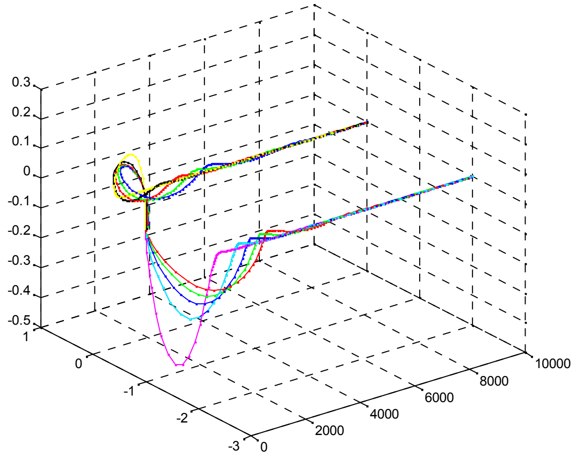

Fig.2. Phase portrait is built $k_1 = 1;\ k_2 = -5;\ k_3 = -2;\ T_1 = 100$

We have built dynamic characteristics for visualization of current processes. They are shown on Figure 3:

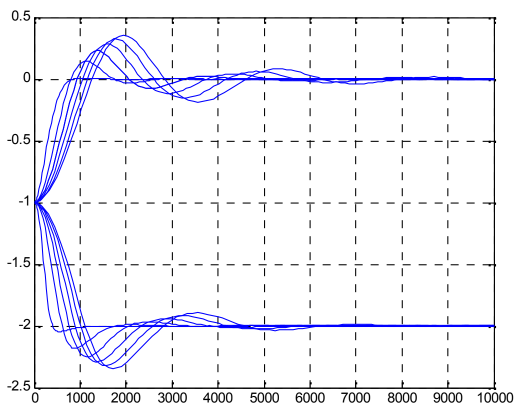

Fig.3. Dynamic characteristics of system

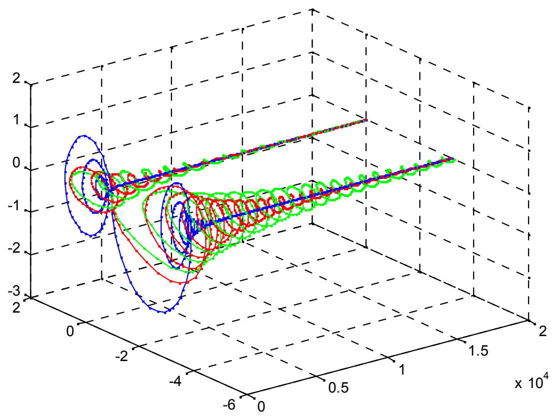

Fig.4. Phase portrait built at the time of inclusion of hyperbolic umbilic

We can easily see existence of two balance points on the picture.

We have used other function of controller from the list of Thom's elementary catastrophes. Namely, hyperbolic umbilic: $u = x_2^3 + x_1^3 + k_1 x_2 x_1 - k_2 x_2 + k_3 x_1$.

We have selected coefficients, made modeling and obtained the following image (Fig.4).

From this result we can also see existence of two balance points.

We have draw up a model Simulink and carried out modeling. At the time of one of the experiments, catastrophe has been fixed on oscilloscope (Figure 5).

Based on above described method, we have considered a real example: problem of pitch control.

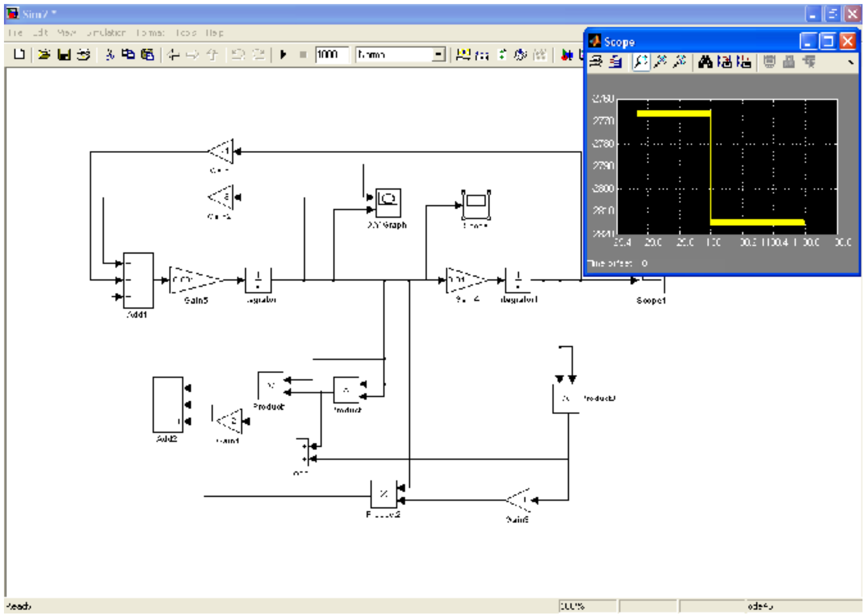

Fig.5. Modeling scheme in Simulink system

3. Research of pitch dynamic with catastrophe theory

Motion of aircraft is united process, although these difficult motions are divided into simple types (pitch, list, motion of center of mass, vertical motion and etc.). In frequent cases it is sufficient to confine to longitudinal and diametrical motions.

Let us consider dynamic of pitch. Generally it has a quite difficult structure and is described with systems of nonlinear differential equation of higher order, but we can pick up dynamic subsystem, variables and parameters of which characterize angles and their connections by dependence with flight control.

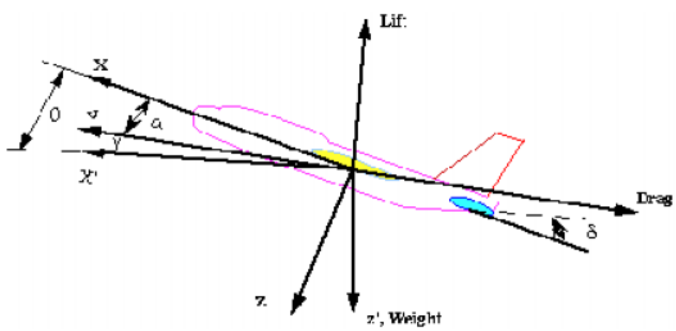

Fig.6. Airplane motion angles

Dynamic of isolated angular motion of airplane is described with the following differential equations:

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Where matrix A, B and C have the following (nominal) meanings:

$$A = \begin{pmatrix} a_y^a & 0 & -a_y^a \\ a_{m_z}^a & -a_{m_z}^{\omega_z} & -a_{m_z}^a \\ 0 & 1 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ a_{m_z}^{\delta_a} \\ 0 \end{pmatrix}, \; C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}, \tag{7}$$

With nominal parameters:

$a_y^a = -2.1$ [s-1], $a_{m_z}^a = 29.4$ [s-2], $a_{m_z}^{\omega_z} = 2.18$ [s-1], $a_{m_z}^{\delta_a} = 60.7$ [s-2], u=$\delta_a(t)$.

If we get incoming signal and learn (7) system dynamic stability, then we will see that it is situated on an edge of stability and is unused for practice. Let us select control law with the following form:

$$u = -\frac{1}{b_2}\left(k_1\left(x_3^2 + x_2^2\right) - k_2 x_3 - k_3 x_2\right). \tag{8}$$

Therefore, (7) system in addition to (8) will receive the appearance:

$$\begin{cases} \dfrac{dx_1}{dt} = a_y^a x_1 - a_y^a x_3 \\ \dfrac{dx_2}{dt} = a_{m_z}^a x_1 - a_{m_z}^{\omega_z} x_2 - a_{m_z}^a x_3 - k_1\left(x_3^2 + x_2^2\right) + k_2 x_3 + k_3 x_2 \\ \dfrac{dx_3}{dt} = x_2 \\ y = x_3 \end{cases}, \tag{9}$$

New nonlinear control system (9) has two balance points:

$$x_1 = 0;\; x_2 = 0;\; x_3 = 0; \tag{10}$$

$$x_1 = x_3 = \frac{k_2}{k_1},\; x_2 = 0 \tag{11}$$

(10) Stability provision of balance point is:

$$\begin{cases} a_{m_z}^{\omega_z} - k_3 - a_y^\alpha > 0, \\ \left(a_{m_z}^{\omega_z} - k_3 - a_y^\alpha\right)\left(a_y^\alpha\left(k_3 - a_{m_z}^{\omega_z}\right) - k_2 + a_{m_z}^\alpha\right) - k_2 a_y^\alpha > 0, \\ k_2 a_y^\alpha > 0. \end{cases} \tag{12}$$

(11) Stability provision of balance point is:

$$\begin{cases} a_{m_z}^{\omega_z} + k_3 - a_y^\alpha > 0, \\ \left(a_{m_z}^{\omega_z} + k_3 - a_y^\alpha\right)\left(a_y^\alpha\left(k_3 - a_{m_z}^{\omega_z}\right) + k_2 + a_{m_z}^\alpha\right) + k_2 a_y^\alpha > 0, \\ -k_2 a_y^\alpha > 0. \end{cases}$$

As we see, last inequalities of both provisions are confrontational. If parameter satisfies one of these provisions, then the system strives to appropriate balance point. If some time later parameter gets satisfactory meaning of second provision, then current balance point becomes unstable and it is disappeared. Then the other stable balance point is created. Therefore, for (9) system stability it is not important which meaning will be received by parameter, except zero, in any case (9) system will be stable.

References:

1. Ten V. Approach to Design of Robust Control Systems by Structurally Stable Functions From Catastrophe Theory. Applied Mathematical Sciences, Vol.3, no.34, pg. 1655-1674.
2. Арнольд В.И. Теория катастроф. –М.: Знание, 1981. -61с.

მართვის სისტემის მოდელირება თვითმფრინავის ფრენის რეჟიმებისათვის

ნინო მჭედლიშვილი, ია მოსაშვილი, დავით ყიფშიძე

საქართველოს ტექნიკური უნივერსიტეტი

რეზიუმე

თვითმფრინავის მართვა ზოგადად წარმოადგენს რთულ აეროდინამიკურ ამოცანას. მიზნის მიღწევა, ეფექტური და უსაფრთხო ფრენა შეუძლებელია მართვის მაღალტექნოლოგიური ავტომატური სისტემების გარეშე. ნაშრომში შესწავლილია თვითმფრინავის ფრენისას თავისუფალი მობრუნება სამი ღერძის ირგვლივ, დასმულია თვითმფრინავის ტანგაჟის მართვის სისტემის სინთეზის ამოცანა, რომელიც ამოხსნილია პროგრამული უზრუნველყოფა Matlab-ის საშუალებით თანამედროვე სინერგეტიკული მეთოდების, კერძოდ, კატასტროფების თეორიის გამოყენებით.

МОДЕЛИРОВАНИЕ СИСТЕМЫ УПРАВЛЕНИЯ РЕЖИМОВ ПОЛЕТА САМОЛЕТА

Мчедлишвили Н., Мосашвили Й., Киршидзе Д.

Грузинский Технический Университет

Резюме

Управление самолетом в общем представляет собой сложную аэродинамическую задачу. Достижение цели, эффективный и безопасный полет невозможен без высокотехнологических автоматических систем управления. В работе изучены свободные повороты вокруг трех осей при полете самолета, поставлена задача синтеза системы управления по тангажу, которая решена с использованием программного обеспечения Matlab и современных методов синергетики, в частности, теории катастроф.