PROPER OSCILLATIONS OF A DIELECTRIC CYLINDER ARISING DUE TO AN AXIAL ALTERNATING CURRENT

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Summary

In presented article the proper oscillations of a dielectric cylinder are considered, when they arise due to existence of an axial alternating current, streaming through the metal bar displaced nearby. The current emits the symmetrical cylindrical wave in (I) area occupied by the cylinder; experiencing the partial reflection from the wall (the border) of the dielectric, and partly penetrates through it into free space (II). As a result of the multiple reflections from the interface of the dielectric cylinder and of free space, in (I) area the regime of standing waves is stated with the discrete spectra of frequencies. In order to determine the transverse wave number, the transcendental dispersive equation is received, while its solution is obtained in an analytical form for big values of the relative dielectric permittivity of the cylinder. Received results may be used for calculation of dielectric permittivity of ferroelectrics (so called variconds).

Keywords: current, oscillations, dielectric, cylinder, frequency.

1. Introduction

Orientation of given cylinder in rectangular and cylindrical coordinate systems is presented in figure 1. Further will be used the designations, listed below:

$a, \varepsilon_r -$ the radius and the relative dielectric permittivity of the cylinder, respectively, $M' -$ observation point with cylindrical coordinates r, φ, z, $QQ' -$ the conductor with a harmonic current $J = J_0 \sin \omega t$, J_0 being the amplitude of the current, $\omega -$ its circular frequency, $r_0 -$ a radius of the current conductor. The conductor points axial-parallel to the axis of the cylinder. The origin is selected in the middle of the conductor (in point 0).

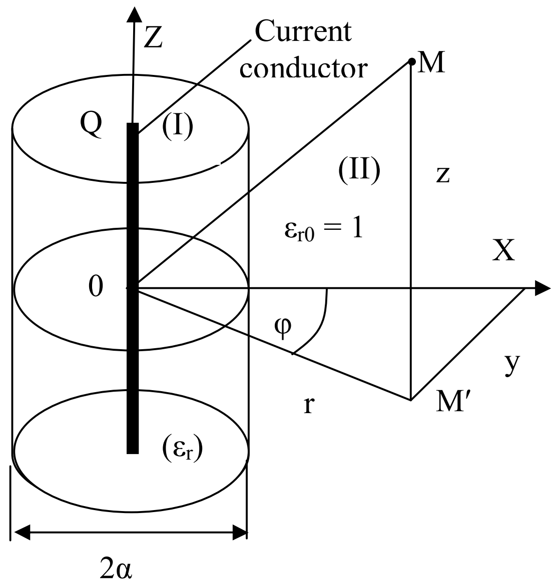

Figure 1.

An electrodynamic process, which we shall investigate further, consists in following: the alternating current streaming through the bar radiates the symmetrical cylindrical wave in (I) area occupied by the cylinder; the wave experiences the partial reflection from the wall (the border) of the dielectric, and partly penetrates through it into free space (II), with the dielectric permittivity $\varepsilon_{r0} = 1$. Resulting the multiple reflections from the interface of the dielectric cylinder and free space, in (I) area the regime of standing waves is stated with the discrete spectra of frequencies f_n $(n = 0,1,2,\ldots)$. Determination of these frequencies as the functions of parameters $a, \varepsilon_r, \lambda$, here λ being the wavelength in vacuum, is our priority problem.

2. RESULTS AND DISCUSSION

2.1. Field Structure in Separate Areas

As to the conductor uniformly radiates the symmetrical electromagnetic wave with the cylindrical front, in (I) area the vertical component of the electric field strength E_{z1} may be presented as superposition of two cylindrical traveling waves, moving in opposite directions relative to each other

$$E_{z1} = AH_0^{(2)}(kr) + BH_0^{(1)}(kr), \; (r_0 \le r \le a) \tag{1}$$

(time dependence is taken as ~ $e^{i\omega t}$), where $A = {k'^2 J_0}/{4\pi\varepsilon}$, $\varepsilon = \varepsilon_{0r}\varepsilon_r$, $k' = k\sqrt{\varepsilon_r}$, $k = {2\pi}/{\lambda}$, B is unknown coefficient, $H_0^{(1)}(kr)$ and $H_0^{(2)}(kr)$ are Hankel's functions of first and second order, respectively, r_0 – the radius of the conductor.

In (II) area the field is given as the cylindrical traveling wave flowing to infinity, thus the component of the electric field strength E_{z2} may be presented as follows:

$$E_{z2} = CH_0^{(2)}(kr), \ (r \geq a) \tag{2}$$

where C is unknown coefficient.

Magnetic components of the fields in (I) and (II) areas should be determined from Maxwell's equations, as a result we get:

$$H_{\varphi 1} = -\frac{k'}{i\omega\mu_0}\left[H_1^{(2)}(k'r)A + H_1^{(1)}(k'r)B\right], \ (r_0 \leq r \leq a) \tag{3}$$

$$H_{\varphi 2} = -\frac{k}{i\omega\mu_0}CH_1^{(1)}(kr), \ (r \geq a) \tag{4}$$

$\mu_0 = 7 \cdot 10^{-7}$ Hn/m, ω – circular frequency.

Unknown coefficients B and C should be found from the border conditions:

$$\left.\begin{aligned} E_{z1} &= E_{z2}, \\ H_{\varphi 1} &= H_{\varphi 2}, \end{aligned}\right\} \text{at r} = \alpha$$

easily reduced to the system

$$\left.\begin{aligned} BH_0^{(1)}(ka) - CH_0^{(2)}(ka) &= -AH_0^{(2)}(k'a), \\ \sqrt{\varepsilon_r}BH_1^{(1)}(k'a) - CH_1^{(2)}(ka) &= -\sqrt{\varepsilon_r}AH_1^{(2)}(k'a), \end{aligned}\right\} \tag{5}$$

and its solution is given as follows:

$$B = A\frac{H_0^{(2)}(k'a)H_1^{(2)}(ka) - \sqrt{\varepsilon_r}H_1^{(2)}(k'a)H_0^{(2)}(ka)}{\sqrt{\varepsilon_r}H_0^{(2)}(ka)H_1^{(1)}(k'a) - H_0^{(1)}(k'a)H_1^{(2)}(ka)}, \tag{6}$$

$$C = \sqrt{\varepsilon_r}A\frac{H_0^{(2)}(k'a)H_1^{(1)}(k'a) - H_0^{(1)}(k'a)H_1^{(2)}(k'a)}{\sqrt{\varepsilon_r}H_1^{(1)}(k'a)H_0^{(2)}(k'a) - H_0^{(1)}(k'a)H_1^{(2)}(ka)}. \tag{7}$$

At the absent of the cylinder, i.e. when k′ = k, the evident equalities B = 0 and C = A should take place. Actually, from (6) and (7) directly follows that $\lim_{k' \to k} B = 0$, $\lim_{k' \to k} C = A$.

2.2. Dispersive Equation of the Problem

Received solutions of (5) system should have a single meaning. To reach it, it is necessary the field inside the cylinder to satisfy one more border condition

$$E_{z1} = 0 \text{ at } r = r_0,$$

i.e.

$$AH_0^{(2)}(k'r_0)+BH_0^{(1)}(k'r_0)=0\,. \tag{8}$$

Inserting here B value from (6), we get

$$A\left[H_0^{(2)}(k'r_0)+F(k'a,ka)H_0^{(1)}(k'r_0)\right]=0\,, \tag{9}$$

where the following designation is implemented:

$$F(k'a,ka)=\frac{H_0^{(2)}(k'a)H_1^{(2)}(ka)-\sqrt{\varepsilon_r}\,H_1^{(2)}(k'a)H_0^{(2)}(ka)}{\sqrt{\varepsilon_r}\,H_0^{(2)}(ka)H_1^{(1)}(k'a)-H_0^{(1)}(k'a)H_1^{(2)}(ka)}\,. \tag{10}$$

Considering the case, when

$$ka\geq 1{,}5\,, \tag{11}$$

it is possible to operate by approximate formulas [2]

$$H_0^{(2)}(x)\simeq\sqrt{2/\pi x}\cdot e^{-i\left(x-\pi/4\right)},\ H_0^{(1)}(x)\simeq\sqrt{2/\pi x}\cdot e^{i\left(x-\pi/4\right)},$$

$$H_1^{(2)}(x)\simeq\sqrt{2/\pi x}\cdot e^{-i\left(x-3\pi/4\right)},\ H_1^{(1)}(x)\simeq\sqrt{2/\pi x}\cdot e^{i\left(x-3\pi/4\right)},$$

and taking in mind as well, that $e^{-i\left(x-3\pi/4\right)}=ie^{-i\left(x-\pi/4\right)}$, $e^{i\left(x-3\pi/4\right)}=-ie^{i\left(x-\pi/4\right)}$, where $x=k'a$ or $x=ka$, instead of (10) we get

$$F(k'a,ka)=-i\frac{\sqrt{\varepsilon_r}-1}{\sqrt{\varepsilon_r}+1}e^{-2i(x'-x)},$$

or

$$F(k'a,ka)=\frac{\sqrt{\varepsilon_r}-1}{\sqrt{\varepsilon_r}+1}e^{-2i\left[ka\left(\sqrt{\varepsilon_r}-1\right)+\pi/2\right]}. \tag{12}$$

Inserting this expression into (9), we arrive to the transcendental equation

$$H_0^{(2)}\left(ka\sqrt{\varepsilon_r}\xi\right)+R\cdot e^{-i\beta(ka)}H_0^{(1)}\left(ka\sqrt{\varepsilon_r}\xi\right)=0\,, \tag{13}$$

where, for brevity, the following designation are involved:

$$\xi=r_0/a\,,\ R=\frac{\sqrt{\varepsilon_r}-1}{\sqrt{\varepsilon_r}+1},\ \beta(ka)=2\left[ka\left(\sqrt{\varepsilon_r}-1\right)+\pi/2\right]. \tag{14}$$

(13) is the dispersive equation of our problem, which serves for determination of kα parameter.

Involving $\eta=ka\sqrt{\varepsilon_r}\xi$ designation, overwrite (13) as follows:

$$J_0(\eta)-iN_0(\eta)+R\left[J_0(\eta)+iN_0(\eta)\right](\cos\beta-i\sin\beta)=0\,,$$

which decomposes into two equations:

$$J_0(\eta)(1+R\cos\beta)=-RN_0(\eta)\sin\beta\,,\ N_0(\eta)(1-R\cos\beta)=-RJ_0(\eta)\sin\beta\,.$$

Dividing them on each other term by term, we get:

$$\frac{J_0^2(\eta)}{N_0^2(\eta)}=\frac{1-R\sin\beta}{1+R\sin\beta},$$

that is just the same, that

$$\frac{J_0(\eta)}{N_0(\eta)}=\sqrt{\frac{1-R\sin\beta}{1+R\sin\beta}}.$$

Returning back to the initial designations, we get

$$\frac{J_0\left(ka\sqrt{\varepsilon_r}\xi\right)}{N_0\left(ka\sqrt{\varepsilon_r}\xi\right)}=\sqrt{\frac{1+R\sin\left[2ka\left(\sqrt{\varepsilon_r}-1\right)\right]}{1-R\sin\left[2ka\left(\sqrt{\varepsilon_r}-1\right)\right]}}. \tag{15}$$

This is transcendental equation relative to kα, which may be solved, in general, only graphically; however, at big ε_r, for example, in case of ferroelectrics (variconds) [2], the approximate solution may be obtained in an analytical form.

Indeed, at $ka\sqrt{\varepsilon_r}\xi \geq 2$, it is possible to use the approximate relations [1] $(R \simeq 1)$

$$J_0\left(ka\sqrt{\varepsilon_r}\xi\right)\simeq\sqrt{\frac{2}{\pi ka\sqrt{\varepsilon_r}\xi}}\cos\left(ka\sqrt{\varepsilon_r}\xi-\pi/4\right),$$

$$N_0\left(ka\sqrt{\varepsilon_r}\xi\right)\simeq\sqrt{\frac{2}{\pi ka\sqrt{\varepsilon_r}\xi}}\sin\left(ka\sqrt{\varepsilon_r}\xi-\pi/4\right),$$

that is why, instead of (15) should be $\dfrac{J_0(\eta)}{N_0(\eta)}=\sqrt{\dfrac{1-R\cos\beta}{1+R\cos\beta}}$.

In general case, for arbitrary taken ε_r $(1\leq\varepsilon_r\leq a)$ this equation should be solved graphically. Denoting the desired value by x = kα and taking in mind that

$\eta = x\sqrt{\varepsilon_r}\xi$, $R=\dfrac{\sqrt{\varepsilon_r}-1}{\sqrt{\varepsilon_r}+1}$ and $\beta = 2x\left(\sqrt{\varepsilon_r}-1\right)+\pi/2$, instead of (15) we get

$$\frac{J_0\left(x\sqrt{\varepsilon_r}\xi\right)}{N_0\left(x\sqrt{\varepsilon_r}\xi\right)}=\sqrt{\frac{1-R\cos\left[2x\left(\sqrt{\varepsilon_r}-1\right)+\pi/2\right]}{1+R\cos\left[2x\left(\sqrt{\varepsilon_r}-1\right)+\pi/2\right]}}. \tag{16}$$

In private case, at $\sqrt{\varepsilon_r} >> 1$ and for moderate ξ, this equation transforms into

$$tg\left(x\sqrt{\varepsilon_r}\xi\right)=\sqrt{\frac{1-\cos\left[2x\left(\sqrt{\varepsilon_r}-1\right)+\pi/2\right]}{1+\cos\left[2x\left(\sqrt{\varepsilon_r}-1\right)+\pi/2\right]}}, \text{ or}$$

$$tg\left(x\sqrt{\varepsilon_r}\xi\right)=tg\left[x\left(\sqrt{\varepsilon_r}-1\right)+\pi/2\right].$$

This equation possesses infinite multiple of solutions

$$x_n=\frac{2n\pi-\pi/4}{\sqrt{\varepsilon_r}(1-\xi)-1}.\ (n=1,2,3,\ldots) \tag{17}$$

Taking in mind that $x_n = k_n a$, finally we get:

$$k_n = \frac{2n\pi - \pi/4}{a\left[\sqrt{\varepsilon_r}(1-\xi)-1\right]}. \quad (n = 1,2,3,\ldots) \tag{18}$$

Transverse wave numbers k_n are related to proper frequencies f_n of electromagnetic oscillations in dielectric as follows:

$$k_n = \frac{2\pi}{c} f_n, \tag{19}$$

c – being the speed of light in vacuum, and from (18) we receive

$$f_n = \frac{\left(2n - 1/4\right)c}{a\left[\sqrt{\varepsilon_r}(1-\xi)-1\right]}. \quad (n = 1,2,3,\ldots) \tag{20}$$

For the basic frequency (mode) at n = 1, we get

$$f_1 = \frac{7}{8} \cdot \frac{c}{a\left[\sqrt{\varepsilon_r}(1-\xi)-1\right]}. \tag{21}$$

3. Conclusions

The formula (21) should be used in the case of dielectrics with big ε_r, the latter being ferroelectrics (variconds) [2].

In figure 2 at $\xi << 1$ the curves of dependences $f_1 = f_1(\varepsilon_r)$ for $a = 3$ and 6 sm are presented. A_j and A_j' points $(j = 1,2,3,4)$ correspond to the values of proper frequencies of the variconds of following types: $(A_1) - BK-6, f_1^{(1)} = 1.02GHz$,

$(A_2) - BK-2, f_1^{(2)} = 0.6GHz$, $(A_3) - ceramics, BaTiO_3, f_1^{(3)} = 0.57GHz$,
$(A_4) - BK-1, f_1^{(4)} = 0.47GHz$, $(A_1') - BK-6, f_1^{(1)}{}' = 0.46GHz$,
$(A_2') - BK-2, f_1^{(2)}{}' = 0.26GHz$, $(A_3') - ceramics, BaTiO_3, f_1^{(3)}{}' = 0.25GHz$,
$(A_4') - BK-1, f_1^{(4)}{}' = 0.24GHz$.

(see in [2] the table on page 48)

Finally notice that from (21) formula follows

$$\sqrt{\varepsilon_r} = 1 + 0{,}875 \frac{c}{af_1}. \tag{22}$$

From it, at given f_1, it is possible to determine the numerous value of the dielectric permittivity ε_r, while for each definite case f_1 should be determined experimentally.

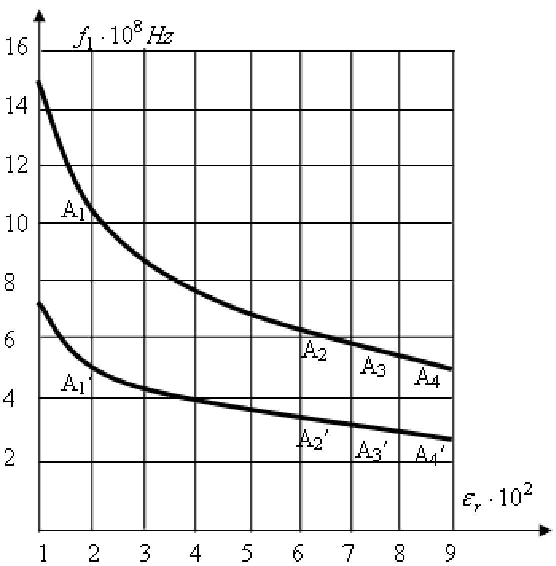

Figure 2. Dependence of basic frequency on ε_r.

R E F E R E N C E S

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СОБСТВЕННЫЕ КОЛЕБАНИЯ ДИЭЛЕКТРИЧЕСКОГО ЦИЛИНДРА, ВОЗНИКАЮЩИЕ БЛАГОДАРЯ АКСИАЛЬНОМУ ПЕРЕМЕННОМУ ТОКУ

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Резюме

В статье рассмотрены собственные колебания диэлектрического цилиндра, возникающие благодаря присутствию аксиального переменного тока, текущего в расположенном неподалеку металлическом стержне. Ток излучает симметричную цилиндрическую волну в занятую цилиндром область (I). При этом волна частично отражается от стенки (границы) диэлектрика, частично же проникает в свободное пространство (II). В результате многочисленных отражений от поверхности диэлектрического цилиндра и свободного пространства, в области (I) устанавливается режим стоячих волн с дискретным спектром частот. Для определения поперечного волнового числа выведено трансцендентное дисперсионное уравнение, решение которого получено в аналитическом виде для больших значений относительной диэлектрической проницаемости цилиндра. Полученные результаты могут быть использованы для вычисления диэлектрических проницаемостей сегнетоэлектриков (варикондов).

დიელექტრიკული ცილინდრის საკუთარი რხევები, აღძრული აქსიალური ცვლადი დენით

ი. ქევანიშვილი, ლ. კოკილაშვილი, ვ. მელაძე,
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რეზიუმე

სტატიაში განხილულია დიელექტრიკული ცილინდრის საკუთარი რხევები, აღძრული აქსიალური ცვლადი დენით, რომელიც სიახლოვეს განლაგებულ ლითონის ღეროში მიედინება. დენი ასხივებს სიმეტრიულ ცილინდრულ ტალღას ცილინდრის მიერ დაკავებულ (I) არეში. ამასთან ტალღა ნაწილობრივ ირეკლება დიელექტრიკის კედლიდან (საზღვრიდან), ნაწილობრივ კი აღწევს თავისუფალ სივრცეში (II). დიელექტრიკული ცილინდრის ზედაპირიდან და თავისუფალი სივრციდან მრავალჯერადი არეკვლის შედეგად (I) არეში მყარდება მდგარი ტალღების რეჟიმი სიხშირეთა დისკრეტული სპექტრით. განივი ტალღური რიცხვის განსაზღვრისთვის გამოყვანილია ტრანსცენდენტული დისპერსიული განტოლება, რომლის ამონახსნი მიიღება ანალიზური სახით ცილინდრის დიელექტრიკული შეღწევადობის დიდი მნიშვნელობებისთვის. მიღებული შედეგები შეიძლება გამოყენებულ იქნას სეგნეტოელექტრიკების (ვარიკონდების) დიელექტრიკული შეღწევადობის გამოსათვლელად.